Impedance Control for Multi-Arm Manipulation

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Abstract

In this paper a geometrically consistent impedance concept is applied to control a multi-arm system. The case of a common rigid object rigidly grasped by the manipulators in the system is considered. A six-DOF impedance behaviour is enforced at the object level so as to keep limited the force/moment due to the interaction with an external environment. Moreover, in order to avoid large internal forces, a six-DOF impedance behaviour is imposed at each end-effector. The adoption of the unit quaternion to describe object frame orientation ensures consistency with the task geometry. The overall control scheme is of inverse dynamics type with an inner motion loop which provides robustness to unmodeled dynamics and disturbances.

1 Introduction

When a multi-arm system is employed for the manipulation of a commonly held object, it is important to control both the variables describing the object motion and the internal stresses. To this aim, the mappings between forces and velocities at the end effector of each manipulator and their counterparts at the manipulated object are to be considered [1]. Control schemes designed in this framework have been proposed for the control of absolute motion and internal force, see e.g. [2, 3] and more recently [4, 5]. Most of the approaches recalled above can be classified as force/motion control schemes, in that they decompose the control action in a motion control loop providing tracking of the desired object motion and a force control loop counteracting the effects of internal loading at the object. An alternative approach can be pursued based on the well-known impedance concept [6]. As a matter of fact, when a manipulation system interacts with the environment and/or other manipulators, large values of the contact forces and moments can be avoided by enforcing a suitable compliant dynamic behaviour between position/orientation displacements and contact force/moment.

Impedance control approaches for multi-arm manipulation have been pursued for, e.g., control of internal forces [7] or control of object/environment interaction [8]. These approaches either are based on the use of minimal representations for the orientation displacements (e.g., Euler angles) or consider simple planar cases. However, as shown in [9], the use of Euler angles may give rise to task geometric inconsistency due to the dependence of the rotational stiffness upon the actual orientation. In this case a spatial impedance based on the unit quaternion can be defined.

The spatial impedance concept is applied in the present work to multi-arm systems manipulating a commonly held rigid object, which may eventually interact with the external environment; differently from previous works, the proposed strategy is able to keep limited both contact and internal forces and moments.

Namely, when the held object interacts with the environment, large contact forces may arise if the planned trajectory is not consistent with the environment constraints. In order to keep the contact forces limited, an impedance is designed between the object's position/orientation displacements and the contact forces. The resulting compliant motion is used to generate the reference motions for the object.

On the other hand, even in the absence of contact with the environment, the interaction between the manipulators and the object may lead to internal forces and moments, i.e., mechanical stresses which do not contribute to the object's motion and may cause damage to the systems and overloading of actuators. In order to counteract building of large internal forces, an impedance is designed between the position/orientation displacements of each manipulator and the end-effector forces contributing solely to the internal forces [7]. The resulting compliant motion is used to generate the reference motions for each end effector.

Tracking of the reference motions is guaranteed by the adoption of an inner control loop [15] with full dynamic compensation, which provides robustness to unmodeled dynamics and disturbances.

2 Modeling

Consider a system of manipulators tightly grasping a common rigid object. Without loss of generality, a system of two manipulators will be considered in the remainder.

For each manipulator ($k = 1, 2$) let Σ_k denote a frame attached to the end effector, whose origin and orientation are characterized by the (3×1) position vector $\boldsymbol{p}_k$ and the (3×3) rotation matrix $\boldsymbol{R}_k$, respectively. Let $\mathcal{Q}_k = \{\eta_k, \boldsymbol{\epsilon}_k\}$ be the unit quaternion corresponding to $\boldsymbol{R}_k$ (see the Appendix). Since the grasp is tight and the object is rigid, the relative orientation between Σ_1 and Σ_2 is constant and can be set so as $\boldsymbol{R}_1 = \boldsymbol{R}_2$. Let also $\boldsymbol{v}_k = [\,\dot{\boldsymbol{p}}_k^{\mathrm{T}} \quad \boldsymbol{\omega}_k^{\mathrm{T}}\,]^{\mathrm{T}}$ be the (6×1) end-effector (linear and angular) velocity vector.

Further, consider a frame Σ_e attached to the object; the origin $\boldsymbol{p}_e$ is chosen so as to coincide with the object's center of mass, while the orientation is chosen so as to coincide with those of the two end-effector frames, i.e., $\boldsymbol{R}_e = \boldsymbol{R}_1 = \boldsymbol{R}_2$. Let also $\mathcal{Q}_e$ be the unit quaternion corresponding to $\boldsymbol{R}_e$.

All quantities are expressed in the common base frame Σ. Hereafter, a superscript will denote the frame to which a quantity (vector or matrix) is referred; the superscript will be dropped whenever a quantity is referred to the base frame.

The dynamics of the two manipulators can be written in compact form as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{d}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} - \boldsymbol{J}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{h}, \quad (1)$$

where the matrices are block-diagonal, e.g. $\boldsymbol{M} = \mathrm{blockdiag}(\boldsymbol{M}_1, \boldsymbol{M}_2)$, and the vectors are stacked, e.g. $\boldsymbol{g} = [\,\boldsymbol{g}_1^{\mathrm{T}} \quad \boldsymbol{g}_2^{\mathrm{T}}\,]^{\mathrm{T}}$. For each manipulator ($k = 1, 2$), $\boldsymbol{M}_k$ is the $(n_k \times n_k)$ symmetric positive-definite inertia matrix, $\boldsymbol{C}_k\dot{\boldsymbol{q}}_i$ is the $(n_k \times 1)$ vector of Coriolis and centrifugal forces, $\boldsymbol{g}_k$ is the $(n_k \times 1)$ vector of gravitational forces, the vector $\boldsymbol{\tau}_k$ represents the joint torques and $\boldsymbol{h}_k = [\,\boldsymbol{f}_k^{\mathrm{T}} \quad \boldsymbol{\mu}_k^{\mathrm{T}}\,]^{\mathrm{T}}$ is the (6×1) vector of generalized forces acting at the end effector of the k-th manipulator. Finally, the term $\boldsymbol{d}$ represents a vector of disturbance terms, due to inaccurate modeling (e.g., joint friction torques) and/or external disturbances.

