Stable Tracking Control for Unmanned Aerial Vehicles using Non-Inertial Measurements.

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Abstract

A number of recent papers have considered trajectory tracking control design for unmanned aerial vehicles (UAVs). One method to obtain an integrated non-linear control for such vehicles is to exploit the underlying cascade structure (input/output linearizability) of the systems. The dynamics of these vehicles have a particular form when expressed in an inertial frame of reference that lends itself to this approach. In practice, an autonomous vehicle has access only to absolute measurements of non-inertial variables such as linear acceleration and angular velocity along with local measurements of inertial quantities such as position, velocity and orientation. Expressing the dynamics of a typical system in body fixed frame coordinates introduces dynamic coupling that appears to destroy the simple structure of the inertial equations. In this paper it is shown that the inherent passivity-like properties of the underlying mechanical system may be exploited to obtain Lyapunov control design for the more general system equations expressed in the body fixed frame of an unmanned aerial vehicle. This avoids a possibly difficult and highly non-robust state reconstruction that would be necessary before existing control designs could be applied.

1 Introduction

There has been a growing interest within the control community towards studying the question of construction and control of Unmanned Aerial Vehicles (UAV) [2]. The high actuation to inertia ratios of such vehicles lead investigators to consider fully integrated non-linear control designs. Some recent developments in this area have been focused on the exciting problem of developing an effective control design for regulation of the flight of a helicopter [10, 5, 8] as well as autonomous control of aeroplanes [4]. The problems encountered in dealing with reduced scale autonomous aircraft are somewhat different to those encountered in tradition aviation control design. The constraints of cost and weight are relatively far more important than the traditional constraint of safety. As a consequence it is of interest to consider control designs that make use of measurements that may be obtained using cheap, light sensor systems in preference to control designs that required expensive, heavy systems. Using devices such as rate gyros it is relatively simple to obtain measurements of the rotational angular velocity since it is a non-inertial variable. Similarly, translational acceleration may be measured using accelerometers, although such measurements tend to be less accurate than those of angular velocity. In contrast the absolute position, absolute velocity and the airframe orientation are all inertial measurements and are often extremely difficult to obtain without using expensive inertial guidance systems. Instead, it is usual that local measurements of the position of the UAV relative to observed features are obtained using sensor architectures such as vision systems, cheap local radar systems, ultrasound, etc. Such data is measured in the body fixed frame of the aircraft and transforming the data into estimates of the absolute state of the system may be impossible or introduce undesirable errors. This motivates the study of control design working directly with the system dynamics in the body fixed frame.

In this paper we consider the case where the desired trajectory is specified relative to the 'local' observed environment. In this case local 'non-inertial' sensing systems provide a measurement of a relative error $\tilde{X}$ in the body fixed frame. Essentially, the error $\tilde{X}$ measures the distance from the UAV to the relative position that it is desired that it should be in. If the mission were one of shadowing a second aeroplane then the error would simply be the observed position of the second aeroplane minus a fixed offset. The principal contribution of this paper is to show that a backstepping control design may be applied directly to the error dynamics for $\tilde{X}$ without the necessity of a coordinate transformation to obtain an inertial representation. This is not immediately clear since the body fixed frame representation of rigid body dynamics does not have triangular structure. To achieve a backstepping control design the passivity-like properties of rigid body motion is exploited to cancel unwanted dependence on the angular velocity in the early stages of the backstepping procedure.

2 Theoretical Model and Control Task

In this section a general dynamic model is developed for an unmanned autonomous (flying) vehicle (UAV).

The airframe of UAV is modeled as a rigid body **A** in three-dimensional space (cf. Figure 1). Let $\mathcal{I} = \{E_x, E_y, E_z\}$ denote an inertial frame, and let $\mathcal{A}$ a body coordinate frame that is fixed with respect to the body. The following notation is used in the sequel.

