Robust H_∞ Control of Singular Continuous-Time Systems with Delays and Uncertainties

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Abstract

In this paper we study the problem of H_∞ control of singular linear continuous-time systems with parametric uncertainty. The system under consideration is subjected to time-delay in both state and control, and norm-bounded parametric uncertainty entering all matrices of the system and output equations. First, the problem of robust stabilization of the underlying system is investigated. Next, we address the problem of robust H_∞ state feedback control in which both robust stability and a prescribed H_∞ performance are required to be achieved irrespective of the uncertainty and time-delay. It is shown that the above control problem can be solved if a Riccati-like inequality has a symmetric positive definite solution.

1. Introduction

Time delay is commonly encountered in various engineering systems, which often occurs in the transmission of information or material between different parts of a system and is frequently a source of instability and poor performance [1]. Transportation systems, communications systems, chemical process, power systems are typical examples of time-delay systems. During the past years, the study of time-delay systems has received considerable interest, see, e.g., Suh and Bien [2]. In the work of Gutman and Palmor [3], nonlinear state feedback controllers have been considered whereas Hasanul Basher *et al.* [4] has focused on memoryless linear state feedback. Recently, Memoryless stabilization and H_∞ control of uncertain continuous-time delay systems have been extensively investigated. For some representative prior work on this general topic, we refer the reader to Shen *et al.* [5], Lee *et al.* [6], Mahmoud and Al-Muthairi [7], Nguang [8], Benjelloun *et al.* [9], Kim *et al.* [10], Moheimani and Petersen [11], and Li and de Souza [12]. The problem of robust stabilization for a class of time varying delay systems with both Lipschitz and non-Lipschitz bounded uncertainties has been studied by Nguang [8] via Riccati equation approach, and a memoryless state feedback controller is designed. In the research conducted by Mahmoud and Al-Muthairi [7], quadratic stabilization of continuous time systems with state-delay and norm-bounded time-varying uncertainties has been considered. More recently, optimal quadratic guaranteed cost control for a class of uncertain linear time-delay systems with norm-bounded uncertainty has been designed by Moheimani and Petersen [11]. The issue of delay-dependent robust stability and stabilization of uncertain linear delay systems has been tackled by Li and de Souza [12] via a linear matrix inequality approach. However, to the best of authors' knowledge, the problems of robust stability and H_∞ control of singular continuous-time delay uncertain systems has not been fully investigated yet.

In this paper, the problem of robust stability and control of a class of singular uncertain systems with unknown time delays in both system state and output equations is addressed. We consider uncertain systems with norm-bounded time-varying parameter uncertainty in all system matrices. We deal with the problems of robust stabilization and robust H_∞ control, where in the latter the controller is required to guarantee both the robust stability and a prescribed robust H_∞ performance, irrespective of the uncertainty and unknown time delay.

Notation. The notation in this paper is quite standard. $\mathbf{R}^n$ and $\mathbf{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "T" denotes the transpose and the notation $X \geq Y$(respectively, $X > Y$) where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix of appropriate dimension. $L_2[0,\infty)$ is the space of square integrable functions over $[0,\infty)$. $\|\cdot\|$ will refer to the Euclidean vector norm.

2. Problem Formulation and Preliminaries

The system considered in this paper is assumed to be a state-space model as follows:

$$E\dot{x}(t) = [A + \Delta A(t)]x(t) + [A_d + \Delta A_d(t)]x(t - d_1(t)) \\ +[B + \Delta B(t)]u(t) + [B_d + \Delta B_d(t)]u(t - d_2(t)) \\ +[B_w + \Delta B_w(t)]w(t) \quad (2.1)$$

$$z(t) = [C + \Delta C(t)]x(t) + [C_d + \Delta C_d(t)]x(t - d_1(t)) \\ +[D + \Delta D(t)]u(t) + [D_d + \Delta D_d(t)]u(t - d_2(t)) \\ +[D_w + \Delta D_w(t)]w(t) \quad (2.2)$$