The dynamics of the object can be described by the equation

$$m_e\ddot{\boldsymbol{p}}_e - m_e\boldsymbol{g}_e = \boldsymbol{f}_e - \boldsymbol{f}_{env}, \quad (2)$$

$$\boldsymbol{J}_e\,{}^e\dot{\boldsymbol{\omega}}_e + \boldsymbol{S}({}^e\boldsymbol{\omega}_e)\,\boldsymbol{J}_e\,{}^e\boldsymbol{\omega}_e = {}^e\boldsymbol{\mu}_e - {}^e\boldsymbol{\mu}_{env}, \quad (3)$$

where $\dot{\boldsymbol{p}}_e$ and $\boldsymbol{\omega}_e$ are the vectors expressing the linear and angular velocity of Σ_e, respectively, $\boldsymbol{m}_e$ is the object mass, $\boldsymbol{J}_e$ is the object's inertia tensor, $\boldsymbol{g}_e$ is the vector of gravitational forces, $\boldsymbol{h}_e = [\,\boldsymbol{f}_e^{\mathrm{T}}\ \boldsymbol{\mu}_e^{\mathrm{T}}\,]^{\mathrm{T}}$ is the vector of the generalized forces exerted by the manipulators on the object, and $\boldsymbol{h}_{env} = [\,\boldsymbol{f}_{env}^{\mathrm{T}}\ \boldsymbol{\mu}_{env}^{\mathrm{T}}\,]^{\mathrm{T}}$ is the vector of generalized forces exerted by the object on the environment.

Since the grasp is tight, each end effector can exert both a force and a moment on the object at the contact point. The mapping of the contact force vector $\boldsymbol{h}$ into the (6×1) force vector $\boldsymbol{h}_e$ is

$$\boldsymbol{h}_e = \begin{bmatrix} \boldsymbol{I}_3 & \boldsymbol{O}_3 & \boldsymbol{I}_3 & \boldsymbol{O}_3 \\ \boldsymbol{S}(\boldsymbol{r}_1) & \boldsymbol{I}_3 & \boldsymbol{S}(\boldsymbol{r}_2) & \boldsymbol{I}_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \end{bmatrix} = \boldsymbol{W}\boldsymbol{h}, \quad (4)$$

where $\boldsymbol{W}$ is the grasp matrix, $\boldsymbol{O}_l$ denotes the $(l \times l)$ null matrix, $\boldsymbol{I}_l$ denotes the $(l \times l)$ identity matrix, $\boldsymbol{S}(\cdot)$ is the skew-symmetric matrix operator performing the cross product and $\boldsymbol{r}_i$ is the (3×1) vector from the i-th end effector to the point fixed on the object (i.e., $\boldsymbol{r}_i$ is the so-called *virtual stick* [1]).

The matrix $\boldsymbol{W}$ is full row rank; then, for a given $\boldsymbol{h}_{ext}$ the inverse solution to (4) is given by

$$\boldsymbol{h} = \boldsymbol{W}^{\dagger}\boldsymbol{h}_e + \boldsymbol{V}\boldsymbol{h}_s = \boldsymbol{W}^{\dagger}\boldsymbol{W}\boldsymbol{h} + \boldsymbol{V}\boldsymbol{V}^{\dagger}\boldsymbol{h}, \quad (5)$$

where $\boldsymbol{W}^{\dagger}$ denotes a pseudoinverse of $\boldsymbol{W}$, $\boldsymbol{V}$ is a full column rank matrix spanning the null space of $\boldsymbol{W}$, and $\boldsymbol{h}_s$ represents the vector of internal forces, i.e., the forces which do not contribute to the motion of the object, but represent mechanical stresses applied to the object. Hence, the vector $\boldsymbol{h}_M = \boldsymbol{W}^{\dagger}\boldsymbol{W}\boldsymbol{h}$ represents the end-effector forces balancing the object's dynamics and the reaction forces due to the environment, while

$$\boldsymbol{h}_I = \begin{bmatrix} \boldsymbol{h}_{I1} \\ \boldsymbol{h}_{I2} \end{bmatrix} = \boldsymbol{V}\boldsymbol{V}^{\dagger}\boldsymbol{h} \quad (6)$$

represents the vector of end-effector forces contributing to the sole internal forces. It has been recognized in [11] that the use in (5) of a generic pseudoinverse of $\boldsymbol{W}$, e.g. the Moore-Penrose pseudoinverse, may lead to internal stresses even if $\boldsymbol{h}_s = \boldsymbol{0}$; to avoid this, $\boldsymbol{W}^{\dagger}$ must be properly chosen.

