

Enhancing Detectability of Bifurcations in DC-DC Converters by Stochastic Resonance

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Abstract

Noise-driven systems tend to display special features in their power spectra when approaching bifurcation or instability. These features, called noisy precursors, can be of significant use since they can serve as a warning signal for the impending loss of stability. Another phenomenon that has received significant attention in the mathematical physics and biology literature is that of stochastic resonance. In stochastic resonance, the signal-to-noise ratio (SNR) is nonmonotonic with respect to noise level. This cannot occur in linear systems. Most past studies of stochastic resonance involve bistable systems with a weak external periodic forcing. However, recently the same basic phenomenon has been observed in autonomous systems operating along a limit cycle. In any case, the combination of precursors and stochastic resonance provides a tool for instability monitoring. In this paper, these observations are applied to a discrete-time DC-DC converter model. Precursors of period doubling instability are shown to occur. Also, stochastic resonance and its impact on precursor strength is demonstrated. The calculations show how the noise level should be selected to enhance detectability of the impending bifurcation.

1 Introduction

In many complex uncertain systems, it is very difficult to delineate stability boundaries in advance for use in system operation. When such systems need to be operated in stressed high performance conditions, the danger of loss of stability becomes significant. In such circumstances, availability of an on-line stability monitoring system could significantly enhance system operability and reliability. Due to model uncertainty, the monitoring system should not be model-based, but rather nonparametric in nature. One such tool is provided by so-called “noisy precursors” of dynamical instabilities [1, 2]. Noisy precursors are features in the power spectrum of a measured output that occur when a noise-driven system approaches an instability as a pa-

rameter is varied. To employ this concept, one can take advantage of naturally occurring noise or introduce an exogenous noise signal at one point in the system and monitor its effects elsewhere in the system.

Noisy precursors have been applied in electronic circuits [3], lasers [4], and compressors [5]. In this paper, noisy precursors are applied to detect impending loss of stability in power electronic systems. Specifically, the paper focuses on detecting impending loss of stability in current controlled DC-DC boost converters. The mechanism for loss of stability in this system is a period doubling bifurcation.

In addition to studying precursors for loss of stability in DC-DC boost converters, the paper also considers the relationship between precursors in this application and stochastic resonance [6, 7, 8], a phenomenon that has received significant attention in the mathematical physics, biology, and electrical engineering literature. In stochastic resonance, the signal-to-noise ratio (SNR) is nonmonotonic with respect to noise level. This cannot occur in linear systems. Most past studies of stochastic resonance involve bistable systems with a weak external periodic forcing. However, recently the same basic phenomenon has been observed in autonomous systems operating along a limit cycle. In any case, the combination of precursors and stochastic resonance provides a tool for instability monitoring. In this paper, these observations are applied to a discrete-time DC-DC converter model. Precursors of period doubling instability are shown to occur. Also, stochastic resonance and its impact on precursor strength is demonstrated. The calculations show how the noise level should be selected to enhance detectability of the impending bifurcation.

2 Precursors and Stochastic Resonance

To illustrate precursors for period doubling bifurcation and stochastic resonance in a familiar context, consider

the following noise-driven logistic map [9]:

$$x_{n+1} = \rho x_n(1 - x_n) + w(n) \quad (1)$$

Here, $x(n)$ is the (scalar) state, ρ is the bifurcation parameter, and $w(n)$ is a noise process. Figure 1 shows the power spectrum for the noise-free logistic map for the parameter value $\rho = 2.91$. Figure 2 shows the power spectrum with noise for the case $\rho = 2.91$. Note the peak at frequency 0.5. In Figure 3, the peak has increased in height as the parameter has approached the critical value for instability; specifically, $\rho = 2.97$ in this figure. The bifurcation for the logistic map occurs for $\rho = 3.0$.

Regarding stochastic resonance for the logistic map, the signal-to-noise (SNR) ratio can be defined [9] as H/W , where H is the height of the power spectrum peak associated with impending instability, and W is its width. In [9], the SNR is numerically shown to first grow and then decrease as the noise level is steadily increased. This is also known as *coherence resonance*, to emphasize the fact that its occurrence does not require an external periodic force, as opposed to traditional stochastic resonance [10]. Coherence resonance implies that the external noise level can be chosen optimally to enhance detectability of the impending instability.

