A Petri Net Scheme for DES State Estimation*

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Abstract— **This paper presents new results on state estimation of discrete event systems modeled by Interpreted Petri nets. First the concepts of event-detectability and observability are introduced. Then necessary and sufficient conditions for observability of systems described by marked graphs and state machines are presented. Finally, a general scheme for state estimation is proposed. This result is illustrated using an example of a manufacturing system.**

Keywords: Discrete Event Systems, Interpreted Petri Nets, Observability, Observer Design

I. Introduction

In some applications, such as state feedback control or monitoring and fault tolerance systems, it is important to have a whole knowledge of the state of the system. Unfortunately, in most of applications, it is not always possible to perform all the required measurements to completely determine the current state of the system. However, if the knowledge of the system internal structure, its inputs and outputs and the total amount of resources involved on it suffices to compute its current state. In this case, the problem is overcame and this kind of systems are called observable.

Petri nets (PN) are proved to be an appropriate modelling tool for the study of the observability issue in discrete event systems because of their modelling power and flexibility, [1]-[3].

The observability and state estimator design problems in DES modeled by a PN has been studied in [4]- [7]. In [4], the observability problem is divided into two sub-problems: computing the initial marking of the net and determining the firing sequence that leads the observer from the initial marking to the current system state given a sequence of input and output words. In that work, algorithms to determine a set of possible initial markings of a PN and the corresponding firing sequence are presented. In [5] and [6] algorithms to determine the current system state are presented, it is assumed that all transitions are accessible, i.e. the state estimator knows exactly which transition was fired. This hypothesis is too restrictive for an actual DES in which internal events occur. In [7] that limitation has been overcame for binary nets. The existence of internal events is allowed, and the initial marking of the observer is defined to be bigger than the system initial marking.

Herein we present a more general state estimator scheme which operates whether the initial marking of the observer is bigger or lower than the system initial marking. In this scheme the estimator is a copy of the system that includes additional input and output transitions to its places allowing to reduce the estimation error.

This paper is organized as follows. Section II presents a definition of Interpreted Petri nets (IPN) and some related basic definitions. Section III reviews the concepts of event-detectability and observability, and a sufficient condition for DES observability. In section IV, it is proved that event detectability is a necessary and sufficient condition for observability in live and bounded marked graphs and state machines. The design of an asymptotic state estimator, which is an extension of those presented in previous works is presented in section V.

II. BACKGROUND DEFINITIONS

In this paper we use IPN [8], an extension of PN which allows to represent the input and output languages of a system to study the observability property.

Definition 1: A Petri Net structure is the 4-tuple $G = (P, T, I, O)$ where $P = \{p_1, p_2, ..., p_n\}$ and $T = \{t_1, t_2, ..., t_m\}$ are finite sets of elements called places and transitions, respectively, $I : P \times T \longrightarrow \{0, 1\}$ is a function representing the arcs going from places to transitions, and $O : P \times T \longrightarrow \{0, 1\}$ is a function representing the arcs going from transitions to places.

Pictorially, places are represented by circles, transitions are represented by rectangles, and arcs are depicted as arrows. The incidence matrix of G is $C[c_{ij}]$, where $c_{ij} = O(p_i, t_j) - I(p_i, t_j)$. The marking is a function $M : P \longrightarrow \{\mathbb{Z}^+\}^n$ that assigns a non negative integer to each place of the net, representing the number of tokens (depicted as dots) residing in them.

Definition 2: A Petri Net (PN) is the pair $N = (G, M_0)$, where G is the PN structure and M_0 is an initial token distribution.

A transition t_j is enabled at a marking M iff $\forall p_i \in P$, $M(p_i) \geqslant I(p_i, t_j)$. An enabled transition t_j can be fired reaching a new marking M' which can be computed by the PN state equation: $M' = M + Cv$, where $v(i) = 0$ for $i \neq j$ and $v(j) = 1$. The reachability set of a PN, $\mathbf{R}(\mathbf{G}, \mathbf{M_0})$, is the set of all possible reachable markings

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from M_0 by firing enabled transitions. A PN is called binary or safe, if there is non reachable marking that assigns more than one token to every place.

Let $t_j \in T$, $\bullet(t_j)$ and $(t_j)\bullet$ denote the sets of all places p_i such that $I(p_i,t_j) \neq 0$ and $O(p_i,t_j) \neq 0$, respectively. Similarly, $\bullet(p_i)$ and $(p_i)\bullet$ denotes the set of all transitions t_j such that $O(p_i,t_j) \neq 0$ and $I(p_i,t_j) \neq 0$, respectively.