ξ	Position of the origin of $\mathcal{A}$ expressed in $\mathcal{I}$
R	Orientation of UAV, $R \in SO(3)$, $R : \mathcal{A} \to \mathcal{I}$.
v	Translational velocity of UAV in inertial frame $v \in \mathcal{I}$.
V	Velocity of UAV expressed in body fixed frame $V \in \mathcal{A}$.
Ω	Angular velocity of UAV in body fixed frame, $\Omega \in \mathcal{A}$.
m	Total mass of UAV.
I	Inertia matrix of UAV relative to body fixed frame.
g	Gravitational constant $g \approx 9.8 m/s^2$.
F	Total external force acting on the UAV.
Γ	Torque control $\Gamma \in \mathbb{R}^3$.

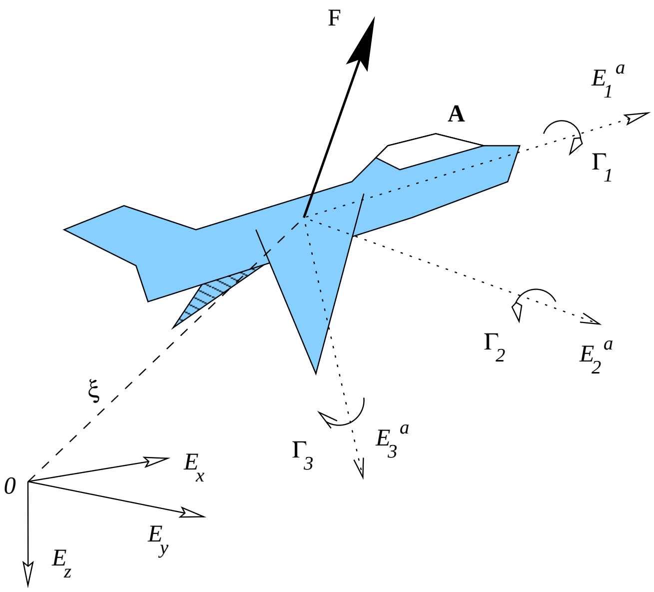

Figure 1: Reference frames, forces and torques for an Unmanned Aerial Vehicle (UAV).

The classical model of a rigid object may be obtained from Newton's equations of motion evolving on the special Euclidean group $SE(3)$. The generalized force inputs considered are; a single translational force (denoted F in Figure 1) along with full torque control (shown by the torques Γ_1, Γ_2 and Γ_3 around axis E_1^a, E_2^a and E_3^a respectively in Figure 1). This is the usual situation encountered in practice for aeroplanes, helicopters and dirigables. The force F combines thrust, lift, gravity and drag components. It may be modeled as a non-linear function

$$F = F(u, W) \in \mathcal{A},$$

where u is the thrust control and $W \in \mathcal{A}$ is the relative wind velocity striking **A** expressed in the body fixed frame. In general the force F may be decomposed into two parts

$$F(u, W) = F_0(u, W) + mgE_z$$

where $E_z = Re_3$ is the direction of gravity. The control u provides a limited control to the external force $F(u, W) \in \mathcal{A}$ usually linked to the magnitude of $F_0(u, W)$ which is normally oriented in a fixed (or nearly fixed) direction relative to the body fixed frame based on the underlying geometry of the UAV considered. Control over the motion of **A** is obtained by using the torque control $\Gamma = (\Gamma_1, \Gamma_2, \Gamma_3)$ to re-orient **A** and align the force F_0 in the direction necessary to track the desired trajectory.

In the absence of external disturbances, and assuming that the system inputs may be accurately modeled then the dynamic equations for a flying vehicle of the nature outlined above may be written[1]

$$\dot{\xi} = v \tag{1}$$
$$m\dot{v} = RF(u, W) = RF_0(u, W) + mge_3 \tag{2}$$
$$\dot{R} = R\,\mathrm{sk}(\Omega), \tag{3}$$
$$\mathbf{I}\dot{\Omega} = -\Omega \times \mathbf{I}\Omega + \Gamma. \tag{4}$$

