Robust Control of Uncertain Systems with Input Delay and Input Sector Nonlinearity

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Abstract

In this paper, we investigate a robust control method for some nonlinear control problems with an input delay. By letting input nonlinearity in the sector bounds as a new diagonal structured uncertainty, we transform the control problems with input nonlinearity into the robust control problems of linear systems with only structured uncertainty. Applying this idea, we obtain linear matrix inequality(LMI) conditions for delay-dependent robust stabilization of structured uncertain systems with input delay and input sector nonlinearity. In addition to LMIs for the fixed input nonlinearity, we also propose an iterative LMI optimization algorithm to find robust input sector bounds such that the given uncertain system is stable for any input nonlinearity in these sector bounds.

Keywords : Robust control, Structured uncertainty, Input sector nonlinearity, Input delay, Linear matrix inequalities

1. Introduction

Input delay is a source of instability, which is frequently encountered in real physical systems since measurement delay and computational delay can be represented by input delay. Currently, robust control problems of state delayed systems have been extensivly studied[1] and results on these can be classified as delay-independent and delay-depenent ones. With regard to input delay, Li et. al.[2] recently provided delay-dependent LMI conditions for robust stability of input delayed systems with norm-bounded unstructured uncertainty.

On the other hand, control problems with input nonlinearity in the sector bounds have been investigated because solutions to these are useful not only in the control problems with input nonlinearity included in the given sector bounds but also in those problems with saturation inputs operating in the restricted zones. In the classical nonlinear control problems, input sector nonlinerity was treated by circle criterion[3, 4], which requires graphical Nyquist plots or solutions of quadratic matrix equations. Recently, Gu et al.[5] provided robust H_∞ control conditions for systems with input sector nonlinearity by Riccati equation approach. Based on the idea[6] that norm-bounded nonlinear uncertainty can be represented by an equivalent linear unstructured uncertainty, Gu et al. regarded sector nonlinearity as norm-bounded unstructured uncertainty and solved the corresponding robust control problems.

In this paper, we treat robust control problems of uncertain systems with input delay and input sector nonlinearity simultaneously. We extend the idea of Gu et. al. to more general structured uncertain systems and use a LMI approach. LMI techniques are attractive tools since a very efficient computational framework[7] exists for solving the control problems formulated as LMIs. While Gu et. al. dealt with only homogeneous sector nonlinearity and assumes no uncertainty in the input matrix, we consider not only elementwise sector nonlinearity but also structured uncertainty in all system matrices. For this purpose, we transform elementwise sector nonlinearity into the diagonal structured uncertainty and treat new composite structured uncertainty composed of the original structured uncertainty, the transformed uncertainty accounting for input nonlinearity in the sector bounds, and unstructured uncertainty resulting from the product of these. From this, we obtain finally structured uncertain systems without input nonlinearity and solve the corresponding robust control problems by applying a LMI optimization method for input delayed systems.

2. Main results

In this section, input nonlinearity is given by the following elementwise sector bounds shown in Figure 1.

$$\mathrm{diag}\begin{bmatrix}\varepsilon_1 & \cdots & \varepsilon_m\end{bmatrix} \le N(u) \le I \tag{1}$$

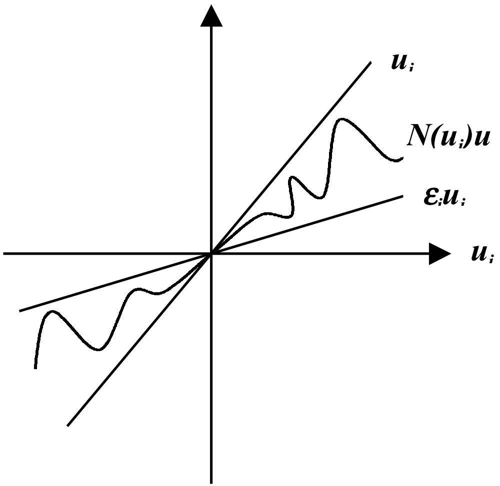

Figure 1. Elementwise input sector nonlinearity

In (1), $N(u) = \mathrm{diag}\left[N_1(u_1) \ \cdots \ N_m(u_m)\right]$ and $0 < \varepsilon_i \leq 1,\ i = 1,\cdots,m$.

From the elementwise sector bounds $\varepsilon_i \leq N_i(u_i) \leq 1$, we obtain (2) by subtracting $\dfrac{\varepsilon_i + 1}{2}$.

$$\left(\varepsilon_i - \frac{\varepsilon_i + 1}{2}\right) = -\frac{1-\varepsilon_i}{2} \leq N_i(u_i) - \frac{\varepsilon_i + 1}{2} \leq \left(1 - \frac{\varepsilon_i + 1}{2}\right) = \frac{1-\varepsilon_i}{2} \quad (2)$$

If we define $\hat{\Delta}_2 = N(u) - \mathrm{diag}\begin{bmatrix} \frac{1+\varepsilon_1}{2} & \cdots & \frac{1+\varepsilon_m}{2} \end{bmatrix}$, $\hat{\Delta}_2$ satisfies (3).

$$\hat{\Delta}_2^T \hat{\Delta}_2 \leq \mathrm{diag}\begin{bmatrix} \frac{(1-\varepsilon_1)^2}{4} & \cdots & \frac{(1-\varepsilon_m)^2}{4} \end{bmatrix} \quad (3)$$

Let $\Delta_2 = \hat{\Delta}_2 \left\{ \mathrm{diag}\begin{bmatrix} \frac{1-\varepsilon_1}{2} & \cdots & \frac{1-\varepsilon_m}{2} \end{bmatrix} \right\}^{-1}$, then

$$\hat{\Delta}_2 = \Delta_2 \left\{ \mathrm{diag}\begin{bmatrix} \frac{1-\varepsilon_1}{2} & \cdots & \frac{1-\varepsilon_m}{2} \end{bmatrix} \right\} \quad (4)$$

where $\Delta_2^T \Delta_2 \leq I$. From the definition of $\hat{\Delta}_2$, we obtain (5).

$$\begin{aligned} N(u) &= \hat{\Delta}_2 + \mathrm{diag}\begin{bmatrix} \frac{1+\varepsilon_1}{2} & \cdots & \frac{1+\varepsilon_m}{2} \end{bmatrix} = \Delta_2 \left(\mathrm{diag}\begin{bmatrix} \frac{1-\varepsilon_1}{2} & \cdots & \frac{1-\varepsilon_m}{2} \end{bmatrix} \right) \\ &+ \mathrm{diag}\begin{bmatrix} \frac{1+\varepsilon_1}{2} & \cdots & \frac{1+\varepsilon_m}{2} \end{bmatrix} = \Delta_2 \Lambda_- + \Lambda_+ \end{aligned} \quad (5)$$

By the relation (5), we can represent input nonlinearity in (1) as diagonal structured uncertainty.

