Robust and Non-fragile H^{∞} Controller Design for Affine Parameter Uncertain Systems

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Abstract: This paper describes the synthesis of robust and non-fragile H^{∞} state feedback controllers for linear time varying systems with affine parameter uncertainties and static state feedback controller with polytopic uncertainty. The sufficient condition of controller existence, the design method of robust and non-fragile H^{∞} static state feedback controller, and the region of controllers which satisfies non-fragility are presented. Also using some change of variables and Schur complements, the obtained condition can be rewritten as parameterized Linear Matrix Inequalities (PLMIs), that is, LMIs whose coefficients are functions of a parameter confined to a compact set. We show that the resulting controller guarantees the asymptotic stability and disturbance attenuation of the closed loop system in spite of controller gain variations within a resulted polytopic region.

I. Introduction

It is generally known that feedback systems designed for robustness with respect to plant parameters, or designed for optimization of a single performance measure, may require very accurate controllers[5]. An implicit assumption that is inherent to those control methodology is that the controller is designed will be implemented exactly. However, the controller implementation is subject to A/D conversio n · D/A conversion, finite word length and round-off errors in numerical computations, in addition to the need of providing the practicing engineer with safe-tuning margins. Therefore, it is necessary that any controller should be able to tolerate some uncertainty in the controller as well as plant uncertainty[3-6,9-13].

In a recent paper, Keel *et al.*[12] have shown that the resulting controllers exhibit a poor stability margin if not implemented exactly. So, some researchers have developed non-fragile controller design algorithms. Dorato *et al.*[5] proposed a non-fragile controller design method via symbolic quantifier elimination. And Haddad *et al.*[9] proposed robust resilient dynamic controller via quadratic Lyapunov bounds. Famularo *et al.*[6] and Jadbabie *et al.*[10] considered LQ robust and non-fragile state feedback controller. However, recent researchers did not consider the structure of controller gain variations or the value of the non-fragility. And they did not consider the effect of disturbances.

In this paper, we propose robust and non-fragile H^{∞} controller design method for linear systems with affine parameter uncertainties and static state feedback controller with polytopic uncertainty. Also the sufficient condition of controller existence, the design method of robust and non-fragile H^{∞} static state feedback controller, and the region of controllers which satisfies non-fragility are presented. The sufficient condition is presented using PLMIs, that is, LMIs whose coefficients are functions of a parameter confined to a compact set. However, in contrast to LMIs, PLMI feasibility problems involve infinitely many LMIs hence are inherently difficult to solve numerically. Therefore PLMIs are transformed into finitely many LMI problems using relaxation techniques[1,14].

The paper is structured as follows. The definition of PLMI and basic lemma are described in section 2. And Section 3 discusses robust and non-fragile H^{∞} controller synthesis. Numerical example illustrating robustness and disturbance attenuation is given in section 4 and our conclusions are discussed in section 5.

II. Preliminaries

We consider parameterized LMIs(PLMIs), that is, LMIs depending on a parameter θ evolving in a compact set. The parameter θ can designate parameter uncertainties or system operating conditions but virtually appears. Here, a special emphasis is put on PLMIs of the form

$$M_0(z) + \sum_{i=1}^{L} \theta_i M_i(z) + \sum_{1 \le i \le j \le L} \theta_i \theta_j M_{ij}(z) < 0 \qquad (1)$$

where z is the decision variable, $M_i(z)$, $M_{ij}(z)$ are affine symmetric matrix-valued functions of z and θ is a parameter confined to either the polytope

$$\theta \in \Gamma :=\{ \theta = (\theta_1, \theta_2, \cdots, \theta_L) : \sum_{i=1}^{L} \theta_i = 1, \theta_i \geq 0, i=1,2,\cdots,L \} \tag{2}$$

or the parameter hyper-rectangle

$$\theta = [p, q];\ p \geq 0,\ q > 0,\ p \in \mathbb{R}^L,\ q \in \mathbb{R}^L. \tag{3}$$

However, PLMI feasibility problems involve infinitely many LMIs according to the variations of parameter hence are very hard to solve numerically and computational efforts for finding feasible points are expected to be far much higher than those of LMIs. In this paper, we use relaxation techniques where PLMIs are replaced by finitely many LMIs. Such approaches are potentially conservative but often provide practically exploitable solutions of difficult problems with a reasonable computational effort.

