L_2-L_2 and L_2-L_∞ Output Feedback Control of Time-Delayed LPV Systems

Kan Tan and Karolos M. Grigoriadis[1]
Department of Mechanical Engineering
University of Houston
Houston, TX 77204

Abstract

We examine the analysis and output feedback synthesis problems for linear parameter-varying (LPV) systems with parameter-varying time delays. It is assumed that the state-space data and the time delays are dependent on parameters that are measurable in real-time and vary in a compact set with bounded variation rates. We explore the stability, the L_2 induced norm performance and the L_2 to L_∞ gain performance of these systems using parameter-dependent Lyapunov-Krasovskii functionals. In addition, the designs of parameter-dependent dynamic output feedback controllers that guarantee stability and desired induced norm performance are examined. Both analysis and synthesis conditions are formulated in terms of linear matrix inequalities (LMIs) that can be solved via efficient interior-point algorithms.

Keywords: Time-delayed systems, Linear Parameter Varying systems, Linear Matrix Inequalities.

Introduction

Control of systems with time delays is a subject of significant practical and theoretical importance, and has been examined extensively in the controls literature using both frequency domain and time domain methods [5][6]. Most work has been concentrated on the stability and control of systems with fixed delays or the robust stability and control of systems with uncertain delays. However, in many engineering applications, the time delays are variable and are known functions of system operating conditions or system parameters that can be measured in real-time. For example, the transport delay in an internal combustion engine is a known function of the engine speed. Similarly, parameter-dependent time delays often appear in many manufacturing and chemical processes, biomedical systems and robotic systems where changes in the system dynamics result in variable delay times. Motivated by the linear parameter–varying (LPV) control theory (e.g., see [1][2][4][9], and the references therein), the stabilization and the dynamic output feedback control synthesis of such LPV systems with parameter-dependent time delays is examined in this work.

In the proposed LPV framework, it is assumed that the state-space system matrices and the time delays are functions of time-varying system parameters that are measured in real-time. We seek to design parameter-varying controllers to stabilize the time-delayed LPV system and to provide disturbance attenuation measured in terms of the L_2 induced norm and the L_2 to L_∞ gain of the system. The proposed approach utilizes parameter-dependent Lyapunov-Krasovskii functionals to obtain sufficient conditions for stabilization, the L_2 induced norm performance and the L_2 to L_∞ gain performance. Using a basis expansion and parameter gridding these conditions are formulated as finite-dimensional linear matrix inequalities (LMIs) that can be solved efficiently using recently developed interior-point optimization algorithms [3]. The present work extends previous work on the state feedback control of time-delayed LPV systems [8] in three directions: A main contribution is the extension to the output feedback case. Also, the L_2 to L_∞ gain delayed LPV stabilization and control is addressed, that has not been examined before in the literature. In addition, a less conservative formulation is developed compared to [8]. In this new formulation both parameter matrices of the Lyapunov-Krasovskii functional are parameter dependent to provide a wider class of Lyapunov functions and reduce conservatism. The proposed results provide delay-rate dependent stabilization, the L_2 gain and L_2-to-L_∞ gain output feedback synthesis conditions for delay LPV systems.

The notation to be used is as follows: Given a real matrix B, the orthogonal complement $B^\perp$ is defined as the (possibly non-unique) matrix with maximum row rank that satisfies $B^\perp B = 0$ and $B^\perp B^{\perp\prime} > 0$. In a symmetric block matrix, the expression $(*)$ will be used to denote the submatrices that lie above the diagonal.

Output Feedback Stabilization

Consider the following time-delayed LPV system

[1]Corresponding author. Tel: 713-743-4387, Fax: 713-743-4503, Email: karolos@uh.edu

$$\begin{array}{rcl} \dot{x}(t) & = & A(p(t))x(t) + A_h(p(t))x(t-h(p(t)) \\ & & +B(p(t))u(t) \\ y(t) & = & C(p(t))x(t) + C_h(p(t))x(t-h(p(t))) \end{array} \quad (1)$$

with initial condition

$$x(t) = \phi(t), \qquad t \in [-h(p(0), 0]. \quad (2)$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^{n_u}$ is the input vector and $y(t) \in \mathbf{R}^{n_y}$ is the measurement vector. The state-space matrices $A(p), A_h(p), B(p), C(p)$ and $C_h(p)$ are assumed to be bounded continuous functions of a time-varying parameter vector $p(t) \in F_{\mathcal{P}}^v$. The set $F_{\mathcal{P}}^v$ is the set of allowable parameter trajectories