According to the above description of the geometry of the grasp, the end-effector quantities corresponding to a given object's motion can be expressed as:

$$\begin{aligned}
\boldsymbol{p}_k &= \boldsymbol{p}_e + \boldsymbol{R}_e\,{}^e\boldsymbol{r}_k, \\
\boldsymbol{R}_k &= \boldsymbol{R}_e, \\
\dot{\boldsymbol{p}}_k &= \dot{\boldsymbol{p}}_e - \boldsymbol{S}(\boldsymbol{R}_e\,{}^e\boldsymbol{r}_k)\boldsymbol{\omega}_e, \\
\boldsymbol{\omega}_k &= \boldsymbol{\omega}_e, \\
\ddot{\boldsymbol{p}}_k &= \ddot{\boldsymbol{p}}_e - \boldsymbol{S}(\boldsymbol{\omega}_e)\boldsymbol{S}(\boldsymbol{R}_e\,{}^e\boldsymbol{r}_k)\boldsymbol{\omega}_e - \boldsymbol{S}(\boldsymbol{R}_e\,{}^e\boldsymbol{r}_k)\dot{\boldsymbol{\omega}}_e, \\
\dot{\boldsymbol{\omega}}_k &= \dot{\boldsymbol{\omega}}_e,
\end{aligned}$$

with $k = 1, 2$.

The above equations define a mapping

$$\mathcal{C} : \{\boldsymbol{p}_e, \boldsymbol{R}_e, \dot{\boldsymbol{p}}_e, \boldsymbol{\omega}_e, \ddot{\boldsymbol{p}}_e, \dot{\boldsymbol{\omega}}_e\} \mapsto \{\boldsymbol{p}_k, \boldsymbol{R}_k, \dot{\boldsymbol{p}}_k, \boldsymbol{\omega}_k, \ddot{\boldsymbol{p}}_k, \dot{\boldsymbol{\omega}}_k\} \quad (7)$$

between end-effector and object motion variables. This represents a set of mechanical constraints on the position and orientation on the manipulator end effectors, and thus they are always fulfilled during system's motion. Any attempt to violate such constraints gives rise to internal forces. A certain amount of internal force may be desired in some cases in order to ensure a firm grasp; however, when the grasp is tight, even small errors leading to violation of the closed-chain constraints may cause large values of internal forces which must be avoided.

3 Inner motion control loop

In the remainder, an inner motion control loop is designed for each manipulator, which guarantees tracking of a reference end-effector position $\boldsymbol{p}_{k_r}$ and orientation $\mathcal{Q}_{k_r}$, as well as of a reference end–effector velocity $\boldsymbol{v}_{k_r}$ and acceleration $\dot{\boldsymbol{v}}_{k_r}$ $(k = 1, 2)$.

To the purpose, an inverse dynamics strategy is adopted, which requires knowledge of the dynamic models of the two robots —assumed to have six joints each. According to the well–known concept of inverse dynamics, the driving torques are chosen as

$$\begin{aligned}\boldsymbol{\tau}_k &= \boldsymbol{M}_k(\boldsymbol{q}_k)\boldsymbol{J}_k^{-1}(\boldsymbol{q}_k)\big(\boldsymbol{a}_k - \dot{\boldsymbol{J}}_k(\boldsymbol{q}_k, \dot{\boldsymbol{q}}_k)\dot{\boldsymbol{q}}_k\big) \\ &\quad + \boldsymbol{C}_k(\boldsymbol{q}_k, \dot{\boldsymbol{q}}_k)\dot{\boldsymbol{q}}_k + \widehat{\boldsymbol{d}}_k(\boldsymbol{q}_k, \dot{\boldsymbol{q}}_k) + \boldsymbol{g}_k(\boldsymbol{q}_k) + \boldsymbol{J}_k^{\mathrm{T}}(\boldsymbol{q}_k)\boldsymbol{h}_k,\end{aligned} \tag{8}$$

where $\boldsymbol{a}_k$ is a new control input, end–effector force and moment measurements are used to compensate for the term $\boldsymbol{h}_k$ in (1), and $\widehat{\boldsymbol{d}}_k$ denotes the available estimate of the disturbance terms due to inaccurate modeling. To this purpose, notice that it is reasonable to assume accurate compensation of the dynamic terms in the model (1), e.g., as obtained by a parameter identification technique [12], except for the friction torques.

Substituting the control law (8) in (1) and accounting for the time derivative of $\boldsymbol{v}_k = \boldsymbol{J}_k(\boldsymbol{q}_k)\dot{\boldsymbol{q}}_k$ gives

$$\dot{\boldsymbol{v}}_k = \boldsymbol{a}_k - \boldsymbol{\delta}_k, \tag{9}$$

that is a resolved end–effector acceleration for which the term $\boldsymbol{\delta}_k = \boldsymbol{J}_k\boldsymbol{M}_k^{-1}(\boldsymbol{d}_k - \widehat{\boldsymbol{d}}_k)$ can be regarded as a disturbance. In the case of mismatching on other terms in the dynamic model (1), such a disturbance would include additional contributions.