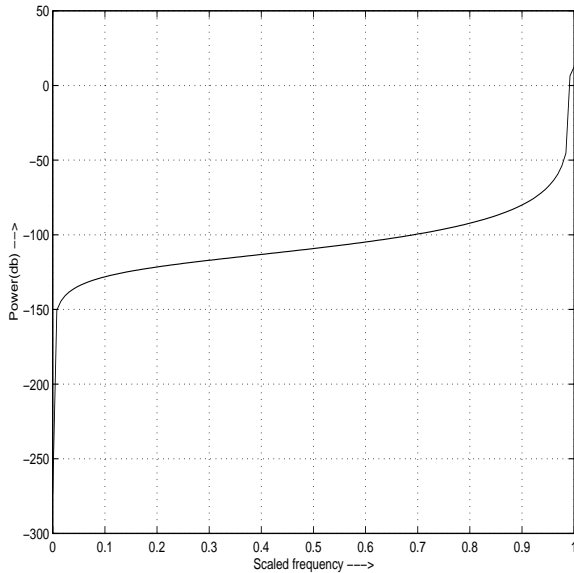


Figure 1: Power spectrum for logistic map without noise; $\rho = 2.91$

3 DC-DC Converter Model and Instability Precursors

Consider the following power electronic circuit. The circuit in Fig. 4 consists of a peak current controlled boost converter feeding a resistive load. The converter is operating in continuous current mode (CCM). In this