Note that $F(u, W) \in \mathcal{A}$ and the inertial force applied to the system $RF(u, W)$ is obtained by multiplication by the rotation matrix R. Equations 1, 2 and 3 are expressed in the inertial frame while Eq. 4 is expressed in the body fixed frame. Expressed in this form, Eqn's 1-4 are in lower triangular form and it is possible to directly apply backstepping design techniques [6]. Setting $x = (x_1, x_2, x_3, x_4) = (\xi, v, R, \Omega)$ then a general expression of Eqn's may be written 1-4

$$\dot{x}_1 = \phi_1(x_2), \quad \dot{x}_2 = \phi_2(x_3, u, W)$$
$$\dot{x}_3 = \phi_3(x_3, x_4), \quad \dot{x}_4 = \phi_4(x_4, \Gamma).$$

where the triangular dependence of the variables is clearly visible. In Mahony and Hamel [7] the particular case of a helicopter is considered and the triangular structure of the system is exploited to design a trajectory tracking control using the backstepping design methodology. In fact, Equations 1-4 are in a form that admits most non-linear control design strategies. For example the system may be input/output linearized between the inputs (u, Γ) and outputs ξ [3]. In common with any feedback linearizable system of this form the outputs ξ are differentially flat outputs [1]. The basic triangular structure of Eqn's 1-4 will also admit forwarding techniques and other sophisticated control techniques [9]. As long as the dynamic system considered has a triangular structure (as above) and all states are measurable then all sensible control designs will generate closed loop structures with much the same properties! However in practice, a major technical difficulty in controlling UAVs is the lack of precise inertial measurements. Indeed, the position $\xi \in \mathcal{I}$, the velocity $v \in \mathcal{I}$ and the rotation $R : \mathcal{A} \to \mathcal{I}$ are all inertial measurements and are often extremely difficult to obtain without using expensive inertial guidance systems. Instead, it is usual that local measurements of the position of the UAV relative to observed features are obtained using sensor architectures such as vision systems, local radar systems, etc. The situation considered in this paper is the case where the desired trajectory (which one wishes to track) is

[1] The notation $\mathrm{sk}(\Omega)$ denotes the skew-symmetric matrix such that $\mathrm{sk}(\Omega)v = \Omega \times v$ for the vector cross-product $\times$ and any vector $v \in \mathbb{R}^3$.

specified relative to the observed environment. In this case local 'non-inertial' sensing systems provide a measurement of a relative error $\tilde{X}$ expressed in the body fixed frame

$$\tilde{X} = R^T(\xi_r - \xi). \quad (5)$$

If the rotation R is known then it is possible to invert this expression to obtain a relative error $\tilde{\xi} = \xi_r - \xi$ expressed in the inertial frame. In many cases, however, the rotation may not be known or only poorly known due to disturbances and it is desirable to work directly with the available error measurements to improve robustness of the associated control laws. Either by using direct measurements or by exploiting knowledge of Ω (along with Eq. 7) one may obtain a reliable estimate of the local error velocity $\tilde{V}$

$$\tilde{V} = R^T(\dot{\xi}_r - \dot{\xi}). \quad (6)$$

It is assumed that the inertial measurements $\xi_r, \xi, \dot{\xi}_r, \dot{\xi} \in \mathcal{I}$ are not available and cannot be used explicitly in the control design.

To obtain a control algorithm based directly on local measurements it is necessary to express the dynamics of the UAV with respect to the available measurements. It is assumed that the external force $F(u, W)$ and the torque Γ are known relative to the body fixed frame. The equations of motion of the UAV with respect to local measurements are

$$\dot{\tilde{X}} = -\Omega \times \tilde{X} + \tilde{V}, \quad (7)$$

$$m\dot{\tilde{V}} = -m\Omega \times \tilde{V} + mR^T\ddot{\xi}_r + F(u, W), \quad (8)$$

$$\dot{R} = R\mathrm{sk}(\Omega), \quad (9)$$

$$\mathbf{I}\dot{\Omega} = -\Omega \times \mathbf{I}\Omega + \Gamma. \quad (10)$$

There are two important blocks to using classical control design techniques on the above equations. Firstly, there is an unknown term $mR^T\ddot{\xi}_r$ which enters into Eq. 8. Secondly, the important triangular structure of Eqn's 1-4 is dependent on the inertial frame representation of position and velocity of the UAV and is destroyed in Eqn's 7-10.