$$F_{\mathcal{P}}^v = \{p : p(t) \in \mathcal{P}, |\dot{p}_i| \le v_i, i = 1, 2, \cdots, s, \forall t \in \mathbf{R}_+\}$$

where $\mathcal{P}$ is a compact set of $\mathbf{R}^s$, $\{v_i\}_{i=1}^s$ are nonnegative numbers and $v = [v_1 \ \cdots \ v_s]'$. That is, we consider bounded parameter trajectories with bounded variation rates. The delay term $h(p(t))$ is assumed to be a differentiable function such that

$$0 \le h(t) \le H < \infty, \quad \dot{h}(t) \le \tau < 1, \ \forall t \in \mathbf{R}_+$$

The following result provides a sufficient condition for the asymptotic stability of the unforced time-delayed LPV system.

Lemma 1 *Consider the unforced time-delayed LPV system*

$$\dot{x}(t) = A(p(t))x(t) + A_h(p(t))x(t-h(p(t))) \quad (3)$$

If there exist matrix functions $P(p) > 0$ and $Q(p) > 0$ such that

$$\begin{bmatrix} \left\{ \begin{array}{c} A(p)'P(p)+P(p)A(p) \\ +\sum_{i=1}^{s}\left(\dot{p}_i \frac{\partial P}{\partial p_i}\right)+Q(p) \end{array} \right\} & (*) \\ A_h'(p)P(p) & -\left[1-\sum_{i=1}^{s}\left(\dot{p}_i \frac{\partial h}{\partial p_i}\right)\right]Q(r) \end{bmatrix} < 0 \quad (4)$$

for all $p, r \in \mathcal{P}, |\dot{p}_i| \le v_i$, then the system (3) is asymptotically stable.

Remark 1 *Since $|\dot{p}_i| \le v_i$, the above inequality involving $\dot{p}_i$ will be satisfied if the following inequalities hold for all $p, r \in \mathcal{P}$*

$$\begin{bmatrix} \left\{ \begin{array}{c} A(p)'P(p)+P(p)A(p) \\ +\sum_{i=1}^{s}\pm\left(v_i \frac{\partial P}{\partial p_i}\right)+Q(p) \end{array} \right\} & (*) \\ A_h'(p)P(p) & -\left[1-\sum_{i=1}^{s}\pm\left(v_i \frac{\partial h}{\partial p_i}\right)\right]Q(r) \end{bmatrix} < 0 \quad (5)$$

The notation $\sum_{i=1}^{s} \pm (\cdot)$ in (5) is used to indicate that every combination of $+(\cdot)$ and $-(\cdot)$ should be included in the inequality. That is, the inequality (5) actually represents 2^s different inequalities that correspond to the 2^s different combinations in the summation. Later on, the other inequalities involving $\dot{p}_i$ can be handled in the same way.

The output feedback stabilization problem is to design a dynamic output feedback LPV controller of the following form

$$\begin{array}{ll} \dot{x}_k(t) & = A_k(p(t))x_k(t) + B_k(p(t))y(t) \\ u(t) & = C_k(p(t))x_k(t) + D_k(p(t))y(t) \end{array} \quad (6)$$

that asymptotically stabilizes the closed-loop system.

The following lemma is needed to provide conditions for the stabilization problem.

Lemma 2 *[7] Consider given matrices $U \in \mathbf{R}^{n\times m}, V \in \mathbf{R}^{k\times n}$ and $W \in \mathbf{S}^{n\times n}$ where W is symmetric and $rank(U) = m$ and $rank(V) = k$. There exists a matrix $G \in \mathbf{R}^{m\times k}$ satisfying the following matrix inequality*

$$UGV + (UGV)' + W < 0, \quad (7)$$

if and only if the following two conditions hold

$$\begin{array}{rcl} U^{\perp}WU^{\perp\prime} & < & 0, \\ V'^{\perp}WV'^{\perp\prime} & < & 0. \end{array}$$

If these conditions are satisfied then one such solution G is given by

$$G = -R^{-1}U'\Phi V'(V'\Phi V')^{-1}, \quad (8)$$

and R is an arbitrary positive definite matrix such that

$$\Phi = (UR^{-1}U' - W)^{-1} > 0.$$

The following result provides sufficient conditions for the output feedback time-delayed LPV stabilization problem.