The new control input for each manipulator $(k = 1, 2)$ can be chosen as $\boldsymbol{a}_k = [\boldsymbol{a}_{p,k}^{\mathrm{T}} \quad \boldsymbol{a}_{\epsilon,k}^{\mathrm{T}}]^{\mathrm{T}}$ where $\boldsymbol{a}_{p,k}$ and $\boldsymbol{a}_{\epsilon,k}$ are chosen as

$$\boldsymbol{a}_{p,k} = \ddot{\boldsymbol{p}}_{k_r} + k_{Vp}(\dot{\boldsymbol{p}}_{k_r} - \dot{\boldsymbol{p}}_k) + k_{Pp}(\boldsymbol{p}_{k_r} - \boldsymbol{p}_k), \tag{10}$$

$$\boldsymbol{a}_{\epsilon,k} = \dot{\boldsymbol{\omega}}_{k_r} + k_{V\epsilon}(\boldsymbol{\omega}_{k_r} - \boldsymbol{\omega}_k) + k_{P\epsilon}\boldsymbol{R}_k\,{}^k\boldsymbol{\epsilon}_{k_r k}, \tag{11}$$

where ${}^k\boldsymbol{\epsilon}_{k_r k}$ is the vector part of $\mathcal{Q}_{k_r k} = \mathcal{Q}_k^{-1} * \mathcal{Q}_{k_r}$. It can be shown [13] that control law (10),(11) ensures asymptotic tracking of the reference position and orientation trajectories, i.e.

$$\begin{aligned}\boldsymbol{p}_k &\to \boldsymbol{p}_{k_r}, \quad \boldsymbol{R}_k \to \boldsymbol{R}_{k_r}, \\ \dot{\boldsymbol{p}}_k &\to \dot{\boldsymbol{p}}_{k_r}, \quad \boldsymbol{\omega}_k \to \boldsymbol{\omega}_{k_r}, \\ \ddot{\boldsymbol{p}}_k &\to \ddot{\boldsymbol{p}}_{k_r}, \quad \dot{\boldsymbol{\omega}}_k \to \dot{\boldsymbol{\omega}}_{k_r},\end{aligned}$$

for $t \to \infty$, when $\boldsymbol{\delta}_k = \boldsymbol{0}$. The control gains k_{Vp}, k_{Pp}, $k_{V\epsilon}$, $k_{P\epsilon}$ can be tuned so as to provide accurate and fast tracking as well as robustness to the disturbance $\boldsymbol{\delta}_k$.

The inner motion control loop represents the basis for the development of the two approaches analyzed in the following. Namely, the control problem for multi-arm manipulation is formulated as that of computing suitable reference position and orientation trajectories for the inner control loop.

4 Positional control

Let the desired object position $\boldsymbol{p}_{e_d}(t)$ and orientation $\mathcal{Q}_{e_d}(t)$ (extracted from $\boldsymbol{R}_{e_d}(t)$) be assigned with the associated linear and angular velocities and accelerations. If a purely positional control strategy is pursued, the reference vectors for each manipulator are generated from the desired trajectory for the object according to the geometry of the grasp, i.e.

$$\begin{bmatrix} \boldsymbol{p}_{k_r} \\ \boldsymbol{R}_{k_r} \\ \dot{\boldsymbol{p}}_{k_r} \\ \boldsymbol{\omega}_{k_r} \\ \ddot{\boldsymbol{p}}_{k_r} \\ \dot{\boldsymbol{\omega}}_{k_r} \end{bmatrix} = \mathcal{C}(\boldsymbol{p}_{e_d}, \boldsymbol{R}_{e_d}, \dot{\boldsymbol{p}}_{e_d}, \boldsymbol{\omega}_{e_d}, \ddot{\boldsymbol{p}}_{e_d}, \dot{\boldsymbol{\omega}}_{e_d}), \tag{12}$$

where $\mathcal{C}$ is the mapping defined in (7) representing the closed-chain constraints.

If the reference trajectories for the two end effectors computed in (12) are consistent with the geometry of the grasp (i.e., $\mathcal{C}$ is an accurate model of the grasp geometry), in the absence of interaction of the object with the environment, asymptotic tracking of $\boldsymbol{p}_{e_d}$, $\mathcal{Q}_{e_d}$ and the corresponding velocity and acceleration is guaranteed by the inner motion control loop.

However, the adoption of a purely positional control strategy may lead to building up of internal forces. In fact, achieving small tracking errors for precise object positioning requires a stiff motion control loop; on the other hand, if the desired trajectories for the two end effectors are not consistent with the geometry of the grasp, they cannot be tracked because of the closed-chain constraints. In sum, the higher are the control gains, the larger are the internal forces. Also, internal forces may arise during fast transients if uncompensated dynamics or external disturbances are present. Finally, it is well-known that if the manipulated object comes in contact with the environment, a purely positional control strategy may fail to achieve limited contact forces.

Motivated by the above arguments, in the following an impedance control strategy is devised aimed at limiting both internal and contact forces.

5 Impedance control

When a robot manipulator interacts with other manipulators and/or with the environment, a suitable compliant behavior has to be ensured so as to keep limited the interaction forces. This is typically achieved by enforcing an equivalent mass-damper-spring behavior under the action of an external force $\boldsymbol{f}$ and moment $\boldsymbol{\mu}$, that can be described by a mechanical impedance [6].

In order to derive the impedance equation, let consider the mutual position between a desired frame Σ_d and a reference frame Σ_r, and the corresponding position displacement vector

$$\Delta \boldsymbol{p}_{dr} = \boldsymbol{p}_d - \boldsymbol{p}_r. \tag{13}$$

The positional part of the impedance equation is typically chosen as

$$\boldsymbol{M}_p \Delta \ddot{\boldsymbol{p}}_{dr} + \boldsymbol{D}_p \Delta \dot{\boldsymbol{p}}_{dr} + \boldsymbol{K}_p \Delta \boldsymbol{p}_{dr} = \boldsymbol{f}, \tag{14}$$

where $\boldsymbol{M}_p$, $\boldsymbol{D}_p$ and $\boldsymbol{K}_p$ are (3×3) positive definite matrices representing the mass, damping, and stiffness characterizing the impedance.

