

# ROBUST INTERVAL-BASED SISO AND SIMO REGULATION FOR A CLASS OF HYGLY UNCERTAIN BIOREACTORS: APPLICATION TO THE ANAEROBIC DIGESTION

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**Abstract:** A robust set-valued regulation law is proposed for the stabilization of an anaerobic digester for the treatment of wastewater. This process exhibits a highly nonlinear dynamic behavior. In addition, it must operate in an uncertain environment with the presence of unknown inputs. Supporting some structural and operational conditions, this regulation law exponentially stabilizes the two most important variables in the anaerobic digester: the Chemical Oxygen Demand concentration and the Volatile Fatty Acids concentration around a desirable steady state in the presence of kinetic uncertainties and input disturbances. Simulations are carried out handling operational conditions close to those used in a real plant.

**Keywords:** Robust nonlinear regulation, interval observers, wastewater treatment processes.

## I – Introduction

The robust control of bioreactors is an emerging domain in which a number of practical difficulties limit the development of efficient control algorithms. This is particularly true in the field of Wastewater Treatment Plants (WWTP) which have the following properties: the presence of important nonlinearities and consequently the impossibility to proceed to a linearization around a nominal point, the lack of reliable measurements and the uncertainties on the model and on exogenous inputs. Because of this lack of knowledge and since very few statistics are available, the only reasonable assumption is to consider all the uncertainties as unknown inputs in  $L_\infty$  with known bounds. However, because the observability of nonlinear systems cannot be established independently of the exogenous inputs, WWTP models are usually not observable. To overcome this difficulty, techniques of interval observers (see [RAPAPORT, 1998], [HADJ-SADOK and GOUZÉ, 1998<sup>a</sup>, 1998<sup>b</sup>, 1999], [HADJ-SADOK *et al.*, 1998] and [ALCARAZ-GONZÁLEZ *et al.*, 1999, 2000<sup>b</sup>]), have been proposed to compute dynamically guaranteed bounds on the unmeasured states instead of reconstructing their precise numerical values. These robust observers have then been used in robust control strategies for regulating a single output of a class of biological reactors (see [RAPAPORT and HARMAND, 1998<sup>a</sup>, 1998<sup>b</sup>], [ALCARAZ-GONZÁLEZ *et al.*, 2000<sup>c</sup>]). The proposed controller, which uses these dynamical bounds, guarantees that a quadratic function of the output is a robust Lyapunov function. This approach has been recently extended to solve the Single-Input-Multi-Output stabilization problem using a modified Lyapunov function [MALOUM and RAPAPORT, 2000]:

$$V(y) = e^{\left| \sum_{i=1}^l g_i (y_i - y_i^r) \right| + \sum_{i=1}^l d_i (y_i - y_i^r)} - 1 \quad (1)$$

where the choice of the parameters  $g$  and  $d$  bring more flexibility than for the quadratic case (their signs can be chosen depending on the operation mode of the bioreactor: consumption or production).

This paper presents the application of this new SIMO control strategy to the control of an anaerobic digestion pilot plant for treating wine vinasses. The paper is organized as follows. First, the general nonlinear model considered in this study is described.

Second, a dynamical nonlinear interval observer is introduced assuming that the nonlinearities are partially known and input disturbances as well as initial conditions are unknown but that they belong to a prescribed bounded set. Then, the partial information provided by this interval observer is used to synthesize a robust regulation law that exponentially stabilizes  $l$  regulated variables around a desirable steady state. Simulation results using operational conditions close to those used in a real anaerobic digestion plant are provided before some conclusions and perspectives are drawn.

## II - The considered model

The following general nonlinear time-varying lumped model is considered:

$$\dot{x}(t) = CT(t)f(x(t), t) + A(D(t), t)x(t) + b(t) \quad (2)$$

where  $x(t) \in \mathfrak{R}_+^n$  is the state vector,  $f(x(t), t) \in \mathfrak{R}^r$  denotes the vector of non negative reaction kinetics,  $C \in \mathfrak{R}^{n \times r}$  and  $T(t) \in \mathfrak{R}^{r \times r}$  represents matrices of coefficients (e.g. stoichiometric, yield or kinetic coefficients) where  $C$  is constant and  $T(t)$  is non negative and possibly time varying. The time varying matrix  $A(t) \in \mathfrak{R}^{n \times n}$  explicits the linear dependency between the state variables where  $D(t) \geq 0$  is the manipulated variable (the dilution rate) while  $b(t) \in \mathfrak{R}^n$  belongs to a vector gathering the inputs (e.g. mass feeding rate vector) and/or other possibly time varying functions (e.g., the gaseous outflow rate vector if any). The following hypotheses are introduced:

### Hypotheses H1 :

- $A(t)$  is known for each  $t \geq 0$ .
- $m$  states are measured on-line and  $l \leq m$  of them are the states that to be regulated.
- $C$  is known.
- $A(t)$  is bounded, i.e., there exist matrices  $\underline{A}$  and  $\bar{A}$  such as  $\underline{A} \leq A(t) \leq \bar{A}$ .
- Guaranteed bounds on the initial conditions of the state vector are known with  $\underline{x}_1(0) \leq x_1(0) \leq \bar{x}_1(0)$ .
- Guaranteed bounds on the unknown inputs are given as  $\underline{b}^- \leq b(t) \leq \bar{b}(t) \leq \bar{b}^+$ .
- Guaranteed bounds on the unknown matrix  $T(t)$  are given as  $\underline{T} \leq T(t) \leq \bar{T}$  (e.g. uncertainties on the kinetic parameters).

### Notes:

- The constant signs of  $C$  mean that the reactions are admitted to be irreversible.
- The sign of the terms of the matrix  $C$  indicates if the species  $i$  is consumed ( $(C_{ij}) < 0$ ), produced ( $(C_{ij}) > 0$ ) or unaffected ( $(C_{ij}) = 0$ ) by the  $j^{\text{th}}$  reaction.
- The operator  $\leq$  applied between vectors and between matrices should be understood as a collection of inequalities between elements.

### Assumptions A1:

- A reaction  $j$  stops (i.e.,  $f_j(x(t), t) = 0$ ) when at least one of the variables  $x_j(t)$  involved in  $f_j$  is equal to zero.



- $mode(i) = c$ :  $H_i C_2^Y < 0$  and  $(\underline{y}_i^{in-} - y_i^r) > 0$   
(consumption of  $y_i$ )
- $mode(i) = p$ :  $H_i C_2^Y > 0$  and  $(\underline{y}_i^{in-} - y_i^r) < 0$   
(production of  $y_i$ )
- $(\hat{y}_i^{in}, \tilde{y}_i^{in}) = \begin{cases} (\underline{y}_i^{in-}, \bar{y}_i^{in+}) & \text{if } mod\ e(i) = c \\ (\bar{y}_i^{in+}, \underline{y}_i^{in-}) & \text{if } mod\ e(i) = p \end{cases}$
- $p_j$  is the number of species involved in the reaction  $j$  and  $p = \sum_j p_j$ .
- $\mathbf{j} = K \max_i \sum_j \max(0, -C_{ij})$  with  $K > 0$ .
- $\mathbf{h}(\mathcal{Q}) = \min_{x \in \mathcal{Q}} \min_i x_i$ , where  $\mathcal{Q}$  is a given subset of  $\mathbf{W}$ .
- $\underline{V}_i$  and  $\bar{V}_i$  are the smallest and largest absolute values respectively of the components of the vector  $H_i C_2^Y$ ,  $\underline{V} = \min_{i=1..l} \underline{V}_i$  and  $\bar{V} = \max_i \bar{V}_i$

Now the following hypotheses are introduced:

**Hypotheses H4:**

- a)  $T(t)f(x(t), t)$  is  $K$ -Lipschitz continuous ( $K > 0$ ) on  $\mathbf{W}$  and there exists a positive number  $\mathbf{b}$  such that:  $T(t)f_j(x(t), t) \geq \mathbf{b} \prod_{i=0}^{p_j} x_i$
- b)  $T(t)f(x_1(t), x_2(t), t)$  is monotonic with respect to each component of  $x_1(t)$ .
- c)  $A_{21}^Y = 0$  and  $A_{22}^Y(t) = -D(t)Y$
- d)  $q_i(t) = D(t)y_i^{in}(t) \quad \forall i = 1 \dots l$