Now, we consider the following structured uncertain system with input delay and input sector nonlinearity.

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + (B + \Delta B(t))N_b(u)u(t) \\ &+ (B_d + \Delta B_d(t))N_d(u)u(t - \tau(t)) \end{aligned} \quad (6)$$

$$x(t) = \phi(t),\ \forall\, t \in [-\bar{\tau}, 0],\ \bar{\tau} > 0 \quad (7)$$

where $\mathrm{diag}\left[\varepsilon_{b1} \ \cdots \ \varepsilon_{bm}\right] \leq N_b(u) \leq I$, $\mathrm{diag}\left[\varepsilon_{d1} \ \cdots \ \varepsilon_{dm}\right] \leq N_d(u) \leq I$, and $\tau(t)$ is a time-varying delay satisfying

$$0 \leq \tau(t) \leq \bar{\tau},\ 0 \leq \dot{\tau}(t) < \mu < 1, \quad (8)$$

$x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, $\phi(\cdot)$ is the initial condition, A, B, B_d are known real constant matrices of appropriate dimensions and the uncertainties $\Delta A(\cdot)$, $\Delta B(\cdot)$, $\Delta B_d(\cdot)$ are assumed to be of the form

$$\Delta A(t) = D_a \Delta_a(t) E_a, \Delta B(t) = D_b \Delta_b(t) E_b, \Delta B_d(t) = D_d \Delta_d(t) E_d \quad (9)$$

where D_a, D_b, D_d, E_a, E_b, E_d are known real constant matrices and $\Delta_a(t) \in \Delta_s$, $\Delta_b(t) \in \Delta_s$, $\Delta_d(t) \in \Delta_s$ are unknown real time-varying matrices such that $\Delta_a(t)^T \Delta_a(t) \leq I$, $\Delta_b(t)^T \Delta_b(t) \leq I$, $\Delta_d(t)^T \Delta_d(t) \leq I$. Δ_s is the set of structured uncertain matrices given in (10).

$$\Delta_s = \left\{ \begin{array}{c} \Delta(t) \in R^{(r_1 + \cdots r_k + f_1 + \cdots + f_l) \times (r_1 + \cdots r_k + f_1 + \cdots + f_l)} : \\ \Delta(t) = \mathrm{blockdiag}\left[\delta_1(t) I_{r_1} \ \cdots \ \delta_k(t) I_{r_k} \ \ \Delta_1(t) \ \cdots \ \Delta_l(t)\right] \\ \delta_i(t) \in R \text{ for } 1 \leq i \leq k \text{ and } \Delta_j \in R^{f_l \times f_l} \text{ for } 1 \leq j \leq l \end{array} \right\} \quad (10)$$

For structured uncertainty given by (10), we use the following scaling matrices in order to reduce conservativeness.

$$\Sigma_R = \left\{ \begin{array}{c} T = \mathrm{blockdiag}\left[T_1 \ \cdots \ T_k \ \ d_1 I_{f_1} \ \cdots \ d_s I_{f_s}\right] \in R^{(r_1 + \cdots r_k + f_1 + \cdots + f_s) \times (r_1 + \cdots r_k + f_1 + \cdots + f_s)} : \\ T_i \in R^{r_i \times r_i},\ T_i > 0 \text{ for } 1 \leq i \leq k \text{ and } d_j \in R,\ d_j > 0 \text{ for } 1 \leq j \leq s \end{array} \right\} \quad (11)$$

The objective of this paper is to design a robust state feedback controller $u = Kx(t)$ and find the maximum bound $\bar{\tau}$ of the time-delay for the structured uncertain system (6) such that the closed-loop system with $u = Kx(t)$ is asymptotically stable for any time delay $\tau(t)$ satisfying $0 \leq \tau(t) \leq \bar{\tau}$, $0 \leq \dot{\tau}(t) < \mu < 1$, $\forall\, t \geq 0$ and for any input nonlinearity in the given sector bounds.

From (6), we let

$$N_b(u) = \Delta_{b2} \Lambda_{b-} + \Lambda_{b+} \text{ and } N_d(u) = \Delta_{d2} \Lambda_{d-} + \Lambda_{d+} \quad (12)$$

where $\Delta_{b2}^T \Delta_{b2} \leq I$, $\Delta_{d2}^T \Delta_{d2} \leq I$ and Λ_{b-}, Λ_{b+}, Λ_{d-}, Λ_{d+} are given as follows.

$$\begin{aligned} \Lambda_{b-} &= \mathrm{diag}\begin{bmatrix} \frac{1-\varepsilon_{b1}}{2} & \cdots & \frac{1-\varepsilon_{bm}}{2} \end{bmatrix}, \\ \Lambda_{b+} &= \mathrm{diag}\begin{bmatrix} \frac{1+\varepsilon_{b1}}{2} & \cdots & \frac{1+\varepsilon_{bm}}{2} \end{bmatrix},\ 0 < \varepsilon_{bi} \leq 1 \end{aligned} \quad (13)$$

$$\begin{aligned} \Lambda_{d-} &= \mathrm{diag}\begin{bmatrix} \frac{1-\varepsilon_{d1}}{2} & \cdots & \frac{1-\varepsilon_{dm}}{2} \end{bmatrix}, \\ \Lambda_{d+} &= \mathrm{diag}\begin{bmatrix} \frac{1+\varepsilon_{d1}}{2} & \cdots & \frac{1+\varepsilon_{dm}}{2} \end{bmatrix},\ 0 < \varepsilon_{di} \leq 1 \end{aligned} \quad (14)$$

With $u = Kx(t)$, the uncertain system (6) are rewritten as (15) by using the relation (5).