$$x(t) = \phi_1(t), \ \forall t \in [-d_1(t), 0] \quad (2.3)$$

$$u(t) = \phi_2(t), \ \forall t \in [-d_2(t), 0] \quad (2.4)$$

where $x(t) \in \mathbf{R}^n$ is the state, $u(t) \in \mathbf{R}^m$ is the control, $w(t) \in \mathbf{R}^p$ is the disturbance from $L_2[0,\infty)$, i.e., square-integrable, $z(t) \in \mathbf{R}^q$ is the controlled output, $A, A_d, B, B_d, B_w, C, C_d, D, D_d$ and D_w are real-valued constant matrices of appropriate dimensions that describe the nominal system, $\Delta A(t)$, $\Delta A_d(t)$, $\Delta B(t)$, $\Delta B_d(t)$, $\Delta B_w(t)$, $\Delta C(t)$, $\Delta C_d(t)$, $\Delta D(t)$, $\Delta D_d(t)$ and $\Delta D_w(t)$ are real time-varying matrix functions representing parameter uncertainties, and the matrix E is a singular matrix with $rank(E) = r \le n$. $d_1(t) \ge 0$ and $d_2(t) \ge 0$ are unknown state and control time delays, $\phi_i(t), \ t \in [-d_i(t), 0], i = 1, 2$, are continuous vector valued initial functions. $d_1(t)$ and $d_2(t)$ satisfy the following conditions:

$$0 \le d_i(t) < \infty, \quad \dot{d}_i(t) \le \beta_i < 1, \ i = 1, 2. \quad (2.5)$$

The *admissible parameter uncertainties* in this paper is assumed to be modelled as

$$\begin{bmatrix} \Delta A(t) & \Delta A_d(t) & \Delta B(t) & \Delta B_d(t) & \Delta B_w(t) \\ \Delta C(t) & \Delta C_d(t) & \Delta D(t) & \Delta D_d(t) & \Delta D_w(t) \end{bmatrix} \\ = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F(t)[E_1 \ \ E_2 \ \ E_3 \ \ E_4, E_5] \quad (2.6)$$

where $H_1, H_2, E_1, E_2, E_3, E_4$ and E_5 are known real constant matrices, and $F(t)$ is an unknown time-varying matrix function satisfying

$$\|F(t)\| \le 1, \ \forall t \in [0, \infty). \quad (2.7)$$

Remark 2.1 *It should be noted that (2.1)-(2.4) encompasses many state space models of delay systems and can be used to represent many important physical systems; for example, power systems [13], singular space perturbation theory [14], circuits theory [15], and also cold rolling mills, wind tunnel and water resources systems, see, e.g., Malek-Zavarei and Jamshidi [1] and the references therein.* □

Remark 2.2 *The parameter uncertainty structure as in (2.6)-(2.7) is an extension of the so-called "matching condition" which has been widely used in the problems of robust control and robust filtering of uncertain systems, see, e.g., [16, 17, 18, 19, 20, 21, 22, 23, 24] and the references therein, and many practical systems possess parameter uncertainties which can be either exactly modelled, or overbounded by (2.7). The matrices $H_1, H_2, E_1, E_2, E_3, E_4$ and E_5 specify how the uncertain parameters in $F(t)$ affects the nominal matrices of system (2.1)-(2.4). Observe that the unknown matrix $F(t)$ in (2.6) can even be allowed to be state-dependent, i.e., $F(t) = F(t, x(t))$, as long as (2.7) is satisfied. It also should be noted that the unit overbound for $F(t)$ does not cause any loss of generality. Indeed, $F(t)$ can be always normalized, in the sense of (2.7), by appropriately choosing the matrices $H_1, H_2, E_1, E_2, E_3, E_4$ and E_5. Furthermore, we may consider the more general structure of the uncertainties in system (2.1)-(2.4), that is,*

$$E\dot{x}(t) = Ax(t) + Bu(t) + A_d x(t - d_1(t)) + B_d u(t - d_2(t)) \\ +B_w w(t) + \Delta_1(t, x, u)$$

$$z(t) = Cx(t) + Du(t) + C_d x(t - d_1(t)) + D_d u(t - d_2(t)) \\ +D_w w(t) + \Delta_2(t, x, u)$$

where

$$\|\Delta_i(t, x, u)\| \le a_i\|x\| + b_i\|u\|, \\ i = 1, 2, \ \forall t \in [0, \infty), x \in \mathbf{R}^n, u \in \mathbf{R}^m \quad (2.8)$$

where $a_i \ge 0$ and $b_i \ge 0$, $i = 1, 2$ are known constant numbers. In the work of Shi and Shue [25], it has been shown that the set of the uncertainties satisfying (2.6)-(2.7) is equivalent to the set of the uncertainties satisfying (2.8) after appropriately choosing the constants a_i, b_i and the matrices $H_1, H_2, E_1, E_2, E_3, E_4$ and E_5. □