Definition 3: An Interpreted Petri Net (IPN) is the 6-tuple $Q = (N, \Sigma, \Phi, \lambda, D, \varphi)$ where

- $N = (G, M_0)$ is a PN,
- $\Sigma = \{\sigma_1, \sigma_2, ..., \sigma_r\}$ is the input alphabet of the net, where σ_i is an input symbol,
- $\Phi = \{\phi_1, \phi_2, ..., \phi_s\}$ is the output alphabet, where ϕ_i is an output symbol,
- $\lambda : T \to \Sigma \cup \{\varepsilon\}$ is a labeling function of transitions with the following restriction: $\forall t_j, t_k \in T$, $j \neq k$ if $I(p_i,t_j) = I(p_i,t_k) \neq 0$ and both $\lambda(t_j)$, $\lambda(t_k) \neq \varepsilon$, then $\lambda(t_j) \neq \lambda(t_k)$, where ε represents an internal system event,
- $D : T \to \top$ is a feed-forward function, where $\top = \tau \cup \{\varepsilon\}$ and τ is the set of symbols of the name of transitions. If a transition t_j fires, then its name t_j or the symbol ε are presented as an output,
- $\varphi : \mathbf{R(G, M_0)} \to \{\Phi \cup \{\varepsilon\}\}^{\mathbf{q}}$ is an output function, where $\mathbf{R(N', M_0)}$ is the reachability set defined as in a PN, ε is the null action.

Remark 4: We will write (Q, M_0) instead of $Q = (N, \Sigma, \Phi, \lambda, D, \varphi)$ to enhance the fact that there exists an initial marking in an IPN.

A transition $t_j \in T$ of an IPN is enabled at a marking M if $\forall p_i \in P$, $M(p_i) \geqslant I(p_i,t_j)$. If $\lambda(t_j) = a_i \neq \varepsilon$ is present and t_j is enabled, then t_j must fire. If $\lambda(t_j) = \varepsilon$ and t_j is enabled then t_j can be fired. If an enabled transition t_j fires at a marking M_k, then a new marking M_{k+1} is reached which can be computed using the dynamical part of the state equation: $M_{k+1} = M_k + Cv_j$, where C and v_j are defined like in a PN. This fact is represented as: $M_k \xrightarrow{t_j} M_{k+1}$.

Definition 5: If $\lambda(t_i) \neq \varepsilon$ the transition t_i is said to be **controllable**, and **non-controllable**, otherwise. A place $p_i \in P$ is said to be **measurable** if $\forall M_j(p_i) \neq M_k(p_i)$ it holds that $\varphi M_j \neq \varphi M_k$, and **non-measurable,** otherwise. A transition t_j is said to be **measurable** if $D(t_j) = t_j$ and **non-measurable,** otherwise. In this paper, the measurable places of an IPN are depicted as clear circles, the non-measurable ones as dark circles, the measurable transitions as clear bars, while the non-measurable ones as dark bars.

This paper focuses on the case where $\Phi = \mathbb{Z}^+$, i.e Φ is the set of the non negative integer, and $\varphi : \mathbb{Z}^{+\mathrm{n}} \longrightarrow \mathbb{Z}^{+\mathrm{q}}$ is linear and can be represented as a $q \times n$ matrix $\varphi = [\varphi_{ij}]$ where q is the number of measurable places and the i-th row vector φ_i of φ is the transpose of the elemental vector e_j ($e_j[i \neq j] = 0$, $e_j[j] = 1$), if p_j is the i-th measurable place ($\varphi_i = e_j^T$), according to the order given by the place labeling. In this case, the state equation of an IPN can be written as:

$$\begin{aligned} M_{k+1} &= M_k + C^{\varepsilon} v_k^{\varepsilon} + C^D v_k^D \qquad (1) \\ y_k &= \varphi \bullet M_k \end{aligned}$$

where $C = [C^{\varepsilon} \vdots C^D]$, C^{ε} and C^D are formed by the columns of the non-measurable and measurable transitions, respectively.