The details of measuring or 'knowing' the non-inertial evolution of the desired trajectory, ξ_r, is not the principal topic of this paper. In the following development we assume that the time-derivatives

$$X_r^{(2)} := R^T\ddot{\xi}_r, \;\; X_r^{(3)} := R^T\xi_r^{(3)}, \;\; X_r^{(4)} := R^T\xi_r^{(4)}$$

are known *a priori*. This condition is far less restrictive than requiring knowledge of all the reference trajectory parameters including $X_r := R^T\xi_r$ and $\dot{X}_r := R^T\dot{\xi}_r$. For example if ξ_r has constant velocity then $\ddot{\xi}_r = 0 = \xi_r^{(3)} = \xi_r^{(4)}$ and $X_r^{(i)} = 0$ for $i = 2, 3, 4$.

In practice knowing $X_r^{(2)}$, $X_r^{(3)}$ and $X_r^{(4)}$ is analogous to a common assumption made in trajectory tracking designs that the desired trajectory and all its derivatives are available for the control design. In the case where exact information is unavailable it may be replaced by estimated versions. An alternative approach is to ignore the contributions of higher order derivatives of ξ_r entirely and rely on the inherent robustness of the tracking control developed to achieve acceptable performance. This approach will work well in practice as long as the gain margins in the control design are sufficiently high. The tracking error will be inversely proportional to the gain margin for a given trajectory.

The second issue raised above is the main topic of this paper. It is clear that the angular velocity Ω of the frame $\mathcal{A}$ enters into Eqn's 7-8. Defining $(x_1, x_2, x_3, x_4) = (\tilde{X}, \tilde{V}, R, \Omega)$ then

$$\dot{x}_1 = \phi_1(x_1, x_2, x_4) \quad \dot{x}_2 = \phi_2(x_2, x_3, x_4, u, W)$$
$$\dot{x}_3 = \phi_3(x_3, x_4) \quad \dot{x}_4 = \phi_4(x_4, \Gamma).$$

and the triangular structure of the system is destroyed. Clearly, the difference between the two representations is simply a change of coordinates and does not change the structural properties of the system. However, in practice reconstructing the inertial coordinates in order to apply a non-linear control design is likely to introduce unacceptable errors into the closed loop system. In the next section it is shown how a non-linear control design can be developed directly for the system expressed in the body fixed frame coordinates by exploiting the passivity-like properties of rigid body kinematics and dynamics.

3 Lyapunov Based Tracking Control Design

In this section a backstepping control design is provided for the model Eqn's 7-10 proposed in the previous section.

The kinematics and dynamics of rigid body motion have a structure which may be interpreted as a form of passivity. Consider Equations 1-4 and observe that

$$\frac{d}{dt}\left(\frac{1}{2}\xi^T\xi\right) = \xi^T v, \quad \frac{d}{dt}\left(\frac{m}{2}v^T v\right) = v^T RF(u, W).$$

Considering V and $RF(u, W)$ respectively as inputs into the two equations then Eq. 1 can be considered passive from v to ξ with respect to the storage function $\xi^T\xi$, while Eq. 2 is passive from RF to v with respect to the storage function $v^T v$. Note that Eqn's 1-2 are not passive as a combined system (the outputs ξ are relative degree two with respect to the inputs $RF(u, W)$) [9]. Moreover, in the presence of gravity the linear velocity dynamics are not truly passive due to the unbounded potential energy available (the gravitational effects are hidden in the term $F(u, W)$). Nevertheless, the passivity-like properties of these equations are extremely important in the following development.