Definition 2.1 *For any given two matrices $E \in \mathbf{R}^{n\times n}$ and $A \in \mathbf{R}^{n\times n}$, the pencil $(E, \ A)$ is said regular if there exists a constant number α such that $|\alpha E + A| \ne 0$ or the polynomial $|sE - A| \ne 0$.*

In this paper, we assume that the nominal system (2.1) is regular, i.e., the pair $(E, A + A_d e^{-sd1})$ is regular, where $d1 = \max_t d_1(t)$. This condition will guarantee the existence and uniqueness of the solution for the nominal system (2.1). In addition, we assume that the nominal system (2.1) is impulse free, which ensures the delay system has no infinite poles.

Throughout this paper, it is assumed that the state is measurable for feedback.

In this paper, we are concerned with the problem of robust state feedback control for the singular uncertain time-delay system (2.1)-(2.4) for all admissible uncertainties. Our attention is to design a state feedback controller $\mathcal{G}$:

$$u(t) = Kx(t) \quad (2.9)$$

such that for a given scalar $\gamma > 0$, for all non-zero $w(t) \in L_2[0, \infty)$ and for all parameter uncertainties satisfying (2.6)-(2.7)

$$\sup_{0 \ne w \in [0,\infty)} \left(\frac{\|z\|_2}{\|w\|_2}\right) < \gamma. \quad (2.10)$$

In this situation, the system of (2.1)-(2.4) with the controller (2.9) is said to have a *robust H_∞ performance* (2.10). More specifically, our objective is to *design a state feedback controller $\mathcal{G}$ such that: the system of (2.1)-(2.4) with $\mathcal{G}$ is robustly stable and has a robust H_∞ performance (2.10).*

Here, *robustly stable* means that the uncertain system (2.1)-(2.4) is asymptotically stable about the origin for all admissible uncertainties.

In the remainder of this section, we will establish stability and H_∞ control results associated with the nominal system of (2.1)-(2.4), i.e., the case when $F(t) = 0$.

First we recall the following lemma.

Lemma 2.1 *(Schur Complements) Given constant matrices M, L and Q of appropriate dimensions with M and Q are symmetric and $Q > 0$, then $M + L^T QL < 0$ if and only if*

$$\begin{bmatrix} M & L^T \\ L & -Q^{-1} \end{bmatrix} < 0, \quad or \quad \begin{bmatrix} -Q^{-1} & L \\ L^T & M \end{bmatrix} < 0.$$

Theorem 2.1 *Consider the singular time-delay system (2.1) with all uncertainties being zero, control input $u(t) = 0$ and disturbance input $w(t) = 0$. Then, system (2.1) is asymptotically stable for all $d_1(t) \geq 0$ satisfying (2.5), if there exist matrices $P > 0$ and $R_1 > 0$ such that the following inequality holds*

$$\begin{bmatrix} E^T PA + A^T PE + R_1 & E^T PA_d \\ A_d^T PE & -\tilde{R}_1 \end{bmatrix} < 0, \qquad (2.11)$$

where $\tilde{R}_1 = (1 - \beta_1)R_1 > 0$.

Proof. Let us define a Lyapunov functional candidate as

$$V(x(t)) \triangleq x^T(t)E^T PEx(t) + \int_{t-d_1(t)}^{t} x^T(\tau)R_1 x(\tau)d\tau. \qquad (2.12)$$

The derivate of the Lyapunov functional (2.12) along the trajectory of (2.1) is

$$\begin{aligned} \dot{V}(x(t)) &= \dot{x}^T(t)E^T PEx(t) + x^T(t)E^T PE\dot{x}(t) \\ &\quad + x^T(t)R_1 x(t) - (1 - \dot{d}_1(t))x^T(t - d_1(t))R_1 x(t - d_1(t)) \\ &= [Ax(t) + A_d x(t - d_1(t))]^T PEx(t) \\ &\quad + x^T(t)E^T P[Ax(t) + A_d x(t - d_1(t))] \\ &\quad + x^T(t)R_1 x(t) - (1 - \dot{d}_1(t))x^T(t - d_1(t))R_1 x(t - d_1(t)). \end{aligned}$$