$$\begin{aligned} \dot{x}(t) &= (A + D_a \Delta_a(t) E_a)x(t) \\ &+ (B + D_b \Delta_b(t) E_b)\{\Delta_{b2} \Lambda_{b-} + \Lambda_{b+}\}Kx(t) \\ &+ (B_d + D_d \Delta_d(t) E_d)\{\Delta_{d2} \Lambda_{d-} + \Lambda_{d+}\}Kx(t - \tau(t)) \\ &= (A + D_a \Delta_a(t) E_a)x(t) + \{B \Lambda_{b+} + D_b \Delta_b(t) E_b \Lambda_{b+} \\ &+ B \Delta_{b2} \Lambda_{b-} + D_b \Delta_b(t) E_b \Delta_{b2} \Lambda_{b-}\}Kx(t) \\ &+ \{B_d \Lambda_{d+} + D_d \Delta_d(t) E_d \Lambda_{d+} + B_d \Delta_{d2} \Lambda_{d-} \\ &+ D_d \Delta_d(t) E_d \Delta_{d2} \Lambda_{d-}\}Kx(t - \tau(t)) \end{aligned} \quad (15)$$

We assume T_{12} and define E_s and E_m as follows.

$$E_s^T = \begin{cases} I_s & : s \geq m \\ [I_s \ 0_{s \times (m-s)}] : s < m \end{cases},\quad E_m = \begin{cases} [I_m \ 0_{m \times (s-m)}] : s \geq m \\ I_m & : s < m \end{cases} \quad (16)$$

Note that $E_s^T E_s = I_s$, $E_m E_m^T = I_m$, and $E_m^T E_m = \begin{cases} \leq I_s : s \geq m \\ I_m \ \ : s < m \end{cases}$ in (16).

Let

$$\hat{\Delta}_{b3} = E_s \Delta_b(t) E_b \Delta_{b2} E_m, \quad (17)$$

then

$$E_s^T \hat{\Delta}_{b3} E_m^T = E_s^T E_s \Delta_b(t) E_b \Delta_{b2} E_m E_m^T = \Delta_b(t) E_b \Delta_{b2},$$

$$\begin{aligned} \hat{\Delta}_{b3}^T \hat{\Delta}_{b3} &\leq E_m^T \Delta_{b2}^T E_b^T \Delta_b^T(t) E_s^T E_s \Delta_b(t) E_b \Delta_{b2} E_m \\ &\leq \lambda_{\max}(E_b^T E_b) E_m^T E_m \leq \lambda_{\max}(E_b^T E_b) I = \rho_b^2 I \end{aligned}$$

where $\rho_b = \sqrt{\lambda_{\max}(E_b^T E_b)}$. Define

$$\Delta_{b3} = \frac{\hat{\Delta}_{b3}}{\rho_b}, \quad \hat{\Delta}_{b3} = \rho_b \Delta_{b3} \tag{18}$$

where $\Delta_{b3}^T \Delta_{b3} \le I$. Similarly as these, for $\Delta_d(t) \in R^s$, let

$$\hat{\Delta}_{d3} = E_s \Delta_d(t) E_d \Delta_{d2} E_m \tag{19}$$

then, we have

$$\Delta_{d3} = \frac{\hat{\Delta}_{d3}}{\rho_d}, \quad \hat{\Delta}_{d3} = \rho_d \Delta_{d3} \tag{20}$$

where $\Delta_{d3}^T \Delta_{d3} \le I$ and $\rho_d = \sqrt{\lambda_{\max}(E_d^T E_d)}$. Note that although s in (17) and s in (19) need not be same, we use the same symbol for simplicity.

From (13), (14), (18), and (20), (15) are reduced to the linear delayed system with only affine structured uncertainty and an input delay.

$$\begin{aligned}
\dot{x}(t) &= (A + D_a \Delta_a(t) E_a) x(t) + \{B\Lambda_{b+} + D_b \Delta_b(t) E_b \Lambda_{b+} \\
&\quad + B\Delta_{b2}\Lambda_{b-} + \rho_b D_b E_s^T \Delta_{b3} E_m^T \Lambda_{b-}\} Kx(t) \\
&\quad + \{B_d \Lambda_{d+} + D_d \Delta_d(t) E_d \Lambda_{d+} + B_d \Delta_{d2} \Lambda_{d-} \\
&\quad + \rho_d D_d E_s^T \Delta_{d3} E_m^T \Lambda_{d-}\} Kx(t - \tau(t)) \\
&= (A + D_a \Delta_a(t) E_a) x(t) \\
&\quad + \left\{ \begin{array}{l} B\Lambda_{b+} + \begin{bmatrix} D_b & B & \rho_b D_b E_s^T \end{bmatrix} \\ \times \begin{bmatrix} \Delta_b & 0 & 0 \\ 0 & \Delta_{b2} & 0 \\ 0 & 0 & \Delta_{b3} \end{bmatrix} \begin{bmatrix} E_b \Lambda_{b+} \\ \Lambda_{b-} \\ E_m^T \Lambda_{b-} \end{bmatrix} \end{array} \right\} Kx(t) \\
&\quad + \left\{ \begin{array}{l} B_d\Lambda_{d+} + \begin{bmatrix} D_d & B_d & \rho_d D_d E_s^T \end{bmatrix} \\ \times \begin{bmatrix} \Delta_d & 0 & 0 \\ 0 & \Delta_{d2} & 0 \\ 0 & 0 & \Delta_{d3} \end{bmatrix} \begin{bmatrix} E_d \Lambda_{d+} \\ \Lambda_{d-} \\ E_m^T \Lambda_{d-} \end{bmatrix} \end{array} \right\} Kx(t - \tau(t))
\end{aligned} \tag{21}$$

In (21), $\rho_b = \sqrt{\lambda_{\max}(E_b^T E_b)}$, $\rho_d = \sqrt{\lambda_{\max}(E_d^T E_d) I}$, $\Delta_a, \Delta_b, \Delta_d \in \Delta_s$ are the original structured uncertainty, Δ_{b2}, Δ_{d2} is the transformed diagonal uncertainty from input nonlinearity, and Δ_{b3}, Δ_{d3} is the unstructured uncertainty resulting from the product of the original uncertainty and the transformed uncertainty accounting for input nonlinearity.

For the structured uncertain system (21) with an input-delay, we obtain Theorem 1 by Lyapunov-Krasovskii approach[8].