Lemma 1[14] The PLMI problem (1) and (2) has a solution z whenever the following quadratic conditions hold,

$$x^T M_0(z)x + \sum_{i=1}^{L} \theta_i x^T M_i(z)x + \sum_{1 \leq i \leq j \leq L} \max\left\{ -x^T M_{ij}(z)x\left(\frac{\theta_i^2+\theta_j^2}{2} - \frac{\theta_i+\theta_j}{2} + 0.125\right),\ x^T M_{ij}(z)x\frac{\theta_i^2+\theta_j^2}{2} \right\} < 0, \quad \forall \|x\| = 1,\ \alpha \in \mathrm{vert}\Gamma. \tag{4}$$

The latter conditions are readily rewritten as LMIs and can be easily expressed as an LMI feasibility problem.

Remark 1 It should be noted that

$$\max\left\{ -x^T M_{ij}(z)x\left(\frac{\theta_i^2+\theta_j^2}{2} - \frac{\theta_i+\theta_j}{2} + 0.125\right),\ x^T M_{ij}(z)x\frac{\theta_i^2+\theta_j^2}{2} \right\} \tag{5}$$

is a tight upper bound of $\theta_i \theta_j x^T M_{ij}(z)x$ with $\theta_i + \theta_j \leq 1$. Therefore, if the set Γ is alternatively defined as

$$\Gamma :=\left\{ \theta = (\theta_1, \theta_2, \cdots, \theta_L) : \sum_{i=1}^{L} \theta_i = a,\ \theta_i \geq 0,\ i=1,2,\cdots,L \right\} \tag{6}$$

with $a > 1$, one can use the change of variable $\overline{\theta}_i = \theta_i / a$ to recover the case $\overline{\theta}_i + \overline{\theta}_j \leq 1$. Analogously, applying the change of variable $\overline{\theta}_i = (p + \theta_i)/(q - p)$ to the constraint (3) yields the relation

$$\overline{\theta} \in [0, 1]^L. \tag{7}$$

Ⅲ. Robust and Non-fragile H^∞ Controller Design

Consider a linear time varying uncertain system

$$\begin{aligned} \dot{x}(t) &= A(t,\alpha)x(t) + B_1(t,\alpha)w(t) + B_2(t,\alpha)u(t) \\ z(t) &= C(t,\alpha)x(t) \end{aligned} \tag{8}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^r$ is the square integrable disturbance input, $z(t) \in \mathbb{R}^q$ is the controlled output, and matrices $A(t,\alpha)$, $B_1(t,\alpha)$, $B_2(t,\alpha)$, and $C(t,\alpha)(t \geq 0)$ contain affine uncertainties of the form

$$\begin{aligned} A(t,\alpha) &= A_0 + \sum_{i=1}^{L} \alpha_i(t) A_i, \\ B_1(t,\alpha) &= B_{10} + \sum_{i=1}^{L} \alpha_i(t) B_{1i}, \\ B_2(t,\alpha) &= B_{20} + \sum_{i=1}^{L} \alpha_i(t) B_{2i}, \\ C(t,\alpha) &= C_0 + \sum_{i=1}^{L} \alpha_i(t) C_i \end{aligned} \tag{9}$$

and it is assumed that

(A1) the state-space data $A(t,\alpha)$, $B_1(t,\alpha)$, $\cdots$ are bounded continuous functions of α,

(A2) the time-varying parameter

$$\alpha(t) := [\alpha_1(t), \cdots, \alpha_L(t)]^T$$

defined at all times and continuous, evolve in hyper-rectangles H, that is,

$$\underline{\alpha_i} \leq \alpha_i(t) \leq \overline{\alpha_i}, \quad 1 \leq i \leq L, \quad \forall t \geq 0. \tag{10}$$

The assumption (A1) and (A2) are general and they secure existence and uniqueness of the solutions.