Definition 6: Let (Q, M_0) be an IPN. A **firing sequence** of (Q, M_0) is a sequence $\sigma = t_i t_j ... t_k$ such that $M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} ... \xrightarrow{t_k} M_w$. The set of all firing sequences is called **firing language** $\pounds(Q, M_0) = \{\sigma | \sigma = t_i t_j ... t_k$ and $M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} ... \xrightarrow{t_k} M_w\}$. The **input language** of (Q, M_0) is $\pounds_{in}(Q, M_O) = \{\lambda(t_i)\lambda(t_j)...\lambda(t_k) | t_i t_j ... t_k \in \pounds(Q, M_O)\}$, while the **output language** is $\pounds_{out}(Q, M_O) = \{\varphi(M_O)\varphi(M_1)...\varphi(M_w)... | M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} ... \xrightarrow{t_k} M_w...$ and $t_i t_j ... t_k \in \pounds(Q, M_O)\}$.

Definition 7: A positive p-invariant of an IPN given by (Q, M_0) is a rational-valued non negative solution of the equation $XC = 0$, where C is the incidence matrix of (Q, M_0).

III. Observability concepts

In this section, we first review the concepts of event-detectability and observability for DES modeled by an IPN; then a sufficient condition for IPN observability is presented based on event-detectability and the synchronic distance defined on the transitions of the net.

Definition 8: An IPN (Q, M_0) described by the state equation (1) is **event-detectable** iff all $\varphi \bullet C^{\varepsilon}$ columns are not null and different from each other.

In other words, since $y_{k+1} - y_k = \varphi M_{k+1} - \varphi M_k = \varphi(M_k + C\overrightarrow{t_k}) - \varphi M_k = \varphi C \overrightarrow{t_k}$, then the firing of any non-measurable t_k can be detected iff the column corresponding to t_k in the matrix φC^{ε} is different from any other column and it is not zero.

Definition 9: A set of Conservative Marking Laws (CML) of an IPN given by (Q, M_0) is a set of equations

$$\begin{gathered} \sum_{j=1}^{n} \alpha_j^1 \bullet M(p_j) = k_1 \\ \vdots \\ \sum_{j=1}^{n} \alpha_j^w \bullet M(p_j) = k_w \end{gathered} \qquad (2)$$

such that $\alpha_j^i \in \mathbb{Z}^+$, $\forall p_j$ non-measurable it occurs that $\alpha_j^i \neq 0$ for at least one equation and $\forall \alpha_j^i \neq 0$, k_i / α_j^i is an integer value.

The following definition of observability is an extension of that used in control theory for continuous systems [9].

Definition 10: An IPN (Q, M_0) is **observable at** k **steps** iff $\forall \omega \in \pounds_{in}(Q, M_0)$ $\exists z$, such that $\omega z \in \pounds_{in}(Q, M_0), |z| < k < \infty$ and the information provided by ωz, the output word generated by ωz, a set of conservative marking laws CML and the structure of the system C suffices to uniquely determine M_0.

A set of conservative marking laws of an IPN does not only depend on its structure but also on its initial marking M_0. Fortunately, it is common to know this set in most of the systems [5] even when the initial marking is unknown, because it can be obtained from the knowledge of the maximum number of available resources (parts that a store can hold, a machine capacity, etc.). In this case, given a CML, it is possible to establish upper and lower bounds for all reachable markings of an IPN in the following way.

Definition 11: Let (Q, M_0) be an IPN, where a CML is defined. Then

$$\begin{array}{cc}
M^{LB}(p_k) = \min M(p_k) & M^{UB}(p_k) = \max M(p_k) \\
s.t. & s.t. \\
\sum_{j=1}^{n} \alpha_j^1 \bullet M(p_j) = k_1 & \sum_{j=1}^{n} \alpha_j^1 \bullet M(p_j) = k_1 \\
\vdots & \vdots \\
\sum_{j=1}^{n} \alpha_j^w \bullet M(p_j) = k_w & \sum_{j=1}^{n} \alpha_j^w \bullet M(p_j) = k_w \\
\forall p_i, M(p_i) \geqslant 0 & \forall p_i, M(p_i) \geqslant 0
\end{array} \tag{3}$$

are the minimum and maximum marking bounds of the place p_k respectively. Also, the maximal marking gap in the place p_k is $\mathbb{D}_k = M^{UB}(p_k) - M^{LB}(p_k)$. These quantities can be arranged in a vector mode as $M^{UB} = [M^{UB}(p_1)...M^{UB}(p_n)]^T$, $M^{LB} = [M^{LB}(p_1)...M^{LB}(p_n)]^T$, $\mathbb{D}= [\mathbb{D}_1...\mathbb{D}_n]^T$.