Theorem 3 *Consider the time-delayed LPV system (1) with initial data (2). If there exist matrix functions $X(p) > 0$, $Y(p) > 0$ and $S(p) > 0$ such that*

$$\begin{bmatrix} B^{\perp}(p) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \left\{ \begin{array}{c} A(p)X(p) + X(p)A'(p) \\ + \sum_{i=1}^{s} \dot{p}_i \frac{\partial X}{\partial p_i} + S(p) \\ + \left[1 - \sum_{i=1}^{s} \dot{p}_i \frac{\partial h}{\partial p_i}\right]^{-1} A_h(p)A_h'(p) \end{array} \right\} & X(p) \\ X(p) & -I \end{bmatrix} \begin{bmatrix} B^{\perp}(p) & 0 \\ 0 & I \end{bmatrix}' < 0 \quad (9)$$

$$\begin{bmatrix} C'(p) \\ C_h'(p) \end{bmatrix}^{\perp} \begin{bmatrix} \left\{ \begin{array}{c} A(p)'Y(p) + Y(p)A(p) \\ + \sum_{i=1}^{s} \dot{p}_i \frac{\partial Y}{\partial p_i} + I \end{array} \right\} & X(p) \\ Y(p)A_h(p) & -\left[1 - \sum_{i=1}^{s} \dot{p}_i \frac{\partial h}{\partial p_i}\right] I \end{bmatrix} \begin{bmatrix} C'(p) \\ C_h'(p) \end{bmatrix}^{\perp\prime} < 0 \quad (10)$$

$$\begin{bmatrix} X(p) & I \\ I & Y(p) \end{bmatrix} > 0 \quad (11)$$

for all $p \in \mathcal{P}, |\dot{p}_i| \leq v_i$, then there exists an output feedback controller (6) that asymptotically stabilizes the time-delayed LPV system. One such controller

$$G(p) = \begin{bmatrix} D_k(p) & C_k(p) \\ B_k(p) & A_k(p) \end{bmatrix}$$

is given by the explicit solution (8) of the matrix inequality

$$U(p)G(p)V(p) + V'(p)G'(p)U'(p) + W(p) < 0 \quad (12)$$

with

$$W(p)=\begin{bmatrix} \left\{ \begin{array}{c} \bar{A}'(p)P(p) + P(p)\bar{A}(p) \\ + \sum_{i=1}^{s} \dot{p}_i \frac{\partial P}{\partial p_i} + Q(p) \end{array} \right\} & P\bar{A}_h(p) \\ \bar{A}_h'(p)P & -(1 - \sum_{i=1}^{s} \dot{p}_i \frac{\partial h}{\partial p_i})I \end{bmatrix}$$

$$U(p) = \begin{bmatrix} P(p)\bar{B}(p) \\ 0 \end{bmatrix}, \qquad V(p)=\begin{bmatrix} \bar{M}(p) & \bar{L}(p) \end{bmatrix}$$

where $\bar{A}(p)$, $\bar{A}_h(p)$, $\bar{B}(p)$, $\bar{M}(p)$, $\bar{M}_h(p)$ are defined as follows

$$\bar{A}(p)=\begin{bmatrix} A(p) & 0 \\ 0 & 0 \end{bmatrix} \quad \bar{A}_h(p)=\begin{bmatrix} A_h(p) & 0 \\ 0 & 0 \end{bmatrix} \quad \bar{B}(p)=\begin{bmatrix} B(p) & 0 \\ 0 & I \end{bmatrix}$$

$$\bar{M}(p)=\begin{bmatrix} C(p) & 0 \\ 0 & I \end{bmatrix} \quad \bar{L}(p)=\begin{bmatrix} C_h(p) & 0 \\ 0 & 0 \end{bmatrix}$$

and $P(p) = R(p)^{-1}$, where $R(p) = \begin{bmatrix} Y(p) & I \\ I & R_{22}(p) \end{bmatrix}$, with

$$R_{22}(p) = X(p) - Y(p)^{-1}$$

and $Q(p) = \begin{bmatrix} I & 0 \\ 0 & Q_{22}(p) \end{bmatrix}$, with $Q_{22}(p) = S(p)$.