Now, for the design of a robust exponential output stabilization law it is necessary to choose some parameters  $\mathbf{g}$ ,  $\mathbf{d}_i$ ,  $\bar{D}$ ,  $I^+$  and  $I^-$  as follows:

$$\begin{aligned} \mathbf{g}_i > \mathbf{d}_i > 0 & \quad \text{if } mod\ e(i) = c \\ \mathbf{g}_i < \mathbf{d}_i < 0 & \quad \text{if } mod\ e(i) = p \end{aligned} \quad (7)$$

with  $\mathbf{d}_i$  sufficiently small to ensure :

$$\mathbf{D} \equiv \sum_{i=1}^l \mathbf{g}_i (\hat{y}_i^{in} - y_i^r) - \max_{\substack{(x_1, y) \in \mathbf{W} \\ \underline{y}^{in-} \leq y_i^r \leq \bar{y}^{in+}}} \sum_{i=1}^l |\mathbf{d}_i| (y_i^{in} - y_i) > 0 \quad (8)$$

$$\bar{D} > \frac{Gr\bar{V} \max_{x \in \mathbf{W}} \|\bar{f}(x(t), t)\|_\infty}{\mathbf{D}} \quad \text{with } \mathbf{G} = \left( \sum_{i=1}^l |\mathbf{g}_i| + |\mathbf{d}_i| \right) \quad (9)$$

$$0 < I^- \leq \mathbf{D}\bar{D} - Gr\bar{V} \max_{x \in \mathbf{W}} \|\bar{f}(x(t), t)\|_\infty \quad (10)$$

$$I^+ > \max \left\{ \bar{\mathbf{J}} \frac{\sum_{i=1}^l (|\mathbf{g}_i| + |\mathbf{d}_i|) |\bar{y}_i^{in} - y_i^r|}{\sum_{i=1}^l (|\mathbf{g}_i| - |\mathbf{d}_i|) |\bar{y}_i^{in} - y_i^r|}, \bar{\mathbf{J}} + \frac{\sum_{i=1}^l (|\mathbf{g}_i| + |\mathbf{d}_i|) |\bar{y}_i^{in} - \bar{y}_i^r|}{\mathbf{D}} \right\}$$

$$\text{with } \bar{\mathbf{J}} = p(\mathbf{j} + \bar{D}) \quad (11)$$

Define for the following the functions:

$$\mathbf{y}(y) = \sum_{i=1}^l \mathbf{g}_i (y_i - y_i^r) \quad \text{and} \quad \mathbf{n}(y) = \sum_{i=1}^l |\mathbf{d}_i| (y_i - y_i^r) \quad (12)$$

**Proposition 1:** Under hypotheses H1 to H4, for any set of initial conditions  $\mathcal{Q}_0 = [x_{10}, \bar{x}_{10}] \times \{y(0)\} \subset \mathbf{W}$  such that  $\mathbf{h}(\mathcal{Q}_0) > 0$  the following output control law exponentially stabilizes the output  $(y_1, \dots, y_l)$  about  $(y_1^r, \dots, y_l^r)$ :

$$D^*(x_1^*(t), y(t)) = \max \left( 0, \min \left( \bar{D}, \bar{D} \left( x_1^*(t), y(t) \right) \right) \right) \quad (13)$$

with

$$\bar{D} \left( x_1^*(t), y(t) \right) = \frac{-\sum_{i=1}^l h_i(y(t)) H_i C_2^Y T^* f^* \left( x_1^*(t), y(t) \right) - \mathbf{I} \mathbf{u}(y(t))}{\sum_{i=1}^l h_i(y(t)) (y_i^{in*}(t) - y_i(t))}$$

$$f^* \left( x_1^*(t), y(t) \right) = \begin{cases} f(x_1(t), y(t)) & \text{if } \mathbf{y}(y(t)) \geq 0 \\ \bar{f}(\bar{x}_1(t), y(t)) & \text{if } \mathbf{y}(y(t)) < 0 \end{cases}$$