The rotational part of the impedance equation is usually defined by extending the formal expression of the equation written for the translational part (14) and using a minimal representation of the end-effector orientation in terms of three Euler angles. However, as shown in [9], the rotational stiffness may become not well defined even for small orientation displacements and cannot be specified in a consistent way with the task geometry. A solution to this problem has been proposed in [9], where a class of geometrically meaningful representations of the mutual orientation between the desired frame and the reference frame is considered.

In detail, the mutual orientation of Σ_d with respect to Σ_r can be expressed in terms of the unit quaternion $\mathcal{Q}_{dr} = \mathcal{Q}_r^{-1} * \mathcal{Q}_d = \{\eta_{dr}, {}^r\boldsymbol{\epsilon}_{dr}\} = \{\cos \frac{\vartheta_{dr}}{2}, \sin \frac{\vartheta_{dr}}{2} {}^r\boldsymbol{r}_{dr}\}$ (see the Appendix). Let $\boldsymbol{M}_\epsilon$, $\boldsymbol{D}_\epsilon$ and $\boldsymbol{K}_\epsilon$ denote (3×3) positive definite matrices representing the inertia, rotational damping, and rotational stiffness. Then, the impedance equation for the orientation displacement is

$$\boldsymbol{M}_\epsilon \Delta^r \dot{\boldsymbol{\omega}}_{dr} + \boldsymbol{D}_\epsilon \Delta^r \boldsymbol{\omega}_{dr} + \boldsymbol{K}'_\epsilon {}^r\boldsymbol{\epsilon}_{dr} = {}^r\boldsymbol{\mu}, \tag{15}$$

where $\Delta^r \boldsymbol{\omega}_{dr} = {}^r\boldsymbol{\omega}_d - {}^r\boldsymbol{\omega}_r$ is the (3×1) angular velocity vector of Σ_d relative to Σ_r, and $\boldsymbol{K}'_\epsilon$ is an equivalent rotational stiffness which is related to $\boldsymbol{K}_\epsilon$ as

$$\boldsymbol{K}'_\epsilon = 2\,(\eta_{dr}\boldsymbol{I} + \boldsymbol{S}({}^r\boldsymbol{\epsilon}_{dr}))\,\boldsymbol{K}_\epsilon. \tag{16}$$

The relationship between $\boldsymbol{K}'_\epsilon$ and $\boldsymbol{K}_\epsilon$ can be further investigated by considering the decomposition of $\boldsymbol{K}_\epsilon$ as

$$\boldsymbol{K}_\epsilon = \boldsymbol{U}_\epsilon \boldsymbol{\Gamma}_\epsilon \boldsymbol{U}_\epsilon^{\mathrm{T}}, \tag{17}$$

where $\boldsymbol{\Gamma}_\epsilon = \mathrm{diag}\{\gamma_{\epsilon 1}, \gamma_{\epsilon 2}, \gamma_{\epsilon 3}\}$ is the eigenvalue matrix and $\boldsymbol{U}_\epsilon = [\,\boldsymbol{u}_{\epsilon 1} \;\; \boldsymbol{u}_{\epsilon 2} \;\; \boldsymbol{u}_{\epsilon 3}\,]$ is the (orthogonal) eigenvector matrix. An orientation displacement $\{\cos(\vartheta_{dr}/2), \sin(\vartheta_{dr}/2)\boldsymbol{u}_{\epsilon i}\}$ about the i-th eigenvector leads to

$$\boldsymbol{K}'_\epsilon {}^r\boldsymbol{\epsilon}_{dr} = \gamma_{\epsilon i} \sin \vartheta_{dr}\; \boldsymbol{u}_{\epsilon i} \tag{18}$$

which represents an elastic moment about the same $\boldsymbol{u}_{\epsilon i}$ axis. This implies that the rotational stiffness matrix $\boldsymbol{K}_\epsilon$ can be expressed in terms of three parameters $\gamma_{\epsilon i}$ representing the stiffness about three principal axes $\boldsymbol{u}_{\epsilon i}$. In turn the rotational stiffness can be specified by suitably choosing the stiffness parameters and the corresponding principal axes according to the geometry of the task. Of course, task geometric consistency holds for the translational stiffness too, in the sense that $\boldsymbol{K}_p$ can be expressed in terms of three parameters γ_{pi} representing the stiffness along three principal axes $\boldsymbol{u}_{pi}$.

Notice that, differently from (14), the rotational part of the impedance equation (15) has been derived in terms of quantities all referred to the reference frame Σ_r; this allows the impedance behavior to be effectively expressed in terms of the relative orientation of Σ_d with respect to Σ_r, no matter what the absolute orientation of Σ_r with respect to Σ is.

The two impedance equations (14) and (15) define the differential mapping

$$\begin{aligned}\mathcal{I} : \{&\boldsymbol{p}_d, \boldsymbol{R}_d, \dot{\boldsymbol{p}}_d, \boldsymbol{\omega}_d, \ddot{\boldsymbol{p}}_d, \dot{\boldsymbol{\omega}}_d, \boldsymbol{f}, \boldsymbol{\mu}\} \mapsto \\ &\{\boldsymbol{p}_r, \boldsymbol{R}_r, \dot{\boldsymbol{p}}_r, \boldsymbol{\omega}_r, \ddot{\boldsymbol{p}}_r, \dot{\boldsymbol{\omega}}_r\},\end{aligned} \tag{19}$$

which can be conveniently exploited to formulate an impedance control strategy for multi-arm manipulation.