By the result of Kreindler and Jameson [26], $\dot{V}(x(t)) < 0$ if

$$\begin{aligned} \dot{\bar{V}}(x(t)) &= [Ax(t) + A_d x(t - d_1(t))]^T PEx(t) \\ &\quad + x^T(t)E^T P[Ax(t) + A_d x(t - d_1(t))] + x^T(t)R_1 x(t) \\ &\quad - (1 - \beta_1)x^T(t - d_1(t))R_1 x(t - d_1(t)) < 0. \qquad (2.13) \end{aligned}$$

By standard matrix manipulation and bearing in mind of (2.11), one has from (2.13)

$$\begin{bmatrix} x(t) \\ x(t - d_1(t)) \end{bmatrix}^T \begin{bmatrix} E^T PA + A^T PE + R_1 & E^T PA_d \\ A_d^T PE & -\tilde{R}_1 \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ x(t - d_1(t)) \end{bmatrix} < 0,$$

which implies the stability of input-free nominal system (2.1). $\nabla\nabla\nabla$

Remark 2.3 *Theorem 2.1 provides a delay independent stability criteria since inequality (2.11) does not include the unknown delay $d_1(t)$. However, we have used the information β_1 on $d_1(t)$, which seems the best we can do. It should be noted that when $E = I$, Theorem 2.1 will reduce to the result in Choi and Chung [27]. Furthermore, when $A_d = 0$ and $E = I$, the inequality (2.11) becomes the standard necessary and sufficient if condition of stability for the non-singular systems without delay.* □

Next, we conduct the H_∞ analysis for the nominal system (2.1)-(2.2) (setting $F(t) = 0$). Note that the closed-loop system of (2.1)-(2.2) under the control law (2.9) is given by

$$\begin{aligned} E\dot{x}(t) &= A_c x(t) + A_d x(t - d_1(t)) + B_d Kx(t - d_2(t)) \\ &\quad + B_w w(t) \qquad (2.14) \\ z(t) &= C_c x(t) + C_d x(t - d_1(t)) + D_d Kx(t - d_2(t)) \\ &\quad + D_w w(t) \qquad (2.15) \end{aligned}$$

where

$$A_c = A + BK, \quad C_c = C + DK.$$

Theorem 2.2 *Consider the singular time-delay system (2.1)-(2.2) with all uncertainties being zero. Then, for a given constant $\gamma > 0$, system (2.1)-(2.2) is asymptotically stable and has an H_∞ performance γ for all $d_i(t) \geq 0, i = 1, 2$ satisfying (2.5) by the controller (2.9), if there exist matrices $P > 0, R_1 > 0$ and $R_2 > 0$ such that the following inequality holds*

$$\begin{bmatrix} E^T PA_c + A_c^T PE + R_1 + K^T R_2 K & E^T PA_d & E^T PB_d & E^T PB_w & C_c^T \\ A_d^T PE & -\tilde{R}_1 & 0 & 0 & C_d^T \\ B_d^T PE & 0 & -\tilde{R}_2 & 0 & D_d^T \\ B_w^T PE & 0 & 0 & -\gamma^2 I & D_w^T \\ C_c & C_d & D_d & D_w & -I \end{bmatrix} < 0, \qquad (2.16)$$

where $\tilde{R}_i = (1 - \beta_i)R_i > 0, \ i = 1, 2.$

Remark 2.4 *Similar to non-singular system state feedback control problem, we can design a suitable control gain K (see for example, Zhou and Khargonekar*

[28]) such that the nominal closed-loop system (2.1)-(2.2) with the controller (2.9) is asymptotically stable and has an H_∞ disturbance attenuation γ. In particular, in Theorem 2.2 if $C_d = 0, D_d = 0, D_w = 0$ and $D^T[C \quad D] = [0 \quad R_2], R_2 > 0$, the control gain K can be chosen as

$$K = -R_2^{-1}B^TPE.$$

In the above, if $E = I$, the controller will become to a standard one for non-singular time-delay systems. □

We finish this section by recalling the following lemmas which will be needed in the proof of our main results in next section.