( $\underline{x}_1$ ,  $\bar{x}_1$  are provided by the interval observer (5))

$$h_i(y(t)) = \begin{cases} \mathbf{g}_i + \mathbf{d}_i & \text{if } \mathbf{d}_i (y_i(t) - y_i^r) \mathbf{y}(y(t)) \geq 0 \\ \mathbf{g}_i - \mathbf{d}_i & \text{if } \mathbf{d}_i (y_i(t) - y_i^r) \mathbf{y}(y(t)) < 0 \end{cases}$$

$$\mathbf{u}(t) = \begin{cases} 1 - e^{-|\mathbf{y}(y(t))| - \mathbf{n}(y(t))} & \text{if } \mathbf{y}(y(t)) \geq 0 \\ e^{-|\mathbf{y}(y(t))| - \mathbf{n}(y(t))} - 1 & \text{if } \mathbf{y}(y(t)) < 0 \end{cases}$$

$$T^* = \begin{cases} \frac{T}{T} & \text{if } \mathbf{y}(y(t)) \geq 0 \\ \frac{T}{T} & \text{if } \mathbf{y}(y(t)) < 0 \end{cases} \quad y_i^{in*}(t) = \begin{cases} \bar{y}_i^{in}(t) & \text{if } \mathbf{y}(y(t)) \geq 0 \\ \underline{y}_i^{in}(t) & \text{if } \mathbf{y}(y(t)) < 0 \end{cases}$$

$$I = \begin{cases} I^+ & \text{if } \mathbf{y}(y(t)) \geq 0 \\ I^- & \text{if } \mathbf{y}(y(t)) < 0 \end{cases}$$

*Proof:* Consider the candidate partial Lyapunov function :

$$V(y(t)) = e^{|\mathbf{y}(y(t))| + \mathbf{n}(y(t))} - 1 \quad (14)$$

$$\text{and define: } g(y(t)) = \begin{cases} e^{|\mathbf{y}(y(t))| + \mathbf{n}(y(t))} & \text{if } \mathbf{y}(y(t)) \geq 0 \\ -e^{|\mathbf{y}(y(t))| + \mathbf{n}(y(t))} & \text{if } \mathbf{y}(y(t)) < 0 \end{cases}$$

then one have:

$$\begin{aligned} \dot{V}(y(t)) &= \sum_{i=1}^l h_i(y(t)) g(y(t)) \left( H_i C_2^Y T^* f^* \left( x_1^*(t), y(t) \right) \right. \\ &\quad \left. + D^* \left( x_1^*(t), y(t) \right) (y_i^{in}(t) - y_i(t)) \right) \leq -IV(y(t)) \end{aligned} \quad (15)$$

So,  $V(y(t))$  is decreasing and exponentially stable, an thus, the claimed result. More details for this proof can be found in [Maloum and Rapaport, 2000]. ■

**Remark 1:** The proposed control law for  $D^*$  is ‘‘bang-bang’’ about  $y_i^r$ , between two expressions:  $D^+(x_1(t), y(t))$  when  $\mathbf{g}_i (y_i(t) - y_i^r) \geq 0$  and  $D^-(x_1(t), y(t))$  when  $\mathbf{g}_i (y_i(t) - y_i^r) < 0$ . It can be smoothed for practical stability (i.e., about a neighborhood  $\{y_i(t) | \mathbf{g}_i (y_i(t) - y_i^r)\} \leq \mathbf{e}$  with  $\mathbf{e} > 0$ ) by the following output feedback law:

$$\begin{aligned} D_e^* \left( x_1^*, y(t) \right) &= D^+(x_1, y(t)) & \text{if } \mathbf{y}(y(t)) > \mathbf{e} \\ &= ((\mathbf{y}(y(t)) + \mathbf{e}) D^+(x_1, y(t)) \\ &\quad - (\mathbf{y}(y(t)) - \mathbf{e}) D^-(x_1, y(t))) / 2\mathbf{e} & \text{if } |\mathbf{y}(y(t))| \leq \mathbf{e} \\ &= D^-(x_1, y(t)) & \text{if } \mathbf{y}(y(t)) < -\mathbf{e} \end{aligned} \quad (16)$$