The following is a definition of the synchronic distance between two transitions, [2].

Definition 12: Given an IPN (Q, M_0), the synchronic distance of a transition t_i with respect to a transition t_j is the maximum number of firings of t_i without firing t_j in all firing sequences. This value is denoted as $SD(Q, M_0; t_i, t_j)$.

Using the above definitions, we can establish the following result, which is an extension of those presented in [5]-[7].

Theorem 13: Let (Q, M_0) be a cyclic, live, bounded and event-detectable IPN, described by the state equation (1), where the initial marking M_0 is unknown, but a CML in the sense of definition 9 can be obtained.

(Q, M_0) is observable, if $\forall p_j$ non-measurable either:
i) $SD(Q, M_0; \bullet(p_j), (p_j)\bullet) \geqslant \mathbb{D}_j$ or
ii) $SD(Q, M_0; (p_j)\bullet, \bullet(p_j)) \geqslant \mathbb{D}_j$.

Proof: Since (Q, M_0) is event-detectable, then any firing sequence η_i, such that $M_0 \xrightarrow{\eta_i} M_i$, can be computed. In order to compute M_0 it is necessary to know M_i; however, it cannot be directly obtained because of the existence of non-measurable places. Let p_k be a non-measurable place:

Assume that *i)* holds, then a firing transition sequence σ_k -such that the number of firings of transitions in $\bullet(p_k)$ without firing any transition in $(p_k)\bullet$ is equal to $\mathbb{D}_k$- exists. If σ_k does not occurs immediately from M_0, since (Q, M_0) is cyclic, then it will return to M_0 and eventually σ_k will occur. σ_k can be split as $\sigma_k = \sigma_1\sigma_2$, such that σ_2 does not contain any transition in $(p_j)\bullet$ and the transitions in $\bullet(p_j)$ appears $\mathbb{D}_k$ times. Then $M_j(p_k) = M_0(p_k) + C(p_k,\bullet)\overrightarrow{\sigma_1} + C(p_k,\bullet)\overrightarrow{\sigma_2} = M_n(p_k) + \mathbb{D}_k = M_n(p_k) + M^{UB}(p_k) - M^{LB}(p_k)$. We claim that, $M_n(p_k) = M^{LB}(p_k)$ and $M_j(p_k) = M^{UB}(p_k)$. To prove it, assume that $M_n(p_k) = M^{LB}(p_k) + \triangle M$, this implies that $\triangle M > 0$, then $M_j(p_k) = M^{LB}(p_k) + \triangle M + M^{UB}(p_k) - M^{LB}(p_k) = M^{UB}(p_k) + \triangle M > M^{UB}(p_k)$, which is a contradiction. Thus, after firing σ_k the marking of p_k is $M^{UB}(p_k)$, which can be determined from the output because (Q, M_0) is event-detectable.

Now, assume that *ii)* holds, then a firing transition sequence σ'_k -such that the number of firings of transitions in $(p_k)\bullet$ without firing any transition in $\bullet(p_k)$ is equal to $\mathbb{D}_k$- exists. Following a similar procedure like in the previous case, after firing σ'_k the marking of p_k can be determined.

Once the actual marking of p_k has been computed, it will remain known for any firing sequence since (Q, M_0) is event-detectable.

Moreover, using this procedure, the marking of the remaining non-measurable places can be determined. Then the whole marking will be known and M_0 can be computed using the fired sequence and the state equation (1). Therefore, (Q, M_0) is observable. ■

IV. OBSERVABILITY IN MARKED GRAPHS AND STATE MACHINES

Unfortunately, the firing sequence η_i of the previous theorem not always exists for any kind of net. In this section, we study the observability property in live, bounded and event-detectable marked graphs and state machines. We will prove that in such PN subclasses it always exists a sequence ω such that, after its firing, it is possible to uniquely determine the whole marking of the net.

Theorem 14: Let (Q, M_0) be a live interpreted strongly connected marked graph, where the total number of tokens of each p-invariant is known. (Q, M_0) is observable iff it is event-detectable.

Proof: ($\longrightarrow$) If (Q, M_0) is not event-detectable then two cases are possible: a) there exists a zero φC^ε column, i.e. there exists a transition t_j that is not connected with measurable places and its firing is not detected, so it is impossible to distinguish M_0 and M'_0, where $M_0 \xrightarrow{t_j} M'_0$; b) there exist two φC^ε columns that are equal, i.e. there exist transitions t_i, t_j such that $\varphi C(\bullet, t_i) = \varphi C(\bullet, t_j)$, so it is impossible to distinguish markings M_0 and M'_0, where $M_0 \xrightarrow{t_i} M_1$ and also $M'_0 \xrightarrow{t_j} M_1$. In both cases, (Q, M_0) is not observable.