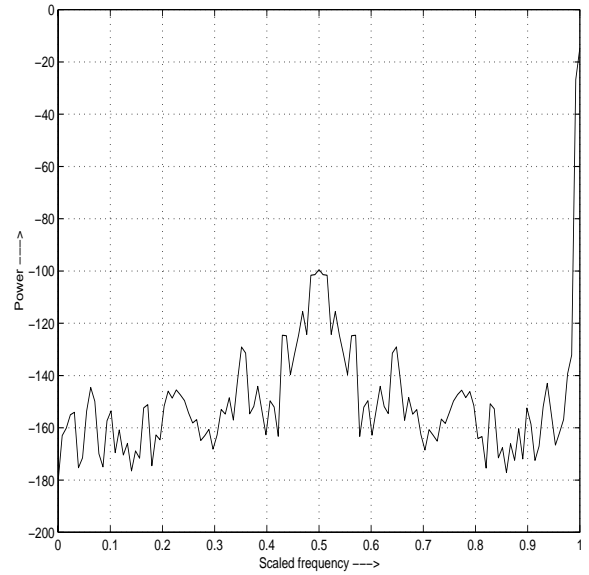


Figure 2: Power spectrum for noisy logistic map; $\rho = 2.91$

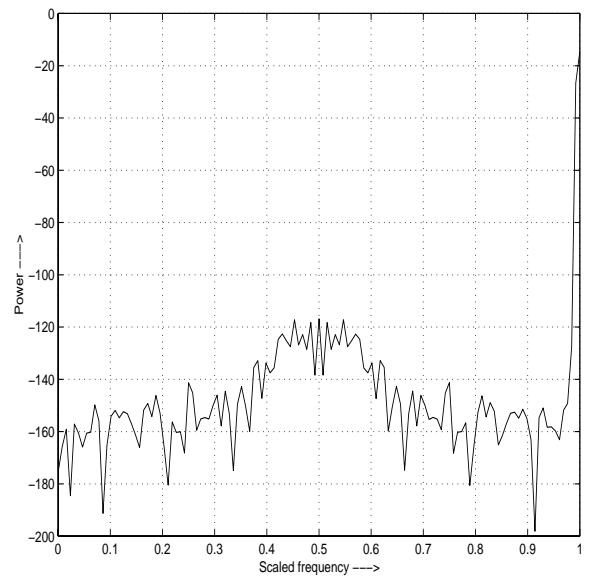


Figure 3: Power spectrum for noisy logistic map; $\rho = 2.97$

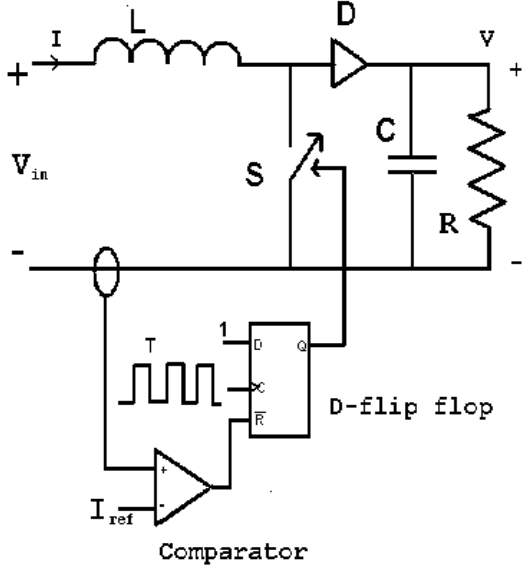


Figure 4: Power electronic circuit

control logic, the switch is turned on by clock pulses that are spaced T seconds apart. When the switch is closed, the inductor current increases till it reaches the specified reference value I_{ref} . The switch opens when $i = I_{ref}$. Any clock pulse arriving during the *on* period is ignored. Once the switch has opened, the next clock pulse causes it to close.

The evolution of the state variables i and v_c during the *on* and *off* periods are described by linear differential equations:

$$\text{'On' period: } \frac{di}{dt} = \frac{V_{in}}{L}, \quad \frac{dv_c}{dt} = -\frac{v_c}{CR} \quad (2)$$

$$\text{'Off' period: } \frac{di}{dt} = \frac{V_{in}}{L} - \frac{v_c}{L}, \quad \frac{dv_c}{dt} = \frac{i}{C} - \frac{v_c}{CR} \quad (3)$$

We normalize the variables using the following definitions (assuming I_{ref} , R , L and C are not zero) :

$$\begin{aligned} I &= \frac{i}{I_{ref}} \\ V &= \frac{v_c}{I_{ref}R} \\ \zeta &= \frac{1}{2R} \sqrt{\frac{L}{C}} \\ T_0 &= \frac{T}{\sqrt{LC}} \\ \rho &= \frac{V_{in}}{I_{ref}R} \\ \omega &= \sqrt{1 - \zeta^2} \\ \gamma &= \frac{T}{RC} \end{aligned}$$

We adopt sampled data modeling in the form of a ‘‘stroboscopic map,’’ where the state variables are observed in synchronism with the clock. Let the normalized state variables at a clock instant be I_n , V_n , and those at the next clock instant be I_{n+1} , V_{n+1} . There are two ways in which the state can move from one clock instant to the next. A clock pulse may arrive before the current reaches I_{ref} . In that case the map can be obtained by solving the equations for the *on*-state with V_n and I_n as the initial conditions.

$$I_{n+1} = \frac{\rho T_0}{2\zeta} + I_n \quad (4)$$

$$V_{n+1} = V_n e^{-\gamma} \quad (5)$$

If the inductor current reaches I_{ref} before the arrival of the next clock pulse, the map would include an *on* and an *off* period. In this case the *on*-period t_{on} is obtained by using the final value $I = 1$ and the states at the next clock instant are found by solving the *off*-time equations for the period $t_{off} = (T - t_{on})$. This gives the map

$$I_{n+1} = e^{-\zeta\tau_n'} \left(C_1 \cos \omega\tau_n' + C_2 \sin \omega\tau_n' \right) + \rho \quad (6)$$

$$V_{n+1} = \rho - e^{-\zeta\tau_n'} \left(K_1 \cos \omega\tau_n' + K_2 \sin \omega\tau_n' \right) \quad (7)$$

where

$$\begin{aligned} \tau_n &= \frac{2\zeta}{\rho} (1 - I_n) \\ \tau_n' &= T_0 - \tau_n \\ V_f &= V_n e^{-2\zeta\tau_n} \\ C_1 &= 1 - \rho \\ C_2 &= \frac{\frac{\rho - V_f}{2\zeta} + \zeta C_1}{\omega} \\ K_1 &= \rho - V_f \\ K_2 &= \frac{2\zeta}{\omega} \left(\frac{V_f - \rho}{2} - C_1 \right) \end{aligned}$$

Also, the borderline between the two cases is given by the value of I_n for which the inductor current reaches I_{ref} exactly at the arrival of the next clock pulse,

$$I_{border} = 1 - \frac{\rho T_0}{2\zeta} \quad (8)$$

Thus (4) and (5) apply if $I_n \leq I_{border}$, while (6) and (7) apply for $I_n \geq I_{border}$.