5.1 Object/environment interaction

In order to keep limited the contact forces exchanged with the environment, an impedance-based strategy can be pursued as in [8]. Let $\boldsymbol{p}_{e_d}$, $\mathcal{Q}_{e_d}$, $\dot{\boldsymbol{p}}_{e_d}$, $\boldsymbol{\omega}_{e_d}$, $\ddot{\boldsymbol{p}}_{e_d}$, $\dot{\boldsymbol{\omega}}_{e_d}$ the motion variables describing the assigned trajectory for the object desired frame Σ_{e_d}. Then, the corresponding reference vectors $\boldsymbol{p}_{e_r}$, $\mathcal{Q}_{e_r}$ $\dot{\boldsymbol{p}}_{e_r}$, $\boldsymbol{\omega}_{e_r}$, $\ddot{\boldsymbol{p}}_{e_r}$, $\dot{\boldsymbol{\omega}}_{e_r}$ can be chosen as the solution to the differential equations:

$$\begin{aligned}&(\alpha_p - 1)\, m_e \ddot{\boldsymbol{p}}_{e_r} + \boldsymbol{D}_{pe}\dot{\boldsymbol{p}}_{e_r} + \boldsymbol{K}_{pe}\boldsymbol{p}_{e_r} = \\ &-\boldsymbol{f}_e - m_e \boldsymbol{g}_e + \alpha_p m_e \ddot{\boldsymbol{p}}_{e_d} + \boldsymbol{D}_{pe}\dot{\boldsymbol{p}}_{e_d} + \boldsymbol{K}_{pe}\boldsymbol{p}_{e_d} \qquad (20)\\ &(\alpha_\epsilon - 1)\, \boldsymbol{J}_e {}^{e_r}\dot{\boldsymbol{\omega}}_{e_r} + \boldsymbol{D}_{\epsilon e} {}^{e_r}\boldsymbol{\omega}_{e_r} - \boldsymbol{K}'_{\epsilon e} {}^{e_r}\boldsymbol{\epsilon}_{e_d e_r} = \\ &-{}^{e_r}\boldsymbol{\mu}_e + \boldsymbol{S}({}^{e_r}\boldsymbol{\omega}_{e_r})\, \boldsymbol{J}_e {}^{e_r}\boldsymbol{\omega}_{e_r} + \alpha_\epsilon \boldsymbol{J}_e {}^{e_r}\dot{\boldsymbol{\omega}}_{e_d} + \boldsymbol{D}_{\epsilon e} {}^{e_r}\boldsymbol{\omega}_{e_d}. \qquad (21)\end{aligned}$$

It is straightforward to prove that asymptotic stability of the equations (20) and (21) is ensured if the scalars $\alpha_p, \alpha_\epsilon > 1$ and the matrix gains $\boldsymbol{D}_{pe}$, $\boldsymbol{K}_{pe}$, $\boldsymbol{D}_{\epsilon e}$ and $\boldsymbol{K}_{\epsilon e}$ are symmetric and positive definite, with $\boldsymbol{K}'_{\epsilon e}$ defined as in (16).

The above choice of the object reference trajectory enforces an impedance behaviour with mass $\alpha_p m_e$ (inertia $\alpha_\epsilon \boldsymbol{J}_e$), translational damping $\boldsymbol{D}_{pe}$ (rotational damping $\boldsymbol{D}_{\epsilon e}$) and translational stiffness $\boldsymbol{K}_{pe}$ (rotational stiffness $\boldsymbol{K}_{\epsilon e}$). In fact, assuming perfect tracking of the object reference trajectory, the frames Σ_e and Σ_{e_r} can be assumed to be aligned. Hence, folding (20),(21) into the object dynamic equations (2),(3) the following impedance equations are obtained

$$\begin{aligned}&\alpha_p\, m_e \Delta \ddot{\boldsymbol{p}}_{e_d e_r} + \boldsymbol{D}_{pe}\Delta \dot{\boldsymbol{p}}_{e_d e_r} + \boldsymbol{K}_{pe}\Delta \boldsymbol{p}_{e_d e_r} = \boldsymbol{f}_{env} \qquad (22)\\ &\alpha_\epsilon\, \boldsymbol{J}_e \Delta^{e_r}\dot{\boldsymbol{\omega}}_{e_d e_r} + \boldsymbol{D}_{\epsilon e}\Delta^{e_r}\boldsymbol{\omega}_{e_d e_r} + \boldsymbol{K}'_{\epsilon e} {}^{e_r}\boldsymbol{\epsilon}_{e_d e_r} = \\ &{}^{e_r}\boldsymbol{\mu}_{env}. \qquad (23)\end{aligned}$$

It is worth noticing that, differently from [8], direct measurement of the object acceleration is not required thanks to the nonlinear feedforward terms in (20),(21) compensating for the object's dynamics. On the other hand, the mass and inertia matrices for the impedance cannot be assigned by the user in an arbitrary way, but they must be set proportional to the mass and inertia, respectively, of the object with a scaling factor greater than 1. This is due to the fact that no direct measurement of the contact force $\boldsymbol{h}_{env}$ or, equivalently, of the object acceleration is available. It is also advisable to choose the scaling factors close to the unity so as to avoid large modifications of the system's mass/inertia characteristics.

Notice that, in view of (19) the reference trajectories can be seen as output of the mapping

$$\mathcal{I}:\{\boldsymbol{p}_{e_d},\boldsymbol{R}_{e_d},\dot{\boldsymbol{p}}_{e_d},\boldsymbol{\omega}_{e_d},\ddot{\boldsymbol{p}}_{e_d},\dot{\boldsymbol{\omega}}_{e_d},\boldsymbol{f}_{env},\boldsymbol{\mu}_{env}\}\mapsto \{\boldsymbol{p}_{e_r},\boldsymbol{R}_{e_r},\dot{\boldsymbol{p}}_{e_r},\boldsymbol{\omega}_{e_r},\ddot{\boldsymbol{p}}_{e_r},\dot{\boldsymbol{\omega}}_{e_r}\}, \quad (24)$$

characterized by the impedance parameters in (22),(23).