($\longleftarrow$) Since (Q, M_0) is a strongly connected marked graph then every place of Q is contained in at least one p-invariant and due to the number of tokens inside each p-invariant is known, then the set of p-invariants forms a CML. The CML can be arranged as follows:

$$\begin{array}{cccc} \alpha_1^1 M(p_1)+ & \dots & +\alpha_n^1 M(p_n) & = k_1 \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_1^w M(p_1)+ & \dots & +\alpha_n^w M(p_n) & = k_w \end{array} \qquad (4)$$

where $\left[\begin{array}{ccc} \alpha_1^i & \cdots & \alpha_n^i \end{array}\right]^T$ is the $i-th$ p-invariant.

For each place p_j, two different markings can be computed: M_k, when p_j reaches its upper marking bound and M_k', when p_j reaches its lower marking bound. The marking of the remaining places $p_l \neq p_j$ can be computed to fulfill the set of equations (4), for both cases M_k and M_k'.

Now, as theorem 3.21 in [3] states, in a live strongly connected marked graph there exists a sequence σ such that $M_0 \xrightarrow{\sigma} M_r$ iff the number of tokens in each p-invariant at M_0 and M_r remain constant. Hence previous solutions M_k, M_k' are reachable markings and since $M_k \xrightarrow{\gamma} M_0$ then $M_0 \xrightarrow{\alpha} M_k$, $M_0 \xrightarrow{\beta} M_k'$. Moreover $\exists\delta$ such that $M_k' \xrightarrow{\delta} M_k$ fulfilling conditions of theorem 13. Since the net is cyclic, the procedure can be repeated for any place, therefore (Q, M_0) is observable. ■

Theorem 15: Let (Q, M_0) be a cyclic, live and bounded interpreted state machine, where a CML is defined as the p-invariants of the net. (Q, M_0) is observable iff it is event-detectable.

Proof: It follows from the proof of the previous theorem since a live state machine has only one p-invariant. ■

V. DYNAMICAL OBSERVERS

In [5]-[7], the issue of designing an estimator for DES modeled by PNs has been already addressed. The observer scheme design technique herein presented is an extension of all of them and is based on the block diagram depicted in figure 1.

Let us begin by giving the following definition.

Definition 16: The system IPN model is

$$N_S = (P_S, T_S, I_S, O_S, \Sigma, \Phi, \lambda, D, \varphi)$$

and the observer net is

$$N_O = (P_S, T_O, I_O, O_O, \Sigma, \Phi, Id, Id, Id)$$

Note that in the observer all transitions are controllable and all places are measurable. The state equation of the IPN system is as the state equation (1) and the state equation of the net observer is:

$$\hat{M}_{k+1} = \hat{M}_k + \left[\begin{array}{ccc} C & \digamma & \Upsilon \end{array}\right] \left[\begin{array}{c} \gamma_k \\ \beta_k \\ \delta_k \end{array}\right] \qquad (5)$$

$$\hat{y}_k = \hat{M}_k$$

where $\digamma[i,i] = -1$ and $\digamma[i,j] = 0, i \neq j$; and $\Upsilon[i,i] = 1$ and $\Upsilon[i,j] = 0, i \neq j$.

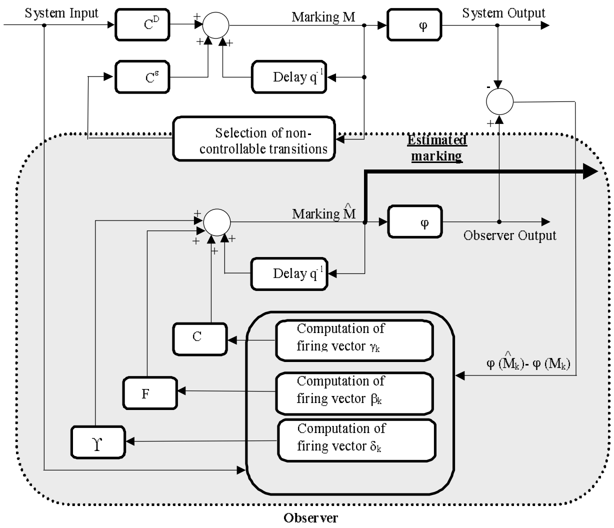

Fig. 1. System and observer scheme.