Lemma 2.2 *[29] Let A, D, E and F be real matrices of appropriate dimensions with $F^TF \le I$. Then*
(a) For any scalar $\varepsilon > 0$, we have

$$DFE + E^TF^TD^T \le \varepsilon DD^T + \varepsilon^{-1}E^TE$$

(b) For any real matrix $P = P^T > 0$ and scalar $\varepsilon > 0$ satisfying $\varepsilon I - EPE^T > 0$, we have

$$\begin{aligned}&(A + DFE)P(A + DFE)^T \\ &\quad \le APA^T + APE^T(\varepsilon I - EPE^T)^{-1}EPA^T + \varepsilon DD^T\end{aligned}$$

(c) For any real matrix $P = P^T > 0$ and scalar $\varepsilon > 0$ satisfying $P - \varepsilon DD^T > 0$, we have

$$\begin{aligned}&(A + DFE)^TP^{-1}(A + DFE) \\ &\quad \le A^T(P - \varepsilon DD^T)^{-1}A + \varepsilon^{-1}E^TE.\end{aligned}$$

Lemma 2.3 *[30] Consider the system*

$$\begin{aligned}\dot{x}(t) &= [A + \Delta A(t)]x(t) + [B_w + \Delta B_w(t)]w(t) \\ &\quad +[B_u + \Delta B_u(t)]u(t) \\ z(t) &= [C_z + \Delta C_z(t)]x(t) + [D_{zw} + \Delta D_{zw}(t)]w(t) \\ &\quad [D_{zu} + \Delta D_{zu}(t)]u(t) \\ y(t) &= [C_y + \Delta C_y(t)]x(t) + [D_{yw} + \Delta D_{yw}(t)]w(t) \\ &\quad [D_{yu} + \Delta D_{yu}(t)]u(t),\end{aligned}$$

where $u(t)$ is the control input, $w(t)$ is the disturbance input, $y(t)$ is the measured output and $z(t)$ is the controlled output, with uncertainties

$$\begin{aligned}&\begin{bmatrix} \Delta A(t) & \Delta B_w(t) & \Delta B_u(t) \\ \Delta C_z(t) & \Delta D_{zw}(t) & \Delta D_{zu}(t) \\ \Delta C_y(t) & \Delta D_{yw}(t) & \Delta D_{yu}(t) \end{bmatrix} \\ &= \begin{bmatrix} H_x \\ H_z \\ H_y \end{bmatrix} \Delta(t)[E_x \quad E_w \quad E_u], \quad \Delta^T(t)\Delta(t) \le I.\end{aligned}$$

This system is stabilizable and has an H_∞ performance $\gamma > 0$ by a linear output feedback control if and only if there exists a $\lambda > 0$ such that the uncertainty-free system

$$\dot{x}(t) = Ax(t) + [B_w \quad \gamma\lambda H_x] \begin{bmatrix} w(t) \\ \hat{w}(t) \end{bmatrix} + B_uu(t)$$

$$\begin{aligned}\begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} &= \begin{bmatrix} C_z \\ \frac{1}{\lambda}E_x \end{bmatrix} x(t) + \begin{bmatrix} D_{zw} & \gamma\lambda H_z \\ \frac{1}{\lambda}E_w & 0 \end{bmatrix} \begin{bmatrix} w(t) \\ \hat{w}(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} D_{zu} \\ \frac{1}{\lambda}E_u \end{bmatrix} u(t)\end{aligned}$$

$$y(t) = C_y(t)x(t) + [D_{yw} \quad \gamma\lambda H_y] \begin{bmatrix} w(t) \\ \hat{w}(t) \end{bmatrix} + D_{yu}u(t)$$

with $u(t)$ the control input, $[w^T(t) \quad \hat{w}^T(t)]^T$ the disturbance input, $y(t)$ the measured output and $[z^T(t) \quad \hat{z}^T(t)]^T$ the controlled output, is stabilizable and has an H_∞ performance γ via an output feedback control.

Lemma 2.3 establishes that the problem of output feedback H_∞ control of uncertainty systems can be converted to a standard H_∞ control problem for systems without parameter uncertainty. Note that the latter can be solved by existing results, see, for example, Safonov *et al.* [31].