Theorem 1. Given scalars $\bar{\tau} > 0$, $\mu > 0$, positive ε_{bi}'s and ε_{di}'s, the structured uncertain system (6) with input delay and input sector nonlineraity satisfying $\mathrm{diag}[\varepsilon_{b1} \ \cdots \ \varepsilon_{bm}] \le N_b(u) \le I$ and $\mathrm{diag}[\varepsilon_{d1} \ \cdots \ \varepsilon_{dm}] \le N_d(u) \le I$ is robustly stabilizable for any time-delay $\tau(t)$ satisfying $0 \le \tau(t) \le \bar{\tau}$ and $0 \le \dot{\tau}(t) < \mu < 1$ if there exist symmetric positive definite matrices X, P_1, P_2, P_3, Q, T_1, T_{21}, T_{22}, t_{23}, T_{31}, T_{32}, t_{33}, T_{41}, T_{42}, t_{43}, T_5, T_{61}, T_{62}, t_{63}, T_{71}, T_{72}, t_{73} and a matrix Y satisfying the following LMIs:

$$\begin{bmatrix}
\Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & \Pi_{16} & \Pi_{17} & \Pi_{18} \\
\Pi_{12}^T & \Pi_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\
\Pi_{13}^T & 0 & \Pi_{33} & 0 & 0 & 0 & 0 & 0 \\
\Pi_{14}^T & 0 & 0 & \Pi_{44} & 0 & 0 & 0 & 0 \\
\Pi_{15}^T & 0 & 0 & 0 & \Pi_{55} & 0 & 0 & 0 \\
\Pi_{16}^T & 0 & 0 & 0 & 0 & \Pi_{66} & 0 & 0 \\
\Pi_{17}^T & 0 & 0 & 0 & 0 & 0 & \Pi_{77} & 0 \\
\Pi_{18}^T & 0 & 0 & 0 & 0 & 0 & 0 & \Pi_{88}
\end{bmatrix} < 0 \tag{22}$$

$$\begin{bmatrix} Q & Y \\ Y^T & X \end{bmatrix} \ge 0, \quad X - P_1 - P_2 - P_3 \ge 0 \tag{23}$$

where

$$\begin{aligned}
\Pi_{11} &= XA^T + AX + Y^T(B\Lambda_{b+} + B_d\Lambda_{d+})^T + (B\Lambda_{b+} + B_d\Lambda_{d+})Y \\
&\quad + D_a T_1 D_a^T + \bar{\tau} B_d \Lambda_{d+} Q \Lambda_{d+}^T B_d^T + D_b T_{21} D_b^T + B T_{22} B^T \\
&\quad + \rho_b^2 t_{23} D_b D_b^T + D_d T_{31} D_d^T + B_d T_{32} B_d^T + \rho_d^2 t_{33} D_d D_d^T \\
&\quad + \bar{\tau} D_d T_{41} D_d^T + \bar{\tau} B_d T_{42} B_d^T + \bar{\tau} \rho_d^2 t_{43} D_d D_d^T
\end{aligned},$$

$\Pi_{12} = XE_a^T$, $\Pi_{13} = \begin{bmatrix} Y^T\Lambda_{b+}^T E_b^T & Y^T\Lambda_{b-}^T & Y^T\Lambda_{b-}^T E_m \end{bmatrix}$,

$\Pi_{14} = \begin{bmatrix} Y^T\Lambda_{d+}^T E_d^T & Y^T\Lambda_{d-}^T & Y^T\Lambda_{d-}^T E_m \end{bmatrix}$,

$\Pi_{15} = \begin{bmatrix} \bar{\tau} B_d \Lambda_{d+} Q \Lambda_{d+}^T E_d^T & \bar{\tau} B_d \Lambda_{d+} Q \Lambda_{d-}^T & \bar{\tau} B_d \Lambda_{d+} Q \Lambda_{d-}^T E_m \end{bmatrix}$,

$\Pi_{16} = \begin{bmatrix} \bar{\tau} XA^T & \bar{\tau} XE_a^T \end{bmatrix}$,

$\Pi_{17} = \begin{bmatrix} \bar{\tau} Y^T \Lambda_{b+}^T B^T & \bar{\tau} Y^T \Lambda_{b+}^T E_b^T & \bar{\tau} Y^T \Lambda_{b-}^T & \bar{\tau} Y^T \Lambda_{b-}^T E_m \end{bmatrix}$,

$\Pi_{18} = \begin{bmatrix} \bar{\tau} Y^T \Lambda_{d+}^T B_d^T & \bar{\tau} Y^T \Lambda_{d+}^T E_d^T & \bar{\tau} Y^T \Lambda_{d-}^T & \bar{\tau} Y^T \Lambda_{d-}^T E_m \end{bmatrix}$,

$\Pi_{22} = -T_1$, $\Pi_{33} = -\mathrm{diag}\begin{bmatrix} T_{21} & T_{22} & t_{23} I \end{bmatrix}$,

$\Pi_{44} = -\mathrm{diag}\begin{bmatrix} T_{31} & T_{32} & t_{33} I \end{bmatrix}$,

$$\Pi_{55} = -\bar{\tau} \left\{ \begin{bmatrix} T_{41} & 0 & 0 \\ 0 & T_{42} & 0 \\ 0 & 0 & t_{43} I \end{bmatrix} - \begin{bmatrix} E_d \Lambda_{d+} \\ \Lambda_{d-} \\ E_m^T \Lambda_{d-} \end{bmatrix} Q \begin{bmatrix} E_d \Lambda_{d+} \\ \Lambda_{d-} \\ E_m^T \Lambda_{d-} \end{bmatrix}^T \right\},$$

$\Pi_{66} = -\bar{\tau}\,\mathrm{diag}\begin{bmatrix} (P_1 - D_a T_5 D_a^T) & T_5 \end{bmatrix}$,

$\Pi_{77} = -\bar{\tau}\,\mathrm{diag}\begin{bmatrix} (P_2 - D_b T_{61} D_b^T - B T_{62} B^T - \rho_b^2 t_{63} D_b D_b^T) & T_{61} & T_{62} & t_{63} I \end{bmatrix}$,

$$\begin{aligned}
\Pi_{88} &= -\bar{\tau}(1-\mu) \\
&\quad \times \mathrm{diag}\begin{bmatrix} (P_3 - D_d T_{71} D_d^T - B_d T_{72} B_d^T - \rho_d^2 t_{73} D_d D_d^T) & T_{71} & T_{72} & t_{73} I \end{bmatrix}.
\end{aligned}$$

In the above LMIs, E_m is given in (16), $\rho_b = \sqrt{\lambda_{\max}(E_b^T E_b)}$, $\rho_d = \sqrt{\lambda_{\max}(E_d^T E_d)}$, Λ_{b-}, Λ_{b+}, Λ_{d-}, Λ_{d+} are given by (13) and (14). T_1, T_5 are structured matrices in Σ_R corresponding to $\Delta_a(t)$, T_{21}, T_{61} are structured matrices in Σ_R corresponding to $\Delta_b(t)$, and T_{31}, T_{41}, T_{71} are structured matrices in Σ_R corresponding to $\Delta_d(t)$. T_{i2}'s for $i = 2,3,4,6,7$ are diagonal matrices and t_{i3}'s for $i = 2,3,4,6,7$ are scalars. Moreover, a stabilizing controller is given by $u(t) = YX^{-1}x(t)$.