Now the observer initial marking $\hat{M}_0$ and the firing vectors γ_k, β_k, δ_k will be defined.

Definition 17: Let N_S be an IPN model of a system and N_O be its observer IPN, where a CML is defined. The initial admissible marking $\hat{M}_0$ of N_O is the following $\hat{M}(p_i) = M(p_i)$ if p_i is measurable and $\hat{M}(p_i)$ is any value fulfilling that $M^{UB}(p_i) \geqslant \hat{M}_0(p_i) \geqslant M^{LB}(p_i)$ if p_i is not measurable.

The state equation of (S, M_0) is defined as the state equation (1) and the state equation of the $(O, \hat{M}_0)$ is given by:

$$\hat{M}_{k+1} = \hat{M}_k + \left[\begin{array}{ccc} C & \digamma & \Upsilon \end{array}\right] \left[\begin{array}{c} \gamma_k \\ \beta_k \\ \delta_k \end{array}\right] \qquad (6)$$

$$\hat{y}_k = \hat{M}_k$$

where $\digamma[i,i] = -1$ and $\digamma[i,j] = 0$, $i \neq j$; and $\Upsilon[i,i] = 1$ and $\Upsilon[i,j] = 0$, $i \neq j$. Now we use the following definition to compute the firing vectors γ_k, β_k, and δ_k of the observer.

Definition 18: Let N_S be an IPN model of the system and N_O be its observer net. If transition $\overrightarrow{t_k}$ fires in N_S then

- $\gamma_k = \begin{cases} v_k & \text{if it is enabled in the observer} \\ 0 & \text{otherwise} \end{cases}$ where $v_k = \overrightarrow{t_k}$,
- $\beta_k = \left[\begin{array}{c} \varpi_1 \\ \vdots \\ \varpi_n \end{array}\right]$, where

$$\varpi_i = \begin{cases} 1 & \textbf{if } \left[\gamma_k = \alpha_k = \overrightarrow{t_k}, \text{ and } \hat{M}_k(p_i) + C(p_i, \bullet)\gamma_k > M^{UB}(p_i)\right] \\ & \textbf{or } \left[\gamma_k \neq \alpha_k = \overrightarrow{t_k}, p_i \in \bullet(t_k), \text{ and } \hat{M}_k(p_i) > M^{LB}(p_i)\right] \\ 0 & \text{otherwise} \end{cases}$$

- $\delta_k = \begin{bmatrix} \upsilon_1 \\ \vdots \\ \upsilon_n \end{bmatrix}$, where

$$\upsilon_i = \begin{cases} 1 & \textbf{if } \Big[\gamma_k = \alpha_k = \overrightarrow{t_k}\text{, and} \\ & \hat{M}_k(p_i) + C(p_i, \bullet)\gamma_k < M^{LB}(p_i)\Big] \\ & \textbf{or } \Big[\gamma_k \neq \alpha_k = \overrightarrow{t_k}, p_i \in (t_k)\bullet\text{, and} \\ & \hat{M}_k(p_i) < M^{UB}(p_i)\Big] \\ 0 & \text{otherwise} \end{cases}$$

The previous definition states that when the number of tokens in the observer are not enough to fire the same transition t_j that fires in the system then the additional input transitions in the observer will be fired to add tokens into the output places of t_j; or when the marking of a place exceeds the known marking bound for that place, those tokens are removed by the firing of the additional output transitions in the observer.

Using this firing vectors the state of the observer will tend to the state of the system as we will prove in the following theorem.

Theorem 19: Let N_S be a cyclic, live, bounded and event detectable IPN modelling a system, where a CML is defined. Let N_O be an observer with an initial marking according to definition 17. If $\forall p_i \in P_S$ either, $SD(N_S, M_0; \bullet(p_i), (p_i)\bullet) = \mathbb{D}_i$ or $SD(N_S, M_0; (p_i)\bullet, \bullet(p_i)) = \mathbb{D}_i$ and the firing vectors of the observer γ_k, β_k and δ_k are computed as in the definition 18, then

$$\lim_{|\alpha| \to \infty} \left\| \hat{M}_{k+1} - M_{k+1} \right\| = 0$$

where α is the transition sequence fired in N_S and $|\alpha|$ is its length.