Intuitively the passivity-like properties of Eqn's 1-4 should not be destroyed by a simple change of the frame of reference. Indeed, it is easily verified that

$$\frac{d}{dt}\left(\frac{1}{2}\tilde{X}^T\tilde{X}\right) = \tilde{X}^T\tilde{V}$$
$$\frac{d}{dt}\left(\frac{m}{2}\tilde{V}^T\tilde{V}\right) = \tilde{V}^T\left(F(u,W) + mX_r^{(2)}\right).$$

since $\Omega \times Z = \text{sk}(\Omega)Z$ for any vectors $\Omega, Z \in \mathbb{R}^3$. A key observation is that any backstepping control design has passivity-like properties of exactly this form [6].

Let $\xi_r = (x_r, y_r, z_r)$ be a reference trajectory with respect to the inertial frame $\mathcal{I}$. The desired control objective is to track the trajectory ξ_r, that is to find a control such that $\tilde{\xi} = \xi - \xi_r \to 0$. Expressed in the body fixed frame this is equivalent to $\tilde{X} \to 0$.

Let S_1 be the first storage function for the backstepping procedure. It is chosen for the full linear dynamics Eqn's 7-8

$$S_1 = \frac{1}{2}c_1|\tilde{X}|^2 + \frac{1}{2}|\tilde{X} + c_2\tilde{V}|^2 \tag{11}$$

where $c_1, c_2 > 0$ are suitable positive constants. Taking the time derivative of S_1 and substituting for Eqn's 7-8 yields

$$\dot{S}_1 = c_1\tilde{X}^T\tilde{V} + \left(\tilde{X} + c_2\tilde{V}\right)^T\left(\tilde{V} + c_2X_r^{(2)} + \frac{c_2}{m}F(u,W)\right) \tag{12}$$

Let G denote a desired value for the scaled linear force input $\frac{c_2}{m}F(u,W)$ expressed in the body fixed frame $\mathcal{A}$. The value of G is thought of as a virtual control for the first storage function and is chosen such that $\dot{S}_1$ would be monotonically decreasing if $\frac{c_2}{m}F(u,W) = G$. Set

$$G := -\left((c_1+1)\tilde{V} + c_2X_r^{(2)} + k_1\left(\tilde{X} + c_2\tilde{V}\right)\right) \tag{13}$$

With the above choice, one has

$$\dot{S}_1 = -c_1c_2|\tilde{V}|^2 - k_1|\tilde{X} + c_2\tilde{V}|^2 + \left(\tilde{X} + c_2\tilde{V}\right)^T\tilde{F} \tag{14}$$

where

$$\tilde{F} := \frac{c_2}{m}F(u,W) - G \tag{15}$$

is the scaled difference between the virtual and true values for the external forces applied to the linear dynamics and is used as the new error in the backstepping procedure.

The virtual control G depends only on measurements made in the body fixed frame and consequently has 'passive' type dynamics

$$\dot{G} = -\Omega \times G + H$$

where

$$H := -(1+c_1+c_2k_1)\left(X_r^{(2)} + \frac{1}{m}F(u,W)) - k_1\tilde{V} - c_2X_r^{(3)}\right)$$

Deriving $\tilde{F}$, substituting for the G dynamics and both adding and subtracting $\Omega \times \frac{c_2}{m}F(u,W)$ one obtains

$$\dot{\tilde{F}} = -\Omega \times \tilde{F} + \frac{c_2}{m}(\dot{F}(u,W) + \text{sk}(\Omega)F(u,W)) - H \tag{16}$$