Proof. Omitted.

Output Feedback L_2 Induced Norm Control

Consider now the following time-delayed LPV system with an exogenous input $w(t) \in \mathbf{R}^{n_w}$ and an output vector $z(t) \in \mathbf{R}^{n_z}$

$$\begin{aligned} \dot{x}(t) &= A(p(t))x(t) + A_h(p(t))x(t - h(p(t))) \\ &\quad +B_1(p(t))w(t) + B_2(p(t))u(t) \\ z(t) &= C_1(p(t))x(t) + C_{1h}(p(t))x(t - h(p(t))) \\ &\quad +D_{11}(p(t))w(t) + D_{12}(p(t))u(t) \\ y(t) &= C_2(p(t))x(t) + C_{2h}(p(t))x(t - h(p(t))) \\ &\quad +D_{21}(p(t))w(t) \end{aligned} \quad (13)$$

and initial data

$$x(t) = 0, \qquad t \in [-h(0), 0] \quad (14)$$

The following result provides sufficient analysis conditions that the for the time delayed LPV system has L_2 induced norm less than a given bound γ, i.e.,

$$\sup_{||w||_2 \neq 0} \frac{||z||_2}{||w||_2} \leq \gamma.$$

Lemma 4 *Consider the following uncontrolled time-delayed LPV system with initial data (14)*

$$\begin{aligned} \dot{x}(t) &= A(p(t))x(t) + A_h(p(t))x(t - h(p(t))) \\ &\quad +B(p(t))w(t) \\ z(t) &= C(p(t))x(t) + C_h(p(t))x(t - h(p(t))) \\ &\quad +D(p(t))w(t) \end{aligned} \quad (15)$$

If there exist matrix functions $P(p) > 0$, $Q(p) > 0$ such that

$$\begin{bmatrix} \left\{ \begin{array}{c} A'(p)P(p) + P(p)A(p) \\ + \sum_{i=1}^{s} \dot{p}_i \frac{\partial P}{\partial p_i} + Q(p) \end{array} \right\} & P(p)A_h(p) & P(p)B(p) & C'(p) \\ A_h'(p)P(p) & -\left[1 - \sum_{i=1}^{s} \dot{p}_i \frac{\partial h}{\partial p_i}\right] Q(r) & 0 & C_h'(p) \\ B'(p)P(p) & 0 & -\gamma I & D'(p) \\ C(p) & C_h(p) & D(p) & -\gamma I \end{bmatrix} < 0 \quad (16)$$

for all $p, r \in \mathcal{P}, |\dot{p}_i| \leq v_i$, then the system (15) is asymptotically stable with L_2 induced norm less than γ.

Proof: Omitted.

The output feedback L_2 induced norm control synthesis problem is to design a dynamic output feedback controller (6) which stabilizes the system (13)-(14) and makes the closed-loop system have an L_2 induced norm less than a given bound γ. The following result provides sufficient solvability conditions for this problem.

Theorem 5 *Consider the LPV time-delayed system (13). If there exist matrix functions $X(p) > 0$, $Y(p) > 0$, $S(p) > 0$ such that*

$$K\begin{bmatrix} \begin{bmatrix} \left\{ \begin{array}{c} A(p)X(p)+X(p)A'(p) \\ + \sum_{i=1}^{s} \dot{p}_i \frac{\partial X}{\partial p_i} + S(p) \end{array} \right\} & X(p)C_1'(p) \\ C_1(p)X(p) & -\gamma I \end{bmatrix} + M(p) & (\star) \\ \begin{bmatrix} X(p) & 0 \end{bmatrix} & -I \end{bmatrix} K' < 0 \quad (17)$$

$$L\begin{bmatrix} \left\{ \begin{array}{c} A'(p)Y(p)+Y(p)A(p) \\ + \sum_{i=1}^{s} \dot{p}_i \frac{\partial Y}{\partial p_i} + I \end{array} \right\} & (\star) & (\star) & (\star) \\ A_h'(p)Y(p) & \left[\sum_{i=1}^{s} \dot{p}_i \frac{\partial h}{\partial p_i} - 1\right] I & (\star) & (\star) \\ B_1'(p) & 0 & -\gamma I & (\star) \\ C_1(p) & C_{1h}(p) & D_{11}(p) & -\gamma I \end{bmatrix} L' < 0 \quad (18)$$