Proof: The initial marking of N_O is $\hat{M}_0 = M_0 + \hat{M}'$, where $\left|\hat{M}'\right| < \mathbb{D}$.

Assume that $SD(N_S, M_0; \bullet(p_k), (p_k)\bullet) = \mathbb{D}_k$ for some p_k, and the firing vectors of the observer γ, β and δ are computed as in definition 18, then a transition firing sequence σ_k -such that the number of firings of transitions in $\bullet(p_k)$ without firing any transition in $(p_k)\bullet$ is equal to $\mathbb{D}_k$- exists. Then, as in theorem 13, when σ_k is fired, a new marking is reached: $M_j(p_k) = M_n(p_k) + \mathbb{D}_k$, where $M_j(p_k) = M^{UB}(p_k)$ and $M_n(p_k) = M^{LB}(p_k)$. Three cases are possible:

a) $\hat{M}'(p_k) = 0$. If a transition $t_j \in \bullet(p_k)$ is fired in σ_k, then by definition 18 either $\gamma = \overrightarrow{t_j}$ or $\upsilon_k = 1$ and $\varpi_k = 0$, then the estimation error of p_k remains zero over the execution of σ_k.

b) $\hat{M}'(p_k) > 0$. If a transition $t_j \in \bullet(p_k)$ is fired in σ_k, then by definition 18 the following subcases are possible:

b.1) $\gamma = \overrightarrow{t_j}$ or $\upsilon_k = 1$ and $\varpi_k = 0$ when $\hat{M}(p_k) < M^{UB}(p_k)$, the estimation error of p_k does not change.

b.2) $\gamma = \overrightarrow{t_j}$ or $\upsilon_k = 1$, and $\varpi_k = 1$ and when $\hat{M}(p_k) \geqslant M^{UB}(p_k)$, then the estimation error of p_k decreases, and will be equal to zero over the execution of σ_k.

Note that, in both cases $\gamma = \overrightarrow{t_j}$ is mutually exclusive with $\upsilon_k = 1$ and the case b.2) occurs exactly $\hat{M}'(p_k)$ times because marking $M^{UB}(p_k)$ is reached and the observer marking is not allowed to exceed it.

c) $\hat{M}'(p_k) < 0$. During the firing of σ_k, the marking $M_n(p_k) = M^{LB}(p_k)$ was reached. By definition 18, when the firing of any transition reduces the marking $\hat{M}(p_k)$ below $M^{LB}(p_k)$, then $\upsilon_k = 1$ freezing this marking in $M^{LB}(p_k)$, so the estimation error of p_k is reduced to zero when the marking $M_n(p_k)$ is reached.

Thus, in all cases the estimation error of p_k is reduced to zero.

Now assume that $SD(N_S, M_0; (p_k)\bullet, \bullet(p_k)) = \mathbb{D}_k$ for some p_k, and the firing vectors of the observer γ, β and δ are computed as in definition 18, then a transition firing sequence σ_k -such that the number of firings of transitions in $(p_k)\bullet$ without firing any transition in $\bullet(p_k)$ is equal to $\mathbb{D}_k$- exists. Using a similar reasoning it can be proved that the estimation error will be equal to zero for place p_k when σ_k has fired.

Since at least one of the conditions on the synchronic distance holds for each place and N_S is a cyclic IPN, then estimation error will be zero in all places when a transition sequence $\alpha = \sum_{i=1..n} \sigma_i$ fires in the system. ■

Example 20: Consider a manufacturing cell, where a product consisting of two parts (Pa, Pb) is processed. The part Pa requires to use the machines $M1$ and $M2$, while the part Pb requires the machines $M3$ and $M2$; both in that sequence. Afterwards both parts are processed, they are assembled and the final product is released and the cell is ready to start another cycle. The IPN of figure 2 is a model of the cell, where a token in the place p_1 represents an idle state of the cell; the transition t_1 represents the start of a cycle. A token in p_2 (p_5) represents that the machine $M1$ ($M3$) is being used. The place p_3 (p_4) represents that the macjime $M1$ ($M3$) is available. The transition t_2 (t_3) represents that the process in $M1$ ($M3$) has finished. A token in p_6 or p_7 represents a part waiting for $M2$ to be available. The transitions t_4 and t_5 represent the start of the process of a part in $M2$. A token in p_9 represents that $M2$ is available. The place p_8 (p_{10}) represents a part PA (PB) being processed in $M2$. The transitions t_6 and t_7 represent that the process in $M2$ has finished. Tokens in p_{11} and p_{12} represent the parts waiting for being assembled. Finally, the transition t_8 represents the assembling and release of the product.