Finally, the corresponding reference trajectories for the inner motion control loop are then computed according to the model of the grasp geometry

$$\begin{bmatrix}\boldsymbol{p}_{k_r}\\ \boldsymbol{R}_{k_r}\\ \dot{\boldsymbol{p}}_{k_r}\\ \boldsymbol{\omega}_{k_r}\\ \ddot{\boldsymbol{p}}_{k_r}\\ \dot{\boldsymbol{\omega}}_{k_r}\end{bmatrix}=\mathcal{C}(\boldsymbol{p}_{e_r},\boldsymbol{R}_{e_r},\dot{\boldsymbol{p}}_{e_r},\boldsymbol{\omega}_{e_r},\dot{\boldsymbol{p}}_{e_r},\boldsymbol{\omega}_{e_r}), \quad (25)$$

which allows projection of the references at the object's level for each end effector.

5.2 Manipulators/object interaction

The impedance paradigm can be keenly exploited to keep limited the internal forces acting at the object as well; to the purpose, a similar strategy as in [7] can be adopted. Namely, an impedance behaviour is imposed between the position/orientation displacements at each end effector and the force and moment contributions $\boldsymbol{f}_{Ik}$ and $\boldsymbol{\mu}_{Ik}$ in (6), i.e., for $k=1,2$

$$\boldsymbol{M}_{pI}\Delta\ddot{\boldsymbol{p}}_{k_dk_r}+\boldsymbol{D}_{pI}\Delta\dot{\boldsymbol{p}}_{k_dk_r}+\boldsymbol{K}_{pI}\Delta\boldsymbol{p}_{k_dk_r}=\boldsymbol{f}_{Ik}, \quad (26)$$

$$\boldsymbol{M}_{\epsilon I}\Delta^{k_r}\dot{\boldsymbol{\omega}}_{k_dk_r}+\boldsymbol{D}_{\epsilon I}\Delta^{k_r}\boldsymbol{\omega}_{k_dk_r}+\boldsymbol{K}'_{\epsilon I}\,{}^{k_r}\boldsymbol{\epsilon}_{k_dk_r}= {}^{k_r}\boldsymbol{\mu}_{Ik}. \quad (27)$$

Therefore, the reference vectors $\boldsymbol{p}_{k_r}$, $\mathcal{Q}_{k_r}$, together with the corresponding velocities and accelerations, are computed by integrating the differential mapping

$$\mathcal{I}:\{\boldsymbol{p}_{k_d},\boldsymbol{R}_{k_d},\dot{\boldsymbol{p}}_{k_d},\boldsymbol{\omega}_{k_d},\ddot{\boldsymbol{p}}_{k_d},\dot{\boldsymbol{\omega}}_{k_d},\boldsymbol{f}_{Ik},\boldsymbol{\mu}_{Ik}\}\mapsto \{\boldsymbol{p}_{k_r},\boldsymbol{R}_{k_r},\dot{\boldsymbol{p}}_{k_r},\boldsymbol{\omega}_{k_r},\ddot{\boldsymbol{p}}_{k_r},\dot{\boldsymbol{\omega}}_{k_r}\}, \quad (28)$$

characterized by the impedance parameters in (26),(27).

In this case the effect of the impedance is that of adjusting the reference trajectories for the two end effectors so as to avoid building up of large internal forces. By following the guidelines in [5], it can be shown that the object motion at steady state is not affected by the impedance on the internal forces, i.e., the object's desired trajectory is reached at steady state even if it is not consistent with the grasp geometry.

Hence, the adoption of the impedance approach on the internal forces allows keeping high gains in the inner motion control loop; this, in turn, ensures accurate tracking of the object's desired motion, while avoiding large internal forces.

6 Conclusion

A geometrically consistent impedance control has been proposed for dual-robot manipulation of a rigid object interacting with a compliant environment. The impedance concept is adopted to control both the contact forces exchanged between the object and the environment and the internal forces at the object. The control scheme has been developed according to an inverse dynamics strategy with closure of an inner motion control loop acting on the tracking errors between a suitably defined reference trajectory and the actual end-effector trajectory for each robot. Experimental validation of the proposed impedance control on a setup composed of two industrial robots is currently in progress.

Acknowledgments

The financial support of *MURST* and *ASI* is gratefully acknowledged.

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Appendix: The unit quaternion

The location of a rigid body in space is typically described in terms of the (3×1) position vector $\boldsymbol{p}$ and the (3×3) rotation matrix $\boldsymbol{R}$ describing the origin and the orientation of a frame attached to the body with respect to a fixed base frame.

The body linear velocity is described by the time derivative of the position vector, i.e. $\dot{\boldsymbol{p}}$, while its angular velocity $\boldsymbol{\omega}$ can be defined through the time derivative of the rotation matrix by the relationship $\dot{\boldsymbol{R}} = \boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{R}$, where $\boldsymbol{S}(\cdot)$ is the skew-symmetric matrix operator performing the cross product between two (3×1) vectors.