$$\begin{bmatrix} X(p) & I \\ I & Y(p) \end{bmatrix} > 0 \quad (19)$$

where

$$M = \begin{bmatrix} A_h(p) & B_1(p) \\ C_{1h}(p) & D_{11}(p) \end{bmatrix} \times \begin{bmatrix} \left(1 - \sum_{i=1}^{s} \dot{p}_i \frac{\partial h}{\partial p_i}\right)^{-1} I & 0 \\ 0 & \gamma^{-1} I \end{bmatrix} \begin{bmatrix} A_h(p) & B_1(p) \\ C_{1h}(p) & D_{11}(p) \end{bmatrix}'$$

$$K = \begin{bmatrix} \begin{bmatrix} B_2(p) \\ D_{12}(p) \end{bmatrix}^{\perp} & 0 \\ 0 & I \end{bmatrix}$$

$$L = \begin{bmatrix} \begin{bmatrix} C_2'(p) \\ C_{2h}'(p) \\ D_{21}'(p) \end{bmatrix}^{\perp} & 0 \\ 0 & I \end{bmatrix}$$

for all $p \in \mathcal{P}$, $|\dot{p}_i| \leq v_i$, then there exists a dynamic output feedback controller (6) such that the closed-loop system is asymptotically stable with L_2 induced norm less than γ. One such controller

$$G(p) = \begin{bmatrix} D_k(p) & C_k(p) \\ B_k(p) & A_k(p) \end{bmatrix}$$

is given by the explicit solution (8) of the following matrix inequality

$$U(p)G(p)V(p) + V'(p)G'(p)U'(p) + W(p) < 0 \quad (20)$$

with

$$W(p) = \begin{bmatrix} \left\{ \begin{matrix} \bar{A}'(p)P(p) + P(p)\bar{A}(p) \\ + \sum_{i=1}^{s} \dot{p}_i \frac{\partial P}{\partial p_i} + Q(p) \end{matrix} \right\} & P(p)\bar{A}_h(p) & P(p)\bar{D}(p) & \bar{C}'(p) \\ \bar{A}_h'(p)P(p) & -\left[1 - \sum_{i=1}^{s} \dot{p}_i \frac{\partial h}{\partial p_i}\right] I & 0 & \bar{C}_h'(p) \\ \bar{D}'(p)P(p) & 0 & -\gamma I & D_{11}'(p) \\ \bar{C}(p) & \bar{C}_h(p) & D_{11}(p) & -\gamma I \end{bmatrix},$$

$$U(p) = \begin{bmatrix} P(p)\bar{B}(p) \\ 0 \\ 0 \\ \bar{H}(p) \end{bmatrix},$$

$$V(p) = \begin{bmatrix} \bar{M}(p) & \bar{L}(p) & \bar{E}(p) & 0 \end{bmatrix},$$

where $\bar{A}(p)$, $\bar{A}_h(p)$, $\bar{B}(p)$, $\bar{C}(p)$, $\bar{C}_h(p)$, $\bar{H}(p)$, $\bar{M}(p)$, $\bar{L}(p)$, $\bar{E}(p)$ denote

$$\bar{A}(p) = \begin{bmatrix} A(p) & 0 \\ 0 & 0 \end{bmatrix} \quad \bar{A}_h(p) = \begin{bmatrix} A_h(p) & 0 \\ 0 & 0 \end{bmatrix}$$

$$\bar{B}(p) = \begin{bmatrix} B_2(p) & 0 \\ 0 & I \end{bmatrix} \quad \bar{D}(p) = \begin{bmatrix} B_1(p) \\ 0 \end{bmatrix}$$

$$\bar{C}(p) = \begin{bmatrix} C_1(p) & 0 \end{bmatrix} \quad \bar{C}_h(p) = \begin{bmatrix} C_{1h}(p) & 0 \end{bmatrix}$$

$$\bar{H}(p) = \begin{bmatrix} D_{12}(p) & 0 \end{bmatrix} \quad \bar{M}(p) = \begin{bmatrix} C_2(p) & 0 \\ 0 & I \end{bmatrix}$$

$$\bar{L}(p) = \begin{bmatrix} C_{2h}(p) & 0 \\ 0 & 0 \end{bmatrix} \quad \bar{E}(p) = \begin{bmatrix} D_{21}(p) \\ 0 \end{bmatrix}$$

and $P(p) = R(p)^{-1}$, where $R(p) = \begin{bmatrix} Y(p) & I \\ I & R_{22}(p) \end{bmatrix}$, with

$$R_{22}(p) = X(p) - Y(p)^{-1}$$

and $Q(p) = \begin{bmatrix} I & 0 \\ 0 & Q_{22}(p) \end{bmatrix}$, with $Q_{22}(p) = S(p)$.