**Remark 2:** The SISO case is covered when  $l = 1$  (obviously) and  $\mathbf{d}_1 = 0$ .

## V – Example: Application to a wastewater treatment process

### V.1 - The anaerobic digester model

In the following, a model of an anaerobic digestion process carried out in a continuous fixed bed reactor for the treatment of

industrial wine distillery vinasses is considered [BERNARD *et al.*, 1998]:

$$\begin{cases} \dot{X}_1 = (\mathbf{m}_{max_1} \mathbf{m}_1 - \mathbf{a}D)X_1 \\ \dot{X}_2 = (\mathbf{m}_0 \mathbf{m}_2 - \mathbf{a}D)X_2 \\ \dot{Z} = D(Z^i - Z) \\ \dot{S}_1 = D(S_1^i - S_1) - k_1 \mathbf{m}_{max_1} \mathbf{m}_1 X_1 \\ \dot{S}_2 = D(S_2^i - S_2) + k_2 \mathbf{m}_{max_1} \mathbf{m}_1 X_1 - k_3 \mathbf{m}_0 \mathbf{m}_2 X_2 \\ \dot{C}_{TI} = D(C_{TI}^i - C_{TI}) + k_7 (k_8 P_{CO_2} + Z - C_{TI} - S_2) \\ \quad + k_4 \mathbf{m}_{max_1} \mathbf{m}_1 X_1 + k_5 \mathbf{m}_0 \mathbf{m}_2 X_2 \end{cases} \quad (17)$$

where  $X_1$ ,  $X_2$ ,  $Z$ ,  $S_1$ ,  $S_2$  and  $C_{TI}$  are respectively the concentrations of acidogenic bacteria, methanogenic bacteria, strong ions, chemical oxygen demand, volatile fatty acids and total inorganic carbon and they are supposed to be positive for any time. The parameter  $\mathbf{a}$  represents a proportionality parameter of experimental determination. In all cases, the upper index  $i$  indicates "influent concentration". The variable  $D = D(t) \geq 0$  is the dilution rate and is supposed to be a persisting input, *i.e.*  $\int_0^\infty D(t)dt > 0$ .

Like in any other mass balance model of biological processes, a strongly nonlinear kinetic behavior is present due to the reaction rates. These rates are given by:

$$\mathbf{m}_1 = \frac{S_1}{K_{S_1} + S_1} \quad \text{and} \quad \mathbf{m}_2 = \frac{S_2}{K_{S_2} + S_2 + \left(\frac{S_2}{K_{I_2}}\right)^2} \quad (18)$$

The  $CO_2$  partial pressure  $P_{CO_2}$  is expressed as function of the states as :

$$P_{CO_2} = \frac{F - \sqrt{F^2 - 4k_8 P_T [CO_2]}}{2k_8} \quad \text{where} \quad (19)$$

$$F = k_8 P_T + [CO_2] + \frac{k_6 \mathbf{m}_0 \mathbf{m}_2 X_2}{k_7} \quad \text{and} \quad [CO_2] = C_{TI} + S_2 - Z$$

Detailed definition of the different functions, parameters and their values can be found in [Bernard *et al.*, 1998]. The nonlinear interval observer and the robust feedback law developed in previous sections is then applied to the dynamic process model (17) defining the state vector  $x_1 = X_1$ ;  $x_2 = X_2$ ,  $x_3 = C_{TI}$ ,  $x_4 = Z$ ;  $x_5 = S_1$ ,  $x_6 = S_2$ , the model (17) can easily be written under the form of the model (2) with appropriate matrix definition.

The observers (4) and (5) have been already successfully applied for the process model (17) in [ALCARAZ-GONZÁLEZ *et al.*, 1999<sup>a-c</sup>]. Indeed, by using  $D$ ,  $P_{CO_2}$  and the two substrate concentrations  $S_1$ ,  $S_2$ , as measurements, the following matrix  $N$ :

$$N = \frac{1}{k_1 k_3} \begin{bmatrix} k_1 k_3 & 0 & 0 & 0 & \vdots & k_3 & 0 \\ 0 & k_1 k_3 & 0 & 0 & \vdots & k_2 & k_1 \\ 0 & 0 & k_1 k_3 & 0 & \vdots & (k_3 k_4 + k_2 k_5) & k_1 k_5 \\ 0 & 0 & 0 & k_1 k_3 & \vdots & 0 & 0 \end{bmatrix} \quad (20)$$

meets the hypotheses H2-3 and then, (5) can be used to estimate guaranteed intervals on  $X_1$ ,  $X_2$ ,  $C_{TI}$  and  $Z$ . Now the robust regulation method depicted in the previous section will be applied to regulate  $S_1$  and  $S_2$  around  $S_1^r < \underline{S}_1^{in-}$  and  $S_2^r < \underline{S}_2^{in-}$

respectively with  $\underline{S}_1^{in-} > S_1(0)$ ,  $\underline{S}_2^{in-} > S_2(0)$ ,  $S_1(0) > S_1^r$ , and  $S_2(0) > S_2^r$ .