Proof. To begin with, we consider the following input delayed system without input sector nonlinearity.

$$\begin{aligned}
\dot{x}(t) &= (A + D_a \Delta_a(t) E_a) x(t) + (B + D_b \Delta_b(t) E_b) u(t) \\
&\quad + (B_d + D_d \Delta_d(t) E_d) u(t - \tau(t))
\end{aligned} \tag{24}$$

With $u(t) = Kx(t)$,

$$x(t-\tau(t)) = x(t) - \int_{-\tau(t)}^{0} \dot{x}(t+\theta)d\theta$$

$$= x(t) - \int_{-\tau(t)}^{0} \begin{bmatrix} (A + D_a\Delta_a(t+\theta)E_a)x(t+\theta) \\ +(B + D_b\Delta_b(t+\theta)E_b)Kx(t+\theta) \\ +(B_d + D_d\Delta_d(t+\theta)E_d)Kx(t-\tau(t+\theta)+\theta) \end{bmatrix} d\theta \tag{25}$$

for $t \geq \bar{\tau}$. Using (25) in (24), we find that $x(t)$ satisfies

$$\begin{aligned} \dot{x}(t) = &\begin{bmatrix} (A + D_a\Delta_a(t)E_a) + (B + D_b\Delta_b(t)E_b)K \\ +(B_d + D_d\Delta_d(t)E_d)K \end{bmatrix} x(t) \\ &-(B_d + D_d\Delta_d(t)E_d)K \\ &\times \int_{-\tau(t)}^{0} \begin{bmatrix} (A + D_a\Delta_a(t+\theta)E_a)x(t+\theta) \\ +(B + D_b\Delta_b(t+\theta)E_b)Kx(t+\theta) \\ +(B_d + D_d\Delta_d(t+\theta)E_d)Kx(t-\tau(t+\theta)+\theta) \end{bmatrix} d\theta. \end{aligned} \tag{26}$$

Consider the following uncertain time-delay system

$$\begin{aligned} \dot{\xi}(t) = &\begin{bmatrix} (A + D_a\Delta_a(t)E_a) + (B + D_b\Delta_b(t)E_b)K \\ +(B_d + D_d\Delta_d(t)E_d)K \end{bmatrix} \xi(t) \\ &-(B_d + D_d\Delta_d(t)E_d)K \\ &\times \int_{-\tau(t)}^{0} \begin{bmatrix} (A + D_a\Delta_a(t+\theta)E_a)\xi(t+\theta) \\ +(B + D_b\Delta_b(t+\theta)E_b)K\xi(t+\theta) \\ +(B_d + D_d\Delta_d(t+\theta)E_d)K\xi(t-\tau(t+\theta)+\theta) \end{bmatrix} d\theta. \end{aligned} \tag{27}$$

$$\xi(t) = \psi(t), \ \forall\, t \in [-2\bar{\tau}, 0] \tag{28}$$

where $\psi(\cdot)$ is the initial condition and $\tau(t)$ is a time delay of the system (6), (7). As noted in [9], the global uniform asymptotic stability of (27), (28) will ensure the global uniform asymptotic stability of (6), (7) since any solution of (6), (7) with $u(t) = Kx(t)$ is also a solution of (27), (28).

Let us take the following Lyapunov functional candidate for the system of (27)

$$V(\xi,t) = \xi^T(t)P\xi(t) + W(\xi,t) \tag{29}$$

where $P > 0$ and $W(\xi,t)$ is given by (30).

$$\begin{aligned} W(\xi,t) = &\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t} \xi^T(s) \begin{bmatrix} A^T(P_1 - D_aT_5D_a^T)^{-1}A + E_a^TT_5^{-1}E_a \\ +K^TB^T(P_2 - D_bT_6D_b^T)^{-1}BK \\ +K^TE_b^TT_6^{-1}E_bK \end{bmatrix} \xi(s)dsd\theta \\ &+\left(\frac{1}{1-\mu}\right)\int_{-\bar{\tau}}^{0}\int_{t-\tau(t+\theta)+\theta}^{t} \xi^T(s) \begin{bmatrix} K^TB_d^T(P_3 - D_dT_7D_d^T)^{-1}B_dK \\ +K^TE_d^TT_7^{-1}E_dK \end{bmatrix} \xi(s)dsd\theta \end{aligned} \tag{30}$$

In (30), $P_1 - D_aT_5D_a^T > 0$, $P_2 - D_bT_6D_b^T > 0$, $P_3 - D_dT_7D_d^T > 0$ and T_5, T_6, T_7 are structured matrices in Σ_R corresponding to $\Delta_a(t)$, $\Delta_b(t)$, $\Delta_d(t)$ respectively. The time-derivative of $\xi^T(t)P\xi(t)$ along the solution of (27), (28) is given by (31)

$$\begin{aligned} &\dot{\xi}^T(t)P\xi(t) + \xi^T(t)P\dot{\xi}(t) \\ &= \xi^T(t)\{\begin{bmatrix} (A + D_a\Delta_a(t)E_a) + (B + D_b\Delta_b(t)E_b)K \\ +(B_d + D_d\Delta_d(t)E_d)K \end{bmatrix}^T P \\ &+P\begin{bmatrix} (A + D_a\Delta_a(t)E_a) + (B + D_b\Delta_b(t)E_b)K \\ +(B_d + D_d\Delta_d(t)E_d)K \end{bmatrix}\}\xi(t) \\ &+\eta_1(\xi,t) + \eta_2(\xi,t) + \eta_3(\xi,t) \end{aligned} \tag{31}$$

where

$$\begin{aligned} \eta_1(\xi,t) = &-2\xi^T(t)P(B_d + D_d\Delta_d(t)E_d)K \\ &\times \int_{-\tau(t)}^{0} \left[(A + D_a\Delta_a(t+\theta)E_a)\xi(t+\theta)\right]d\theta \end{aligned} \tag{32}$$

$$\begin{aligned} \eta_2(\xi,t) = &-2\xi^T(t)P(B_d + D_d\Delta_d(t)E_d)K \\ &\times \int_{-\tau(t)}^{0} \left[(B + D_b\Delta_b(t+\theta)E_b)K\xi(t+\theta)\right]d\theta \end{aligned} \tag{33}$$

$$\begin{aligned} \eta_3(\xi,t) = &-2\xi^T(t)P(B_d + D_d\Delta_d(t)E_d)K \\ &\times \int_{-\tau(t)}^{0} \left[(B_d + D_d\Delta_d(t+\theta)E_d)K\xi(t-\tau(t+\theta)+\theta)\right]d\theta \end{aligned} \tag{34}$$