Proof. Omitted.

Output Feedback L_2 to L_∞ gain Control

Consider again the time-delayed LPV system (13)-(14) with $D_{11} = 0$ and $D_{21} = 0$. The output feedback L_2 to L_∞ gain control problem is to design a dynamic output feedback controller (6) which stabilizes the system (13)-(14) and makes the closed-loop system have L_2 to L_∞ gain less than or equal to a given bound γ, i.e.,

$$\sup_{||w||_2 \neq 0} \frac{||z||_\infty}{||w||_2} \leq \gamma.$$

In the following, we will assume for simplicity the following normalized conditions

$$D_{21}(p) \begin{bmatrix} B_1'(p) & D_{21}'(p) \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix},$$
$$D_{12}'(p) \begin{bmatrix} C_1(p) & D_{12}(p) \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}.$$

The following result provides conditions for the L_2 to L_∞ gain analysis problem.

Lemma 6 *Consider the following uncontrolled time-delayed LPV system with initial data (14)*

$$\begin{aligned} \dot{x}(t) &= A(p(t))x(t) + A_h(p(t))x(t-h(p(t))) + B(p(t))w(t) \\ z(t) &= C(p(t))x(t) + C_h(p(t))x(t-h(p(t))) \end{aligned} \quad (21)$$

If there exist matrix functions $P(p) > 0$ and $Q(p) > 0$

such that

$$\begin{bmatrix} \left\{ \begin{array}{c} A'(p)P(p)+P(p)A(p) \\ +\sum_{i=1}^{s}\dot{p}_i\frac{\partial P}{\partial p_i}+Q(p) \end{array}\right\} & P(p)A_h(p) & P(p)B(p) \\ A_h'(p)P(p) & -\left[1-\sum_{i=1}^{s}\dot{p}_i\frac{\partial h}{\partial p_i}\right]Q(r) & 0 \\ B'(p)P(p) & 0 & -I \end{bmatrix} < 0 \quad (22)$$

$$\begin{bmatrix} -\frac{1}{2}\gamma^{-2}I & 0 & C(p) & 0 \\ 0 & -\frac{1}{2}\gamma^{-2}I & 0 & C_h(p) \\ C'(p) & 0 & -P(p) & 0 \\ 0 & C_h'(p) & 0 & -P(r) \end{bmatrix} < 0 \quad (23)$$

for all $p, r \in \mathcal{P}$, $|\dot{p}_i| \le v_i$, then the system is asymptotically stable with L_2 to L_∞ gain less than or equal to γ.

Proof: Omitted.

The corresponding synthesis conditions are given by the following result.

Theorem 7 *Consider the LPV time-delayed system (13) with initial data (14). If there exist matrix functions $X(p) > 0$, $Y(p) > 0$, $S(p) > 0$ and $K(p)$ such that*

$$\begin{bmatrix} \left\{ \begin{array}{c} A(p)X(p)+X(p)A'(p) \\ +\sum_{i=1}^{s}\dot{p}_i\frac{\partial X}{\partial p_i}+S(p) \\ +B_2(p)K(p)+K'(p)B_2(p)+B_1(p)B_1'(p) \end{array}\right\} & (\star) & (\star) \\ A_h'(p) & -\left[1-\sum_{i=1}^{s}\dot{p}_i\frac{\partial h}{\partial p_i}\right]I & (\star) \\ X(p) & 0 & -X(p) \end{bmatrix} < 0 \quad (24)$$

$$\begin{bmatrix} \left\{ \begin{array}{c} Y(p)A(p)+A'(p)Y(p) \\ +\sum_{i=1}^{s}\dot{p}_i\frac{\partial Y}{\partial p_i}+I-C_2'(p)C_2(p) \end{array}\right\} & (\star) & (\star) \\ A_h'(p)Y(p) & -\left[1-\sum_{i=1}^{s}\dot{p}_i\frac{\partial h}{\partial p_i}\right]I & (\star) \\ B_1'(p)Y(p) & 0 & -I \end{bmatrix} < 0 \quad (25)$$