The term

$$\dot{F}(u,W) + \text{sk}(\Omega)F(u,W) \approx \frac{\partial}{\partial u}F(u,W)\dot{u} - \text{sk}(F(u,W))\Omega + \frac{\partial}{\partial W}F(u,W)\dot{W}$$

provides control authority for the next stage of backstepping process via the virtual control inputs $\dot{u}$ and Ω. The angular velocity Ω provides only two degrees of control due to the rank of $\text{sk}(F(u,W))$ while dynamic extension of the control input u provides the remaining authority providing $\frac{\partial}{\partial u}F(u,W)$ does not lie in the column space of $\text{sk}(F(u,W))$. Since u is typically the magnitude of the vector force F, this requirement is true for most systems. Let J denote the virtual control for the next stage of the backstepping procedure and choose

$$J := H - k_2\tilde{F} + \left(\tilde{X} + c_2\tilde{V}\right). \tag{17}$$

This leads to the backstepping error

$$\delta := \frac{c_2}{m}(\dot{F}(u,W) + \text{sk}(\Omega)F(u,W)) - J \tag{18}$$

that forms the final error term used in the backstepping procedure. The storage function associated with the backstepping procedure up to and including the error $\tilde{F}$ is

$$S_2 = S_1 + \frac{1}{2}|\tilde{F}|^2 \tag{19}$$

Deriving and substituting for Eqn's 17-14 yields

$$\dot{S}_2 = -c_1c_2|\tilde{X}|^2 - k_1|\tilde{X} + c_2\tilde{V}|^2 - k_2|\tilde{F}|^2 + \tilde{F}^T\delta. \tag{20}$$

To complete the final stage of the backstepping procedure the dynamics of δ are computed

$$\dot{\delta} = \frac{c_2}{m}\left(\ddot{F}(u,W) + \text{sk}(\dot{\Omega})F(u,W) + \text{sk}(\Omega)\dot{F}(u,W)\right) - \dot{J}. \tag{21}$$

At this stage the actual control inputs enter into the calculations through $\dot{\Omega}$ (via Eq. 4) and a further dynamic extension of the control input u (resulting in a new control $v = \ddot{u}$).

The storage function associated with this stage of the backstepping and representing the Lyapunov function of the system is

$$\mathcal{L} = S_2 + \frac{1}{2}|\delta|^2$$

To stabilize the full Lyapunov function $\mathcal{L}$, and in the process ensure asymptotic tracking of the trajectory ξ_r, one chooses

$$\ddot{F}(u,W) + \text{sk}(\dot{\Omega})F(u,W) = -\text{sk}(\Omega)\dot{F}(u,W) + \frac{m}{c_2}(\dot{J} + \tilde{F} + k_3\delta) \tag{22}$$

With this choice the derivative of the Lyapunov function may be rewritten

$$\dot{\mathcal{L}} = -c_1c_2|\tilde{X}|^2 - k_1|\tilde{X} + c_2\tilde{V}|^2 - k_2|\tilde{F}|^2 - k_3|\delta|^2$$

Lemma 1 *Consider the full error dynamics system given by Eqn's 7-10 along with a dynamic extension of the control input* u

$$\ddot{u} = v.$$

Let J *be defined by Eq. 17 and note that the time derivative of* J *may be computed from known signals. Assume that* $\mathrm{sk}(F(u,W))$ *is rank two (that is* $F(u,W) \neq 0$*) and that* $\frac{\partial}{\partial u}F(u,W)$ *is linearly independent from the column space of* $\mathrm{sk}(F(u,W))$*. Then the control inputs* v *and* Γ *may be chosen such that*

$$\begin{aligned}\left[\frac{\partial}{\partial u}F(u,W)\right]v - \left[\mathrm{sk}(F(u,W))\mathbf{I}^{-1}\right]\Gamma := &-\frac{\partial^2}{\partial u^2}F(u,W)\dot{u}^2 \\ &- \mathrm{sk}(F(u,W))\mathbf{I}^{-1}\mathrm{sk}(\Omega)\mathbf{I}\Omega - \mathrm{sk}(\Omega)\dot{F}(u,W) \\ &- \mathrm{sk}(\Omega)\frac{\partial}{\partial u}F(u,W)\dot{u} + \frac{m}{c_2}(\dot{J} + \tilde{F} + k_3\delta) \\ &- \frac{d}{dt}\left(\frac{\partial}{\partial W}F(u,W)\dot{W}\right).\end{aligned}$$

With the above choice of control then $\tilde{X}$ *is exponentially convergent to zero.*