The time-derivative of $W(\xi,t)$ along the solution of (27), (28) is given by (35)

$$\begin{aligned} \dot{W}(\xi,t) = &\bar{\tau}\xi^T(t) \begin{bmatrix} A^T(P_1 - D_aT_5D_a^T)^{-1}A + E_a^TT_5^{-1}E_a \\ +K^TB^T(P_2 - D_bT_6D_b^T)^{-1}BK \\ +K^TE_b^TT_6^{-1}E_bK \\ +\left(\frac{1}{1-\mu}\right)\begin{Bmatrix} K^TB_d^T(P_3 - D_dT_7D_d^T)^{-1}B_dK \\ +K^TE_d^TT_7^{-1}E_dK \end{Bmatrix} \end{bmatrix} \xi(t) \\ &-\int_{-\bar{\tau}}^{0} \xi^T(t+\theta)\left[A^T(P_1 - D_aT_5D_a^T)^{-1}A + E_a^TT_5^{-1}E_a\right]\xi(t+\theta)d\theta \\ &-\int_{-\bar{\tau}}^{0} \xi^T(t+\theta)\begin{bmatrix} K^TB^T(P_2 - D_bT_6D_b^T)^{-1}BK \\ +K^TE_b^TT_6^{-1}E_bK \end{bmatrix}\xi(t+\theta)d\theta \\ &-\int_{-\bar{\tau}}^{0} \begin{Bmatrix} \xi^T(t-\tau(t+\theta)+\theta)\begin{bmatrix} K^TB_d^T(P_3 - D_dT_7D_d^T)^{-1}B_dK \\ +K^TE_d^TT_7^{-1}E_dK \end{bmatrix} \\ \times\xi(t-\tau(t+\theta)+\theta) \end{Bmatrix} d\theta \end{aligned} \tag{35}$$

For any $n \times n$ symmetric matrices $P_1 > 0, P_2 > 0, P_3 > 0$, applying Lemma 1 and Lemma 2 in Appendix to η_1, η_2, η_3 and using the relations

$$P^{-1} - P_1 - P_2 - P_3 \geq 0,\ \ Q - KP^{-1}K^T \geq 0, \tag{36}$$

the time-derivative of $V(\xi,t)$ along the solution of (27), (28) is obtained from (31) and (35).

$$\begin{aligned} \dot{V}(\xi,t) \leq \xi^T(t)\{&[(A + BK + B_dK)^TP \\ &+P(A + BK + B_dK) \\ &+P(D_aT_1D_a^T + D_bT_2D_b^T + D_dT_3D_d^T)P \\ &+E_a^TT_1^{-1}E_a + K^TE_b^TT_2^{-1}E_bK + K^TE_d^TT_3^{-1}E_dK \\ &+\bar{\tau}P\begin{bmatrix} B_dQB_d^T + B_dQE_d^T(T_4 - E_dQE_d^T)^{-1}E_dQB_d^T \\ +D_dT_4D_d^T \end{bmatrix}P \\ &+\bar{\tau}[A^T(P_1 - D_aT_5D_a^T)^{-1}A + E_a^TT_5^{-1}E_a \\ &+K^TB^T(P_2 - D_bT_6D_b^T)^{-1}BK + K^TE_b^TT_6^{-1}E_bK \\ &+\left(\frac{1}{1-\mu}\right)K^TB_d^T(P_3 - D_dT_7D_d^T)^{-1}B_dK \\ &+\left(\frac{1}{1-\mu}\right)K^TE_d^TT_7^{-1}E_dK]\}\xi(t) \end{aligned} \tag{37}$$

where T_1, T_2 are structured matrices in Σ_R corresponding to $\Delta_a(t)$, $\Delta_b(t)$ respectively and T_3, T_4

are structured matrices in Σ_R corresponding to $\Delta_d(t)$. Letting $X = P^{-1}$, (36) and (37) are equivalent to (38) by using Schur complements and setting $KX = Y$.

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} \\ \Sigma_{12}^T & \Sigma_{22} & 0 & 0 \\ \Sigma_{13}^T & 0 & \Sigma_{33} & 0 \\ \Sigma_{14}^T & 0 & 0 & \Sigma_{44} \end{bmatrix} < 0, \quad \begin{bmatrix} Q & Y \\ Y^T & X \end{bmatrix} \geq 0,$$

$$X - P_1 - P_2 - P_3 \geq 0 \tag{38}$$

where

$$\Sigma_{11} = XA^T + AX + Y^T(B + B_d)^T + (B + B_d)Y$$
$$+ D_a T_1 D_a^T + D_b T_2 D_b^T + D_d T_3 D_d^T + \bar{\tau} B_d Q B_d^T + \bar{\tau} D_d T_4 D_d^T$$

$$\Sigma_{12} = \begin{bmatrix} XE_a^T & Y^T E_b^T & Y^T E_d^T \end{bmatrix}, \ \Sigma_{13} = \bar{\tau} B_d Q E_d^T$$

$$\Sigma_{14} = \begin{bmatrix} \bar{\tau} XA^T & \bar{\tau} XE_a^T & \bar{\tau} Y^T B^T & \bar{\tau} Y^T E_b^T & \bar{\tau} Y^T B_d^T & \bar{\tau} Y^T E_d^T \end{bmatrix}$$

$$\Sigma_{22} = -\mathrm{diag}\begin{bmatrix} T_1 & T_2 & T_3 \end{bmatrix}, \ \Sigma_{33} = -\bar{\tau}(T_4 - E_d Q E_d^T)$$

$$\Sigma_{44} = -\bar{\tau}\mathrm{diag}\begin{bmatrix} \begin{matrix}(P_1 - D_a \\ \times T_5 D_a^T)\end{matrix} & T_5 & \begin{matrix}(P_2 - D_b \\ \times T_6 D_b^T)\end{matrix} & T_6 & \begin{matrix}(1-\mu) \\ \times (P_3 - D_d \\ \times T_7 D_d^T)\end{matrix} & \begin{matrix}(1-\mu) \\ \times T_7\end{matrix} \end{bmatrix}.$$

In (38), T_1, T_5 are structured matrices in Σ_R corresponding to $\Delta_a(t)$, T_2, T_6 are structured matrices in Σ_R corresponding to $\Delta_b(t)$, and T_3, T_4, T_7 are structured matrices in Σ_R corresponding to $\Delta_d(t)$. Moreover, a stabilizing controller is given by $u(t) = YX^{-1}x(t)$.