## V.2 - Simulation results

Simulations were carried out using the parameter values from [Bernard *et al.*, 1998]. These simulations were carried out over a 100 days period at different dilution rates and at different input substrate concentrations. In addition it was considered that input concentrations (see Figures 1 to 4) as well as  $\mathbf{m}_{max_1}$  ( $1.14 \leq \mathbf{m}_{max_1} \leq 1.26$ ) and  $\mathbf{m}_0$  ( $0.66 \leq \mathbf{m}_0 \leq 0.73$ ) (see Figures 5 to 6) were unknown (dashed line) and only guaranteed intervals on them were known (continuous line). In order to add some realism to these simulations, small fluctuations as well as drastic step perturbations were alternatively introduced in the input concentrations (see Figures 1 to 4). The variables  $X_1^i$  and  $X_2^i$  are supposed to be negligible in the model (17). The "real input concentrations" as well as the "real evolution of the parameters" shown in Figures 1 to 6 were only used to simulate the model (17) from which the measurements  $S_1$  and  $S_2$  and the partial  $CO_2$  pressure  $P_{CO_2}$  were taken directly (see Figures 9, 11 and 12). Upper and lower estimated states (continuous line) for the unmeasured variables  $X_1$  and  $X_2$  are presented in Figures 7 and 8. "Predictions of the model" values (dashed line) in this graphic were also directly obtained from the model. In agreement with the section IV, the dilution rate  $D$  was used for the regulation of  $S_1$  and  $S_2$  about the reference values  $S_1^r = 2$  g/l and  $S_2^r = 10$  mmol/l respectively. The time evolution of  $D$ ,  $S_1$  and  $S_2$  are shown in Figures 10 to 12.

**Remark 3:** According with (7), the dynamics of  $S_1$  operates in *mode (i) = c* (see (17)). For  $S_2$ , this is not obvious, however, under the operational conditions depicted above (close to those used in a real plant), the dynamics of  $S_2$  operates also in *mode (i) = c* in a global manner.

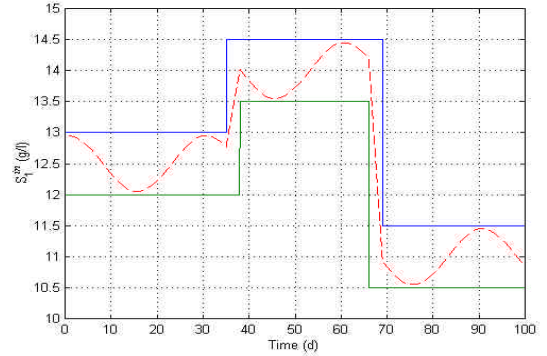


Figure 1: Uncertainty on  $S_1^{in}$ .

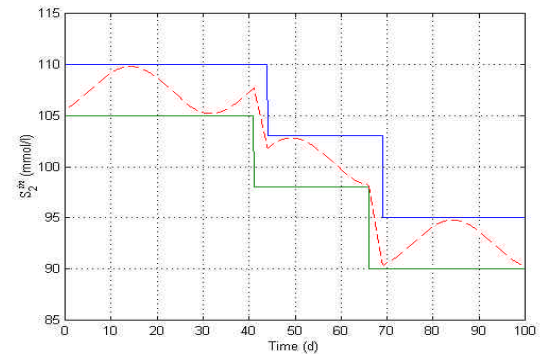


Figure 2: Uncertainty on  $S_2^{in}$ .

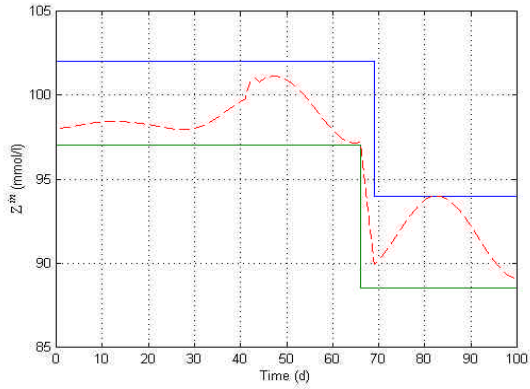


Figure 3: Uncertainty on  $Z^{in}$ .