In the expressions above we have assumed an oscillatory solution ($\zeta < 1$) of the second order differential equation in the *off* period. The normal design procedures, based on obtaining continuous current mode and low output

voltage ripple, usually give parameter values which satisfy this condition.

In Figure 5, the power spectrum of the inductor current is depicted in the absence of noise for parameter values near instability. The power spectrum for the case in which noise is present is depicted in Figure 6.

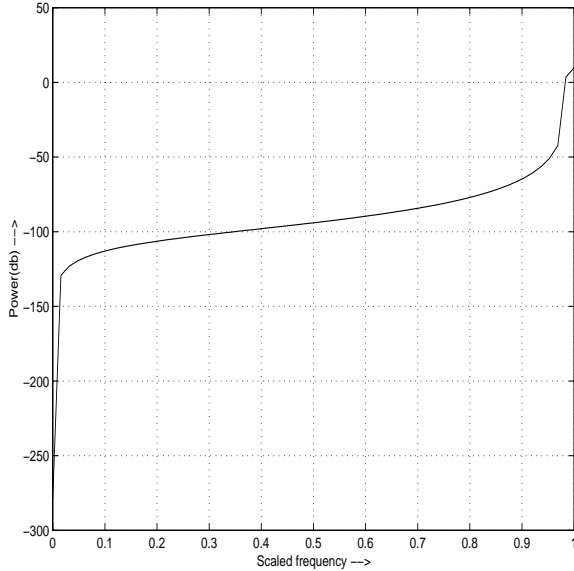


Figure 5: Power spectrum of the inductor current without noise

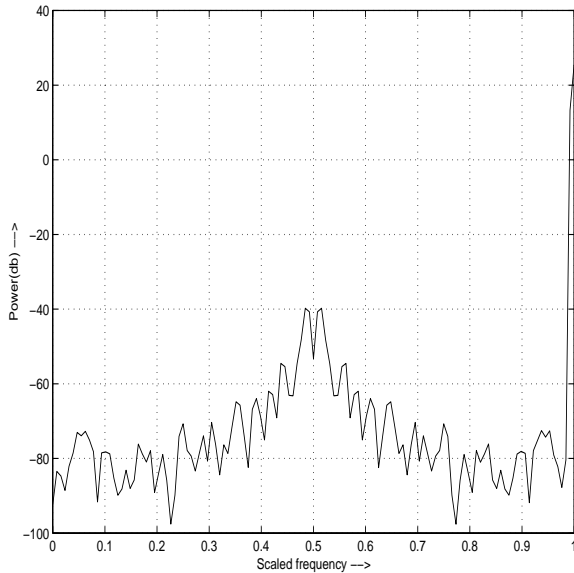


Figure 6: Power spectrum of the inductor current with noise

In Figure 7, the signal-to-noise ratio of the peak predicting instability is plotted versus the input noise level for the DC-DC converter.

Further numerical work has been done to examine the role of stochastic resonance for the circuit under study. The results show that there is an optimal range for the

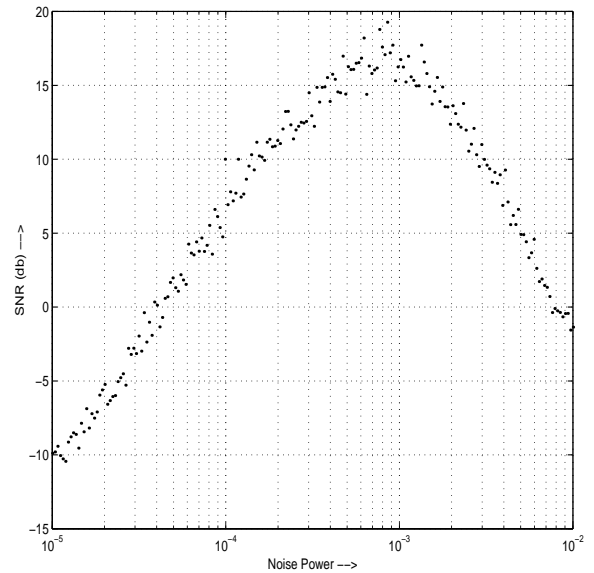


Figure 7: Signal-to-noise ratio at precursor peak

noise level that broadens as the critical parameter value (i.e., instability) is approached. Thus, to detect impending instability early, it is important to know what range of noise excitation should be used.

Acknowledgments

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