$$\begin{bmatrix} \frac{1}{2}\gamma^{-2}I & (\star) & (\star) \\ X(p)C_1'(p)+K'(p)D_{12}'(p) & X(p) & (\star) \\ C_1'(p) & I & Y(p) \end{bmatrix} > 0 \quad (26)$$

$$\begin{bmatrix} \frac{1}{2}\gamma^{-2}I & (\star) & (\star) \\ X(r)C_{1h}'(p) & X(r) & (\star) \\ C_{1h}'(p) & I & Y(r) \end{bmatrix} > 0 \quad (27)$$

for all $p, r \in \mathcal{P}, |\dot{p}_i| \le v_i$, then there exists a dynamic output feedback controller (6) such that the closed-loop system is asymptotically stable with L_2 to L_∞ gain less than or equal to γ. One such controller is given by

$$\begin{aligned} A_k(p) &= \{Y^{-1}(p)A'(p)+\textstyle\sum_{i=1}^{s}(\dot{p}_i\frac{\partial Y^{-1}}{\partial p_i}) \\ &\quad +(A(p)-B_k(p)C_2(p))X(p) \\ &\quad +[1-\textstyle\sum_{i=1}^{s}(\dot{p}_i\frac{\partial h}{\partial p_i})]^{-1}A_h(p)A_h'(p) \\ &\quad +B_1(p)B_1'(p)\}(X(p)-Y^{-1}(p))^{-1} \\ &\quad +Y^{-1}(p)X(p)+B_2(p)C_k(p) \\ B_k(p) &= Y^{-1}(p)C_2'(p) \\ C_k(p) &= K(p)(X(p)-Y^{-1}(p))^{-1} \\ D_k(p) &= 0. \end{aligned} \quad (28)$$

Proof. Omitted.

Numerical Example

Consider the following time-delayed system

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1+0.2p(t) \\ -2 & -3+0.1p(t) \end{bmatrix} x(t) \\ &+ \begin{bmatrix} 0.2p(t) & 0.1 \\ -2+0.1p(t) & -0.3 \end{bmatrix} x(t-(1+0.5p(t)) \\ &+ \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix} w(t) + \begin{bmatrix} 2p(t) \\ 0.1+0.1p(t) \end{bmatrix} u(t) \\ z(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \end{aligned}$$

We assume that the functional representation of the plant parameter $p(t)$ is not known *a priori*, but it can be measured in real-time, and $p(t) \in [-1, 1]$, $|\dot{p}| \le 1$. The time delay $h(t) = 1+0.5p(t)$ is varying from 0.5 to 1.5 and the condition $\frac{dh}{dt} < 1$ holds. To design an output feedback controller to make the L_2 induced norm from w to z less than a given bound γ, we consider the parameter matrices X, Y and S in Theorem 5 be first order polynomial functions of the plant parameter p, i.e.,

$$X(p)=X_0+pX_1, \quad Y(p)=Y_0+pY_1, \quad S(p)=S_0+pS_1.$$

We grid the parameter space using a 20 points grid. Solving the LMIs in Theorem 5 we get the minimum bound $\gamma = 0.585$ for the following parameter matrices

$$X_0=\begin{bmatrix} 0.3335 & -0.3118 \\ -0.3118 & 1.1612 \end{bmatrix} \quad X_1=\begin{bmatrix} 0.0106 & 0.0025 \\ 0.0025 & -0.1166 \end{bmatrix}$$

$$Y_0=\begin{bmatrix} 7.1397 & 6.2236 \\ 6.2236 & 128.2638 \end{bmatrix} \quad Y_1=\begin{bmatrix} 1.5755 & 1.8350 \\ 1.8350 & 1.6540 \end{bmatrix}$$

$$S_0=\begin{bmatrix} 0.0792 & -0.1865 \\ -0.1865 & 0.8941 \end{bmatrix} \quad S_1=\begin{bmatrix} 0.0614 & -0.0548 \\ -0.0548 & -0.0935 \end{bmatrix}$$

In simulation, we let $p(t) = \sin(t)$. For an initial condition $(x_1(0), x_2(0)) = (2, -1)$ and a unit step disturbance $w(t)$, we simulate the closed-loop behavior of the system using the LPV output-feedback controller given by Theorem 5. The states and control input profile are shown in Figures 1 and 2. Note that both states x_1, x_2 decrease rapidly.