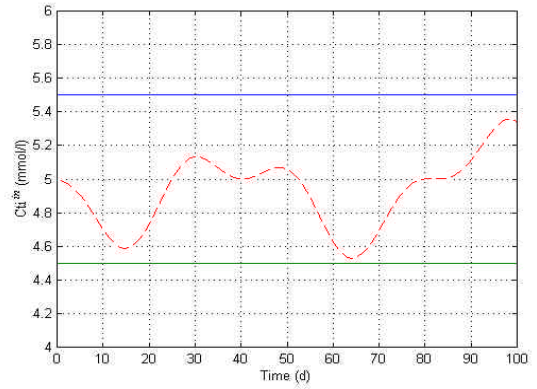


Figure 4: Uncertainty on  $C_{T1}^{in}$ .

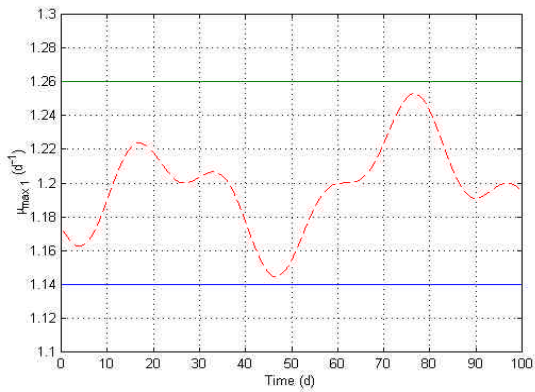


Figure 5: Uncertainty on  $m_{max1}$ .

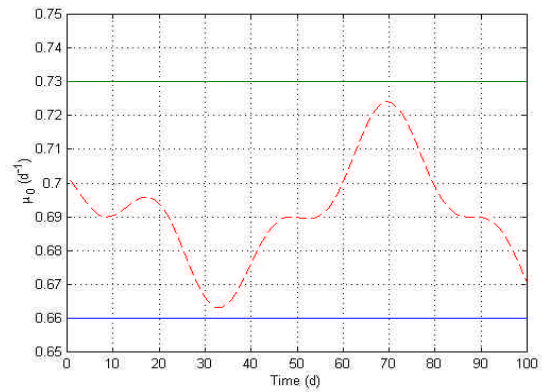


Figure 6: Uncertainty on  $m_b$ .

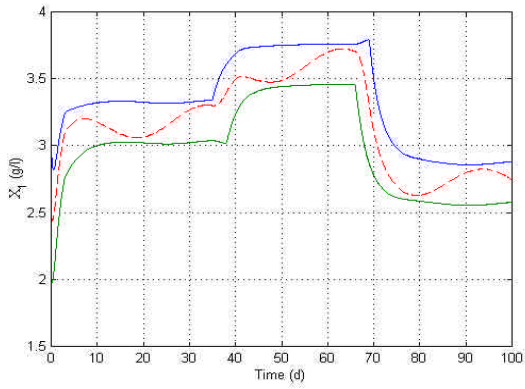


Figure 7: Estimation of  $X_1$ .

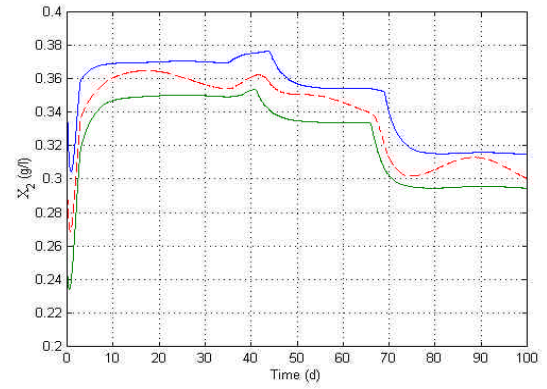


Figure 8: Estimation of  $X_2$ .