Representation singularities typically arise in certain configurations if a minimal description of the orientation is adopted (e.g., Euler angles). Also, the use of minimal descriptions of the orientation do not possess a clear geometrical meaning. These drawbacks can be overcome by a four-parameter representation; namely, the *Euler parameters* or *unit quaternion* defined as [14]:

$$\eta = \cos\frac{\vartheta}{2}, \qquad \boldsymbol{\epsilon} = \sin\frac{\vartheta}{2}\,\boldsymbol{r}, \tag{29}$$

where ϑ and $\boldsymbol{r}$ respectively are the rotation angle and the unit vector of an equivalent angle/axis description of the orientation. Notice that the scalar part and the vector part of the Euler parameters are constrained by $\eta^2 + \boldsymbol{\epsilon}^{\mathrm{T}}\boldsymbol{\epsilon} = 1$ and $\eta \geq 0$ for $\vartheta \in [-\pi, \pi]$.

The rotation matrix corresponding to a given set of Euler parameters is

$$\boldsymbol{R}(\eta, \boldsymbol{\epsilon}) = (\eta^2 - \boldsymbol{\epsilon}^{\mathrm{T}}\boldsymbol{\epsilon})\boldsymbol{I} + 2\boldsymbol{\epsilon}\boldsymbol{\epsilon}^{\mathrm{T}} - 2\eta\boldsymbol{S}(\boldsymbol{\epsilon}). \tag{30}$$

The relationship between the time derivative of the Euler parameters and the body angular velocity $\boldsymbol{\omega}$ is established by the so-called propagation rule:

$$\dot{\eta} = -\frac{1}{2}\boldsymbol{\epsilon}^{\mathrm{T}}\boldsymbol{\omega}\dot{\boldsymbol{\epsilon}}, \qquad \frac{1}{2}\boldsymbol{E}(\eta, \boldsymbol{\epsilon})\boldsymbol{\omega} \tag{31}$$

with

$$\boldsymbol{E}(\eta, \boldsymbol{\epsilon}) = \eta\boldsymbol{I} - \boldsymbol{S}(\boldsymbol{\epsilon}). \tag{32}$$

Consider now two frames, conventionally labeled 1 and 2. Let $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ respectively denote the rotation matrices expressing the orientation of the two frames with respect to the base frame. Then, the mutual orientation between the two frames can be described by the rotation matrix

$$^{1}\boldsymbol{R}_2 = \boldsymbol{R}_1^{\mathrm{T}}\boldsymbol{R}_2. \tag{33}$$

As usual, a superscript denotes the frame to which a quantity (vector or matrix) is referred; the superscript is dropped whenever a quantity is referred to the base frame. The Euler parameters describing the mutual orientation can be either extracted directly from $^{1}\boldsymbol{R}_2$ or computed by composition of the unit quaternions $\mathcal{Q}_1^{-1} = \{\eta_1, -\boldsymbol{\epsilon}_1\}$ and $\mathcal{Q}_2 = \{\eta_2, \boldsymbol{\epsilon}_2\}$ that can be extracted from $\boldsymbol{R}_1^{\mathrm{T}}$ and $\boldsymbol{R}_2$, respectively, i.e.

$$\eta_{21} = \eta_1\eta_2 + \boldsymbol{\epsilon}_1^{\mathrm{T}}\boldsymbol{\epsilon}_2 \tag{34}$$

$$^{1}\boldsymbol{\epsilon}_{21} = \eta_1\boldsymbol{\epsilon}_2 - \eta_2\boldsymbol{\epsilon}_1 - \boldsymbol{S}(\boldsymbol{\epsilon}_1)\boldsymbol{\epsilon}_2 \tag{35}$$

where the double subscript denotes that a mutual orientation is of concern. It can be shown that the unit quaternion $\mathcal{Q}_{21} = \{\eta_{21}, {}^{1}\boldsymbol{\epsilon}_{21}\}$ can be obtained via quaternion product (composition) $\mathcal{Q}_{21} = \mathcal{Q}_1^{-1} * \mathcal{Q}_2$, where $*$ denotes the quaternion product operator [10]. The vector part $^{1}\boldsymbol{\epsilon}_{21}$ has been referred to frame 1, but it is easy to prove that $^{1}\boldsymbol{\epsilon}_{21} = {}^{2}\boldsymbol{\epsilon}_{21}$. Note that $\{\eta_{21}, {}^{1}\boldsymbol{\epsilon}_{21}\}$ and $\{-\eta_{21}, -{}^{1}\boldsymbol{\epsilon}_{21}\}$ represent the same orientation; also, Σ_1 is aligned with Σ_2 as long as $\eta_{21} = \pm 1$ and $^{1}\boldsymbol{\epsilon}_{21} = \boldsymbol{0}$. Let $\Delta^{1}\boldsymbol{\omega}_{21} = {}^{1}\boldsymbol{\omega}_2 - {}^{1}\boldsymbol{\omega}_1 = \boldsymbol{R}_1^{\mathrm{T}}(\boldsymbol{\omega}_2 - \boldsymbol{\omega}_1)$ be the angular velocity of frame 2 relative to frame 1, which has been referred to frame 1; the operator Δ has been introduced to denote that a vector difference has been taken. Accordingly, the differential kinematics relationship corresponding to (31) becomes

$$\dot{\eta}_{21} = -\frac{1}{2}{}^{1}\boldsymbol{\epsilon}_{21}^{\mathrm{T}}\Delta^{1}\boldsymbol{\omega}_{21}, \quad {}^{1}\dot{\boldsymbol{\epsilon}}_{21} = \frac{1}{2}\boldsymbol{E}(\eta_{21}, {}^{1}\boldsymbol{\epsilon}_{21})\Delta^{1}\boldsymbol{\omega}_{21} \tag{36}$$

with $\boldsymbol{E}$ defined as in (32).