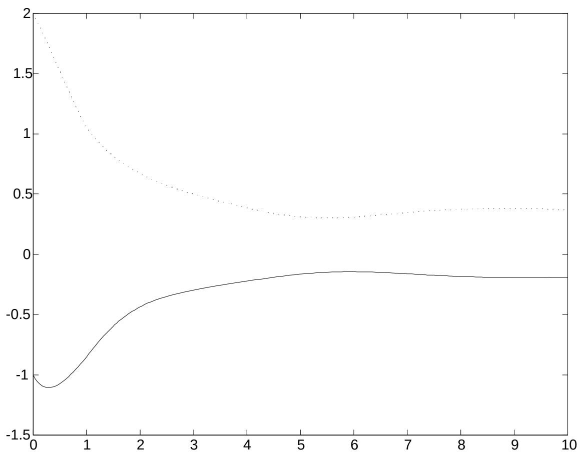

Figure 1: System response: x_1 (dashed) and x_2(solid)

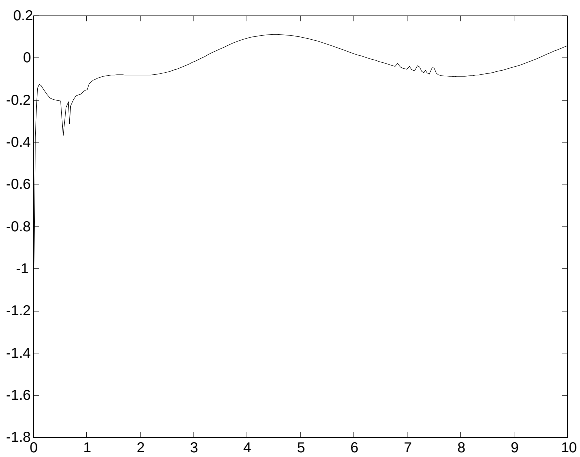

Figure 2: Control input

Conclusions

In this paper, the analysis and output-feedback control synthesis problems for LPV systems with parameter-dependent state delays are addressed. The corresponding analysis and synthesis conditions for stabilization, the L_2 induced norm performance and L_2 to L_∞ gain performance are expressed in terms of LMIs that can be solved efficiently using recently developed interior-point algorithms. These results provide a systematic procedure to address parameter-dependent time-delays in a gain-scheduling control design framework for time-delayed systems. The results can be easily extended to the case of systems with multiple delays.

References

[1] P. Apkarian and P. Gahinet, "A convex characterization of gain-scheduled H_∞ controller", *IEEE Trans. Automat. Contr.*, vol. 40, pp.853-864, 1995.

[2] G. Becker and A.K. Packard, "Robust performance of linear parametrically varying systems using parametrically-dependent linear feedback", *Syst. Contr. Lett.*, vol. 23, pp. 205-215, 1994.

[3] S.P. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*, SIAM, Philadelphia, PA, 1994.

[4] P. Gahinet, P. Apkarian, and M. Chilali, "Affine parameter-dependent Lyapunov functions and real parametric uncertainty," *IEEE Trans. Automat. Contr.,* vol.41, pp.436–442, 1996.

[5] L. Gugard and E.I. Verriest, *Stability and Control of Time-delay Systems, Springer,* London, 1997.

[6] E.T. Jeung, D.C. Oh, J.H. Kim and H.B. Park, "Robust controller design for uncertain systems with time delays: LMI approach", *Automatica,* vol.32, pp.1229-1231, 1996.

[7] R.E. Skelton, T. Iwasaki and K.M. Grigoriadis, *A Unified Algebra Approach to Linear Control Design, Taylor & Francis*, 1998.

[8] F. Wu and K.M. Grigoriadis, "LPV system with parameter-varying time delays: analysis and control ," *Automatica*, to appear. Also in *Proc. 36th IEEE Conf. Dec. Contr.,* San Diego, California, 1997.

[9] F. Wu, X.H. Yang, A.K. Packard, and G. Becker, "Induced L^2 norm control for LPV systems with bounded parameter variation rates", *Int. J. Robust Non. Contr.*, vol.6, pp.983-998, 1996.