Now, we consider the structured uncertain system (6) with input delay and input sector nonlinearity. Since (6) can be represented by (21) and (21) is in the form of (24), we apply the above results in (38) to (21). By replacing B, D_b, E_b, B_d, D_d, E_d with $B\Lambda_{b+}$, $\begin{bmatrix} D_b & B & \rho_b D_b E_s^T \end{bmatrix}$, $\begin{bmatrix} (E_b \Lambda_{b+})^T & (\Lambda_{b-})^T & (E_m^T \Lambda_{b-})^T \end{bmatrix}^T$, $B_d \Lambda_{d+}$, $\begin{bmatrix} D_d & B_d & \rho_d D_d E_s^T \end{bmatrix}$, $\begin{bmatrix} (E_d \Lambda_{d+})^T & (\Lambda_{d-})^T & (E_m^T \Lambda_{d-})^T \end{bmatrix}^T$, we obtain the conditions of the theorem by using scaling matrices T_1, T_5 corresponding to the structured uncertainty Δ_a, $\mathrm{diag}\begin{bmatrix} T_{i1} & T_{i2} & t_{i3} I \end{bmatrix}$, $i = 2, 6$ corresponding to the structured uncertainty $\mathrm{diag}\begin{bmatrix} \Delta_b & \Delta_{b2} & \Delta_{b3} \end{bmatrix}$ and $\mathrm{diag}\begin{bmatrix} T_{i1} & T_{i2} & t_{i3} I \end{bmatrix}$, $i = 3, 4, 7$ corresponding to the structured uncertainty $\mathrm{diag}\begin{bmatrix} \Delta_d & \Delta_{d2} & \Delta_{d3} \end{bmatrix}$ respectively. In the scaling matrices, T_{i2}'s are given by diagonal matrices and t_{i3}'s are given by scalars. ❑

For fixed ε_{bi}'s and ε_{di}'s, an upper bound of $\bar{\tau}$ satisfying $0 \leq \tau \leq \bar{\tau}$, $0 \leq \dot{\tau}(t) < \mu < 1$ can be found by solving the following quasi-convex optimization problem based on Theorem 1 via LMI optimizations.

Maximize $\bar{\tau}$

subject to $X > 0$, $P_1 > 0$, $P_2 > 0$, $P_3 > 0$, $Q > 0$, $T_1 > 0$, $T_{21} > 0$, $T_{22} > 0$, $t_{23} > 0$, $T_{31} > 0$, $T_{32} > 0$, $t_{33} > 0$, $T_{41} > 0$, $T_{42} > 0$, $t_{43} > 0$, $T_5 > 0$, $T_{61} > 0$, $T_{62} > 0$, $t_{63} > 0$, $T_{71} > 0$, $T_{72} > 0$, $t_{73} > 0$, Y, (22) and (23)

In the case of homogeneous nonlinearity, we can obtain similar conditions as the above by letting $\varepsilon_{bi} = \varepsilon_{di} = \varepsilon$ in Theorem 1. In addition to this fact, we can see that if there is not input sector nonlinearity, the LMI conditions of Theorem 1 are reduced to those given in [2] with $\varepsilon_{bi} = \varepsilon_{di} = 1$, $\mu = 0$ and scaling constants instead of structured scaling matrices in (22), (23). That is, in the case of a constant input delay and norm-bounded unstructured uncertainty without input sector nonlinearity, Theorem 1 is the same as Theorem 3.1 in [2]. Hence, Theorem 1 is an extension of Theorem 3.1 in [2] with input sector nonlinearity and structured uncertainty added. Moreover, Theorem 1 can be easily extended to the case of multiple input delays.

In Theorem 1, we show LMI conditions for robust state feedback control of uncertain systems with input nonlinearity satisfying $\mathrm{diag}\begin{bmatrix} \varepsilon_1 & \cdots & \varepsilon_m \end{bmatrix} \leq N(u) \leq I$ for positive ε_i's. Modifying these conditions with some variables fixed, we can also find robust input sector bounds ε_i's such that the given uncertain system is stable for any input nonlinearity in these sector bounds by solving the following successive LMI optimization problems.

Algorithm 1

STEP 1. Given scalars $\bar{\tau} > 0$ and $\mu > 0$, for some values ε_{bi}'s and ε_{di}'s, which are chosen as $\varepsilon_{bi} = \varepsilon_{di} = 1$ initially or found in Step 2, solve the LMI problems of Theorme 1 for the corresponding LMI variables such that the feasible LMIs are maximally negative or positive.

STEP 2. Given Y and $Q > 0$ in Step 1, minimize $\sum_{i=1}^{m} \varepsilon_{bi} + \sum_{j=1}^{m} \varepsilon_{dj}$ subject to the following modified LMI conditions for the LMI variables with ε_{bi}'s and ε_{di}'s added.

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & 0 & \Gamma_{16} & \Gamma_{17} & \Gamma_{18} & \bar{\tau} B_d \Lambda_{d+} Q \\ \Gamma_{12}^T & \Gamma_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Gamma_{13}^T & 0 & \Gamma_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ \Gamma_{14}^T & 0 & 0 & \Gamma_{44} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_{55} & 0 & 0 & 0 & \bar{\tau}\begin{bmatrix} E_d \Lambda_{d+} \\ \Lambda_{d-} \\ E_m^T \Lambda_{d-} \end{bmatrix} Q \\ \Gamma_{16}^T & 0 & 0 & 0 & 0 & \Gamma_{66} & 0 & 0 & 0 \\ \Gamma_{17}^T & 0 & 0 & 0 & 0 & 0 & \Gamma_{77} & 0 & 0 \\ \Gamma_{18}^T & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_{88} & 0 \\ \bar{\tau} Q \Lambda_{d+}^T B_d^T & 0 & 0 & 0 & \bar{\tau} Q \begin{bmatrix} \begin{matrix}\Lambda_{d+}^T \\ \times E_d^T\end{matrix} & \Lambda_{d-}^T & \begin{matrix}\Lambda_{d-}^T \\ \times E_m\end{matrix} \end{bmatrix} & 0 & 0 & 0 & -\bar{\tau} Q \end{bmatrix} < 0 \tag{39}$$

$$\begin{bmatrix} Q & Y \\ Y^T & X \end{bmatrix} \geq 0, \quad X - P_1 - P_2 - P_3 \geq 0 \tag{40}$$