3. Robust Control Design

This section will provides, on the basis of Theorems 2.1 and 2.2, the robust stability and robust control results for uncertain singular system (2.1)-(2.2).

We first deal with the problem of robust stability for system (2.1).

Theorem 3.1 *Consider the singular time-delay system (2.1) with input-free($u(t) = 0$ and $w(t) = 0$). Then, system (2.1) is asymptotically stable for all $d_1(t) \ge 0$ satisfying (2.5) and all admissible parameter uncertainties, if there exist two matrices $P > 0$, $R_1 > 0$ and a scalar $\varepsilon > 0$ satisfying $\tilde{R}_1 - \varepsilon E_2^TE_2 > 0$ such that the following inequality holds*

$$\begin{bmatrix} E^TPA + A^TPE + R_1 + \varepsilon E^TPH_1H_1^TPE + \varepsilon^{-1}E_1^TE_1 & E^TPA_d & H_1 \\ A_d^TPE & -(\tilde{R}_1 - \varepsilon E_2^TE_2) & 0 \\ H_1^T & 0 & -\varepsilon^{-1}I \end{bmatrix} < 0,$$

where $\tilde{R}_1 = (1 - \beta_1)R_1 > 0$.

Inspired by Lemma 2.3, we can have the following lemma. The proof is almost identical, so we omit it here.

Lemma 3.1 *Consider the system*

$$\begin{aligned}\dot{x}(t) &= [A + \Delta A(t)]x(t) + [A_d + \Delta A_d(t)]x(t - d_1(t)) \\ &\quad +[B_u + \Delta B_u(t)]u(t) + [B_d + \Delta B_d(t)]u(t - d_2(t)) \\ &\quad +[B_w + \Delta B_w(t)]w(t) \\ z(t) &= [C_z + \Delta C_z(t)]x(t) + [C_d + \Delta C_d(t)]x(t - d_1(t)) \\ &\quad +[D_{zu} + \Delta D_{zu}(t)]u(t) + [D_d + \Delta D_d(t)]u(t - d_2(t)) \\ &\quad +[D_{zw} + \Delta D_{zw}(t)]w(t),\end{aligned}$$

where $u(t)$ is the control input, $w(t)$ is the disturbance input, and $z(t)$ is the controlled output, with uncertainties

$$\left[\begin{array}{ccccc} \Delta A(t) & \Delta A_d & \Delta B_u(t) & \Delta B_d(t) & \Delta B_w(t) \\ \Delta C_z(t) & \Delta C_d(t) & \Delta D_{zu}(t) & \Delta D_d(t) & \Delta D_{zw}(t) \end{array}\right] = \left[\begin{array}{c} H_x \\ H_z \end{array}\right] \Delta(t)[E_x\ E_{dx}\ E_u\ E_{du}\ E_w]$$

$$\Delta^T(t)\Delta(t) \leq I.$$

This system is stabilizable and has an H_∞ performance $\gamma > 0$ by a linear state feedback control if and only if there exists a $\lambda > 0$ such that the uncertainty-free system

$$\dot{x}(t) = Ax(t) + A_d x(t - d_1(t)) + B_u u(t) + B_d u(t - d_2(t)) + [B_w \quad \gamma\lambda H_x] \left[\begin{array}{c} w(t) \\ \hat{w}(t) \end{array}\right]$$

$$\left[\begin{array}{c} z(t) \\ \hat{z}(t) \end{array}\right] = \left[\begin{array}{c} C_z \\ \frac{1}{\lambda}E_x \end{array}\right] x(t) + \left[\begin{array}{c} C_d \\ \frac{1}{\lambda}E_{dx} \end{array}\right] x(t - d_1(t)) + \left[\begin{array}{c} D_{zu} \\ \frac{1}{\lambda}E_u \end{array}\right] u(t) + \left[\begin{array}{c} D_d \\ \frac{1}{\lambda}E_{du} \end{array}\right] u(t - d_2(t)) + \left[\begin{array}{cc} D_{zw} & \gamma\lambda H_z \\ \frac{1}{\lambda}E_w & 0 \end{array}\right] \left[\begin{array}{c} w(t) \\ \hat{w}(t) \end{array}\right],$$

with $u(t)$ the control input, $[w^T(t) \quad \hat{w}^T(t)]^T$ the disturbance input, and $[z^T(t) \quad \hat{z}^T(t)]^T$ the controlled output, is stabilizable and has an H_∞ performance γ via a state feedback control.