Proof: The proof follows directly from the earlier discussion along with a substitution for expressions for $\ddot{F}(u,W) + \mathrm{sk}(\dot{\Omega})F(u,W)$. ■

In the case where $F(u,W) = F_0(u,W) + mgE_z$ it is necessary to know the direction of gravity $E_z = R^T e_3 \in \mathcal{A}$ in order to compute the linear dynamics Eq. 8. It is not necessary, however, to know explicitly the full rotation matrix R in order to apply the proposed control design. In practice, the Force $F(u,W)$ may be measured using accelerometers and inclinometers and used to estimate the direction of the gravity and modify the applied control to obtain the desired behaviour. The above control design ensures that the tracking error $\tilde{X}$ converges asymptotically to zero. Since most UAVs are under-actuated systems it is necessary to use several degrees of freedom associated with the rotation dynamics to regulate the position. However, there is always at least one remaining degree of liberty in the orientation dynamics. For example, when regulating the position of a helicopter in hover, the 'yaw' angle may be changed without moving the centre of mass! Sensible control of the remaining freedom in orientation is often linked to mission requirements and local configuration of the system considered. A suitable approach is to consider a feed-forward control based on anticipated mission requirements. The aim of the feed-forward control is to anticipate possible system configurations which are necessary for effective flight of the UAV. The feed-forward control should be augmented by a passive damping control to stabilize the orientation of the UAV and remove any accumulated errors. Further details of this aspect of the control design are omitted since they are not the principal topic of this paper and often depend strongly on the individual configuration of the UAV considered. If there is no gravitational force (such as is the case for a blimp or submarine where the gravitational force is overcome by buoyancy forces) then the system becomes non-holonomic and a stabilization task (for inertial trajectory $\xi_r = 0$ becomes difficult due to the loss of controllability close to the stationary point. In particular, the force $F(u,W) \to 0$ and the conditions of Lemma 1 no longer hold. In contrast the path tracking task (for $\dot{\xi}_r \neq 0$) may be solved regardless of the model as long as the vehicle velocity does not approach zero.

4 Autonomous Control of a Reduced Scale Helicopter

In this section the control design presented in Section 3 is applied to a highly idealized model of an autonomous helicopter.

An idealized model of an autonomous helicopter expressed in terms of the local error may be written by combining Eqn's 7-10 with the generalized force and couple vectors defined by

$$F(u,W) = -ue_3 + mgR^T e_3 + D_v(W) \quad (23)$$
$$\Gamma = Q(u) + D_\Omega(W) + w \quad (24)$$

Here ue_3 represents the main heave force derived from collective pitch on the main rotor that sustains the helicopter in flight. It is oriented along the vertical axis of the helicopter body frame. The input w represents the torque control over the airframe derived from cyclic pitch inputs to the main rotor. The terms $Q(u) := |Q_M(u)|e_3 - |Q_T(u)|e_2$ are parasitic torques on the airframe due air resistance on the main and tail rotors. The term $D_{(.)}(W)$ is the drag on the airframe (both linear and rotational) and depends on the relative wind velocity striking **A** expressed in the body fixed frame. For the purposes of the following derivation the wind disturbance is assumed to be constant.

Recalling the control law defined by Eq. 22, and introducing the force vector by its expression, it follows that

$$\left(\ddot{u}e_3 + \mathrm{sk}(\dot{\Omega})e_3 u\right) = -\mathrm{sk}(\Omega)e_3\dot{u} - \frac{m}{c_2}(\dot{J} + \tilde{F} + k_3\delta) \quad (25)$$

Using the following transformation of equation Eq. 10.

$$\tilde{w} := \mathbf{I}^{-1}(-\Omega \times \mathbf{I}\Omega + \Gamma) \quad (26)$$

and knowing that $\mathrm{sk}(e_3)$ is of rank two with entries only in the first and second columns one obtains

$$ve_3 + u\mathrm{sk}(\dot{\Omega})e_3 = \begin{pmatrix} 0 & u & 0 \\ -u & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{w}^1 \\ \tilde{w}^2 \\ v \end{pmatrix}, \quad (27)$$

where $v = \ddot{u}$. It is clear that as long as $u \neq 0$ then the vector value of Eq. 27 may be arbitrarily assigned and the proposed control may be implemented.