Two different signals are displayed when the machines $M1$ and $M3$ are being used, respectively, and $M2$ display two different signals depending on the piece that is being processed. Therefore p_2, p_5, p_8 and p_{10} are measurable places and, since the releasing of a product can be detected, also t_8 is measurable.

The initial marking shown in figure 2 makes the model to be a binary IPN where a CML is given by

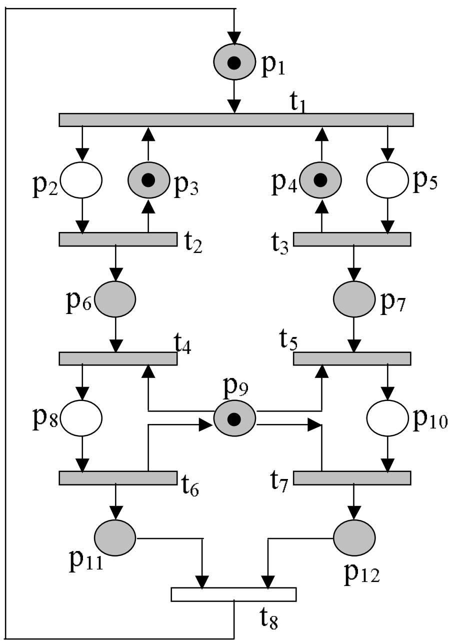

Fig. 2. Manufacturing cell IPN model.

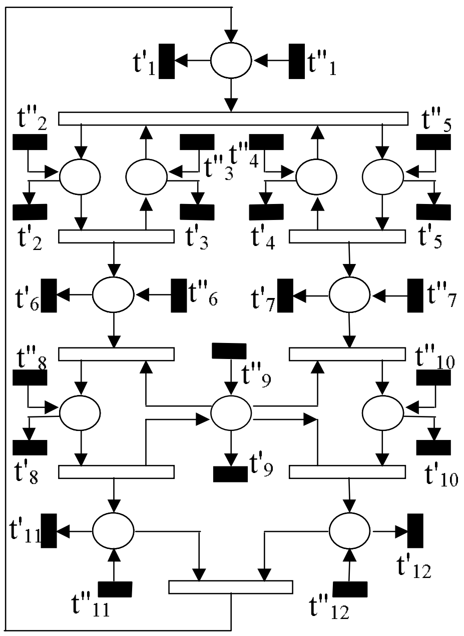

Fig. 3. Observer IPN for manufacturing cell model.

the following p-invariants:

$$\begin{aligned} M(p_2)+M(p_3)&=1 \qquad (7)\\ M(p_4)+M(p_5)&=1\\ M(p_8)+M(p_9)+M(p_{10})&=1\\ M(p_1)+M(p_2)+M(p_6)+M(p_8)+M(p_{11})&=1\\ M(p_1)+M(p_5)+M(p_7)+M(p_{10})+M(p_{12})&=1 \end{aligned}$$

For this model all columns in the matrix

$$\varphi C^{\varepsilon}=\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0\\ 1 & 0 & -1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & -1 & 0\\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

are not null and different, then the net is event-detectable and the firing of any transition can be determined. Using the information provided by the CML (7) for the initial marking depicted in the figure 2 the transition firing sequence $\sigma = t_1$ is enough to compute the system markings M_1 and M_0 since it is known that p_2 and p_5 are marked at M_1 and using 7 the number of tokens in all other places can be computed.

If the observer shown in figure 3 is used with the initial marking depicted in the figure, which is according to definition 17 then, for example, if in the system the sequence t_1, t_2, t_3, t_4 is executed and in consequence $t_2'', t_5'', t_2, t_3, t_8'', t_6'$ is executed in the observer, then the markings in both IPNs become equal.

VI. CONCLUSIONS AND FUTURE WORK

This paper presented three new results. The first one, presented as a theorem, is a characterization of observable IPN; this result was particularized to the strongly connected marked graphs and state machines cases, where necessary and sufficient conditions on the structure of these nets were found. Finally an asymptotic observer and its convergence conditions were presented; this result is an improvement of previous ones because it is applied in more general IPN classes.

Current research addresses the application of this observer scheme to state feedback control and fault tolerant systems. Furthermore, the study of minimal sensor configurations that preserve the observability property is a work in process.

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