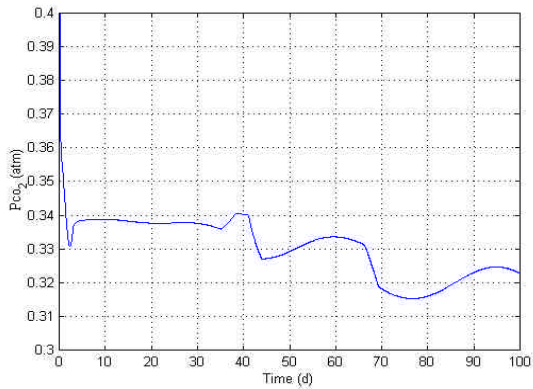


Figure 9: Measurement on  $P_{CO_2}$ .

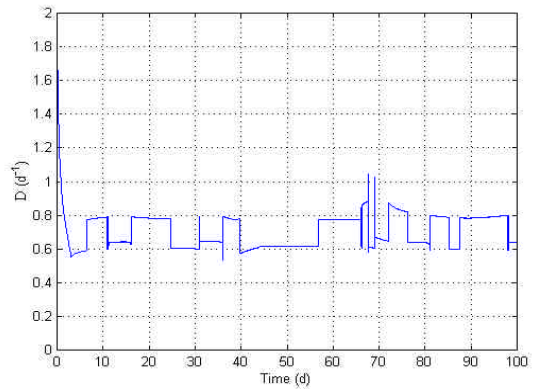


Figure 10: Dilution rate (the regulation law).

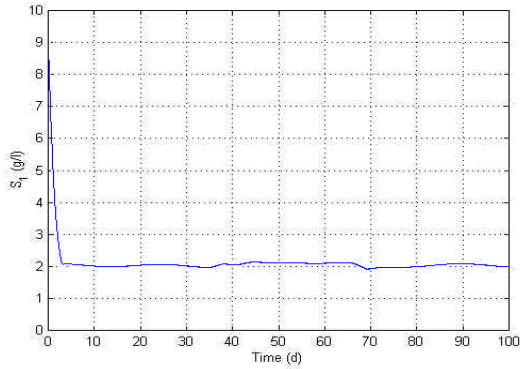


Figure 11: Effluent concentration of  $S_1$  (measured regulated variable).

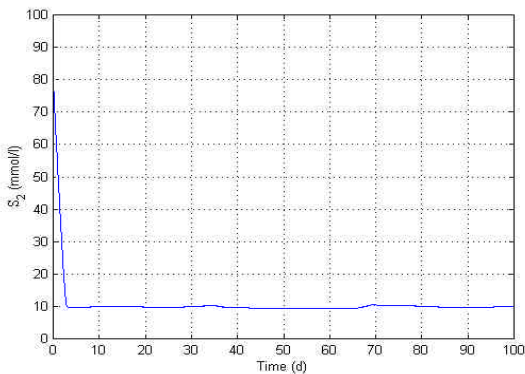


Figure 12: Effluent concentration of  $S_2$  (measured regulated variable).

In Figures 10 to 12 it is shown how the dilution rate  $D$  operates quickly to drive the regulated variables  $S_1$  and  $S_2$  towards the reference values  $S_1^r = 2$  g/l and  $S_2^r = 10$  mmol/l respectively. Also, in these Figures it is shown how, once the regulation goal is achieved, the dilution rate  $D$  performs adequately to keep  $S_1$  and  $S_2$  around their nominal values despite the highly uncertain environment (e.g., fluctuations and drastic step perturbations of the unknown input concentrations, uncertainties on  $m_{max_1}$  and  $m_0$  and uncertainties on the estimated values of  $X_1$  and  $X_2$ ).

## VI – Conclusions and perspectives

In this paper, a robust set-valued SISO and SIMO regulation law has been proposed for an anaerobic digester for the wastewater treatment whose behavior is described by a highly nonlinear dynamic system. Simulations were carried out handling operational conditions close to those used in a real plant. This regulation law presented an excellent performance keeping the regulated two important variables towards their nominal values for a certain steady state even under a highly uncertain environment (e.g., fluctuations and drastic step perturbations on the unknown input concentrations, uncertainties on the kinetic parameters and uncertainties on state variables involved in the kinetic rates). A logical extension of this approach, now under study, is the MIMO case based upon the same philosophy. Because of the large interest of this approach at the experimental scale, the authors are currently working on the experimental validation of a real WWTP.

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