The above control design leaves $\tilde{w}^3$ free to stabilize the yaw angle to a desired value. In particular, the yaw dynamic

$$\dot{\Omega}^3 = \tilde{w}^3$$

are fully decoupled from the evolution of the closed-loop system. In this paper we are not interested in assigning a particular motion to the yaw angle dynamics and a simple stabilizing control is chosen

$$\tilde{w}^3 = -c\Omega^3$$

where $c > 0$ is a positive constant.

A simulation is presented to illustrate the performance of the proposed control strategy. In this experiment the target trajectory considered is a helix, involving motion in all three coordinate directions. The parameters used for the helicopter are given by $m = 18.085$, $I = \mathrm{diag}(1.667, 2.341, 1.197)$ and $g = 9.8$. These values are based on measurements of the model helicopter used by the Swiss research group at the Measurement and Control Laboratory, ETH, Switzerland, along with estimates drawn from the literature. The air resistance and relative wind effects are estimated by the following constants ($|Q_M| = 0.02, |Q_T| = 0.0002, D_v = 0.3, D_\Omega = 0.02$). The magnitude of the initial force input is chosen to be $u_0 = gm \approx 177$ corresponding to the fact that the helicopter is initially in hover flight. The initial position is

$$\xi_0 = (0, 0, -2)^T,\ \phi_0 = 0,\ \dot{\xi}_0 = \dot{\phi} = 0.$$

The followed trajectory is defined by:

$$\xi_r(0) = (1, 1, -5)^T,\ \phi_r(0) = \frac{\pi}{4}.$$
$$\dot{\xi}_r = (0.06\cos\phi_r, 0.06\sin\phi_r, -0.05)^T,\ \dot{\phi}_r = 0.1.$$

For the given value of the initial condition and the helicopter parameters, we have used the following control gains $c_1 = 1/3$, $c_2 = 1$, $k_1 = 2$, $k_2 = 2$ and $k_3 = 2$.

Trajectories for the closed loop system dynamics are shown in Figure 2. Note that the strong exponential convergence of the trajectories ensures that the tracking control will be highly robust to disturbances. The trajectory chosen is a trimming trajectory, one for which the forces and torques acting on the vehicle, expressed in the body fixed frame, are zero. For a flying aircraft, this corresponds to trimmed flight. Note that there are still centripetal forces acting on the body in the inertial frame. A consequence of the trajectory choice is that the roll and pitch angles ψ and θ converge to constant values associated with the inclination of the vehicle required to follow the ascending spiral. The convergence of these values in Figure 2 indicates the overall convergence of the system to the desired trajectories. The gains chosen are not particularly large and the time-response of the tracking algorithm could be improved by increasing the gain margins in the system.

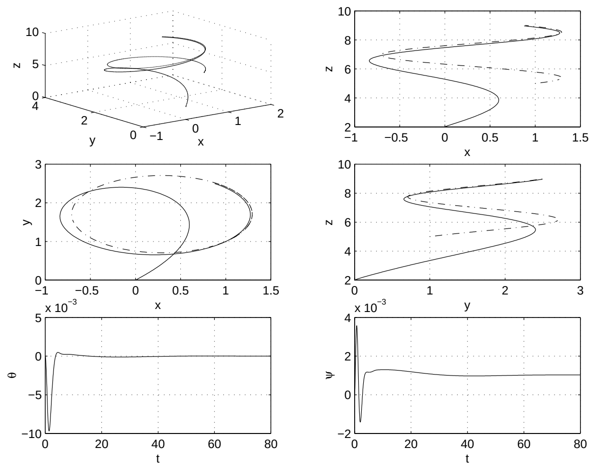

Figure 2: Position and orientations trajectories of the helicopter.

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