where

$$\begin{aligned}\Gamma_{11} &= XA^T + AX + Y^T(B\Lambda_{b+} + B_d\Lambda_{d+})^T + (B\Lambda_{b+} + B_d\Lambda_{d+})Y \\ &\quad + D_aT_1D_a^T + D_bT_{21}D_b^T + BT_{22}B^T + \rho_b^2 t_{23}D_bD_b^T \\ &\quad + D_dT_{31}D_d^T + B_dT_{32}B_d^T + \rho_d^2 t_{33}D_dD_d^T + \bar{\tau}D_dT_{41}D_d^T \\ &\quad + \bar{\tau}B_dT_{42}B_d^T + \bar{\tau}\rho_d^2 t_{43}D_dD_d^T\end{aligned},$$

$$\Gamma_{12} = XE_a^T, \Gamma_{13} = \begin{bmatrix} Y^T\Lambda_{b+}^T E_b^T & Y^T\Lambda_{b-}^T & Y^T\Lambda_{b-}^T E_m \end{bmatrix},$$

$$\Gamma_{14} = \begin{bmatrix} Y^T\Lambda_{d+}^T E_d^T & Y^T\Lambda_{d-}^T & Y^T\Lambda_{d-}^T E_m \end{bmatrix},$$

$$\Gamma_{16} = \begin{bmatrix} \bar{\tau}XA^T & \bar{\tau}XE_a^T \end{bmatrix},$$

$$\Gamma_{17} = \begin{bmatrix} \bar{\tau}Y^T\Lambda_{b+}^T B^T & \bar{\tau}Y^T\Lambda_{b+}^T E_b^T & \bar{\tau}Y^T\Lambda_{b-}^T & \bar{\tau}Y^T\Lambda_{b-}^T E_m \end{bmatrix},$$

$$\Gamma_{18} = \begin{bmatrix} \bar{\tau}Y^T\Lambda_{d+}^T B_d^T & \bar{\tau}Y^T\Lambda_{d+}^T E_d^T & \bar{\tau}Y^T\Lambda_{d-}^T & \bar{\tau}Y^T\Lambda_{d-}^T E_m \end{bmatrix},$$

$$\Gamma_{22} = -T_1, \quad \Gamma_{33} = -\mathrm{diag}\begin{bmatrix} T_{21} & T_{22} & t_{23}I \end{bmatrix},$$

$$\Gamma_{44} = -\mathrm{diag}\begin{bmatrix} T_{31} & T_{32} & t_{33}I \end{bmatrix},$$

$$\Gamma_{55} = -\bar{\tau}\begin{bmatrix} T_{41} & 0 & 0 \\ 0 & T_{42} & 0 \\ 0 & 0 & t_{43}I \end{bmatrix}, \Gamma_{66} = -\bar{\tau}\mathrm{diag}\begin{bmatrix} (P_1 - D_aT_5D_a^T) & T_5 \end{bmatrix},$$

$$\Gamma_{77} = -\bar{\tau}\mathrm{diag}\begin{bmatrix} (P_2 - D_bT_{61}D_b^T - BT_{62}B^T - \rho_b^2 t_{63}D_bD_b^T) & T_{61} & T_{62} & t_{63}I \end{bmatrix},$$

$$\begin{aligned}\Gamma_{88} &= -\bar{\tau}(1-\mu) \\ &\times \mathrm{diag}\begin{bmatrix} (P_3 - D_dT_{71}D_d^T - B_dT_{72}B_d^T - \rho_d^2 t_{73}D_dD_d^T) & T_{71} & T_{72} & t_{73}I \end{bmatrix}\end{aligned}.$$

STEP 3. Iterate Step 1 and Step 2 until no significant change occurs in the criterion $\sum_{i=1}^{m}\varepsilon_{bi} + \sum_{j=1}^{m}\varepsilon_{dj}$. In the case of homogeneous nonlinearity, this algorithm can be also applied to obtain robust input sector bound ε with criterion ε instead of $\sum_{i=1}^{m}\varepsilon_{bi} + \sum_{j=1}^{m}\varepsilon_{dj}$ in STEP 2.

Note that the smaller ε_{bi}'s and ε_{di}'s are obtained, the broader input sector bounds are found to guarantee stability of the closed-loop system with a state feedback controller.

3. Conclusion

In this paper, we have dealt with a robust control method for some nonlinear control problems with an input delay. By letting input nonlinearity in the sector bounds as a new diagonal structured uncertainty, we transformed the control problems with input nonlinearity into the robust control problems of linear systems with only structured uncertainty. Applying this idea, we obtained linear matrix inequality(LMI) conditions for delay-dependent robust stabilization of structured uncertain systems with input delay and input sector nonlinearity. In addition to LMIs for the fixed input nonlinearity, we also propose an iterative LMI optimization algorithm to find robust input sector bounds such that the given uncertain system is stable for any input nonlinearity in these sector bounds.

4. References

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Appendix

Lemma 1[9]. For any $z, y \in R^n$ and for any symmetric positive definite matrix $X \in R^{n \times n}$, (A.1) holds.

$$-2z^T y \le z^T X^{-1} z + y^T X y \tag{A.1}$$

Lemma 2[10]. Let A, D, E, $\Delta(t)$ be real matrices of appropriate dimension and $\Delta(t)$ be an element of Δ_s such that $\Delta(t)^T\Delta(t) \le I$. Then the following three matrix inequalities hold.

(a) For any block-structured matrix $T \in \Sigma_R$ in (4.6), (A.2) holds.

$$D\Delta(t)E + E^T\Delta(t)^T D^T \le DTD^T + E^T T^{-1} E \tag{A.2}$$

(b) For any matrix $P = P^T > 0$ and block-structured matrix $T \in \Sigma_R$ such that $T - EPE^T > 0$, (A.3) holds.

$$\begin{aligned}&(A + D\Delta(t)E)P(A + D\Delta(t)E)^T \\ &\le APA^T + APE^T(T - EPE^T)^{-1}EPA^T + DTD^T\end{aligned} \tag{A.3}$$

(c) For any matrix $P = P^T > 0$ and block-structured matrix $T \in \Sigma_R$ such that $P - DTD^T > 0$, (A.4) holds.

$$\begin{aligned}&(A + D\Delta(t)E)^T P^{-1}(A + D\Delta(t)E) \\ &\le A^T(P - DTD^T)^{-1}A + E^T T^{-1} E\end{aligned} \tag{A.4}$$