Now, we are ready to present our main result in this paper.

Theorem 3.2 *Consider the singular time-delay system (2.1)-(2.2). Then, for a given constant $\gamma > 0$, the closed-loop system (2.1)-(2.2) is asymptotically stable and has an H_∞ performance γ for all $d_i(t) \geq 0, i = 1, 2$ satisfying (2.5) by the controller (2.9), if there exist matrices $P > 0, R_1 > 0$, $R_2 > 0$ and a scalar $\lambda > 0$ such that the following inequality holds*

$$\left[\begin{array}{ccccc} E^T PA_c + A_c^T PE + R_1 + K^T R_2 K & E^T PA_d & E^T PB_d & E^T P\bar{B}_w & \bar{C}_c^T \\ A_d^T PE & -\tilde{R}_1 & 0 & 0 & \bar{C}_d^T \\ B_d^T PE & 0 & -\tilde{R}_2 & 0 & \bar{D}_d^T \\ \bar{B}_w^T PE & 0 & 0 & -\gamma^2 I & \bar{D}_w^T \\ \bar{C}_c & \bar{C}_d & \bar{D}_d & \bar{D}_w & -I \end{array}\right] < 0,$$

where

$$A_c = A + BK, \quad \bar{B}_w = [B_w \quad \gamma\lambda H_1],$$

$$\bar{C}_c = \left[\begin{array}{c} C \\ \frac{1}{\lambda}E_1 \end{array}\right] + \bar{D}K, \quad \bar{D} = \left[\begin{array}{c} D \\ \frac{1}{\lambda}E_3 \end{array}\right]$$

$$\bar{C}_d = \left[\begin{array}{c} C_d \\ \frac{1}{\lambda}E_2 \end{array}\right], \quad \bar{D}_d = \left[\begin{array}{c} D_d \\ \frac{1}{\lambda}E_4 \end{array}\right],$$

$$\bar{D}_w = \left[\begin{array}{cc} D_w & \gamma\lambda H_2 \\ \frac{1}{\lambda}E_5 & 0 \end{array}\right]$$

$$\tilde{R}_i = (1 - \beta_i)R_i > 0, \quad i = 1, 2.$$

Proof. The desired result can be carried out by using Lemma 3.1, together with Theorem 2.2. ∇∇∇

Remark 3.1 *Similar to Remark 2.4, we can design a suitable state feedback control gain K (see for example, Zhou and Khargonekar [28]) such that the uncertain closed-loop system (2.1)-(2.2) with the controller (2.9) is robustly asymptotically stable and has a robust H_∞ disturbance attenuation γ. In particular, in Theorem 3.2, if $C_d = 0, D_d = 0, D_w = 0$ and $D^T[C, \quad D] = [0, \quad R_2], R_2 > 0$, the control gain K can be chosen as*

$$K = -\tilde{R}_2^{-1}(B^T PE + E_3^T E_1)$$

where $\tilde{R}_2 = R_2 + E_3^T E_1$. If system (2.1) is not singularly perturbed, i.e., $E = I$, in the above, the controller will become to a standard one for non-singular uncertain time-delay systems. □

Remark 3.2 *Theorem 3.2 provides a sufficient condition to ensure the closed-loop system (2.1)-(2.2) with the control law (2.9) is asymptotically stable and satisfy the H_∞ performance (2.10). Note that when $E = I$, and time-delays $d_1(t) = 0$ and $d_2(t) = 0$ in (2.1)-(2.2), Theorem 3.2 will cover the robust H_∞ state feedback control result as a special case, for example, the work by Xie [32]. Furthermore, if uncertainties also disappear in (2.1)-(2.2), Theorem 3.2 will reduce to standard H_∞ state feedback control result, e.g. Doyle* et al. *[33].* □

4. Conclusion

This paper dealt with the class of singular continuous-time systems with delay. Under the norm bounded uncertainties the asymptotic stability, the stabilizability and the H_∞ control problems and their robustness have been studied. Delay independent sufficient conditions have been provided to solve all the problems. Presently we are working on the more general delay-dependent conditions for these problems.

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