Although one finds the controller $u(t) = Kx(t)$, the actual controller implemented is assumed as

$$\begin{aligned} &u(t) = K(t,\beta)x(t), \\ &K(t,\beta) = \sum_{j=1}^{L} \beta_j(t) K_j, \quad \sum_{j=1}^{L} \beta_j(t) = 1 \end{aligned} \tag{11}$$

where $K(t,\beta)$ is the region of controller variations and K_j is the vertices of polytope. Here, we choose the center of polytope

$$K_0 = \frac{1}{L} \sum_{j=1}^{L} K_j \tag{12}$$

as nominal controller gain. And the region of controller variations is rewritten as

$$K(t,\beta) = K_0 + \sum_{j=1}^{L} \beta_j(t)\tilde{K}_j, \qquad (13)$$
$$\sum_{j=1}^{L}\beta_j(t) = 1, \quad \tilde{K}_j = K_j - K_0.$$

Here, the values of $\tilde{K}_j$ indicate the measure of non-fragility against controller gain variations. Now, the closed loop system from (8) and (11) is given by

$$\begin{aligned} \dot{x}(t) &= [A(t,\alpha) + B_2(t,\alpha)K(t,\beta)]x(t) + B_1(t,\alpha)w(t) \\ z(t) &= C(t,\alpha)x(t). \end{aligned} \qquad (14)$$

Our controller design objective is described as follows: The closed loop system (14) is asymptotically stable with disturbance attenuation γ and non-fragility, if the following is fulfilled for affine parameter uncertainty of systems satisfying (9) and polytopic uncertainty of controller satisfying (11) or (13):

i) The closed loop system (14) is asymptotically stable.
ii) The closed loop system guarantees, under zero initial conditions, $\|z(t)\|_2 \le \gamma\|w(t)\|_2$ for all non-zero $w(t) \in L_2[0, \infty)$.

Therefore, the objective of this paper is designing a robust and non-fragile H^∞ state feedback controller K_0 in the presence of affine parameter uncertainty of system and polytopic uncertainty of controller. Also the controller guarantees disturbance attenuation of the closed loop system from $w(t)$ to $z(t)$.

Lemma 2 Consider a closed loop system (14) and suppose that the disturbance input is zero for all time. If there exist positive definite matrix P and controller gain $\tilde{K}_j (i = 1,2,\cdots,L)$ satisfying

$$\begin{aligned} A(t,\alpha)^T P + PA(t,\alpha) + PB_2(t,\alpha)K(t,\beta) \\ + K(t,\beta)^T B_2(t,\alpha)^T P < 0 \end{aligned} \qquad (15)$$

then the closed loop system is asymptotically stable.
Proof The Lyapunov derivative corresponding to the closed loop system with Lyapunov functional $V(x(t),t) = x(t)^T Px(t)$ is represented as

$$\begin{aligned} \frac{d}{dt}V(x(t),t) = x(t)^T\{A(t,\alpha)^T P + PA(t,\alpha) \\ + PB_2(t,\alpha)K(t,\beta) + K(t,\beta)^T B_2(t,\alpha)^T P\}x(t) \end{aligned} \qquad (16)$$

Therefore when $\frac{d}{dt}V(x(t),t) < 0$, the closed loop system is asymptotically stable. ■

Lemma 3 If there exist positive definite matrix P and the vertices of the controller variation polytope $\tilde{K}_j(i=1,2,\cdots,L)$such that

$$\begin{bmatrix} U & PB_1(t,\alpha) \\ B_1(t,\alpha)^T P & -\gamma^2 I \end{bmatrix} < 0 \qquad (17)$$

then the closed loop system is asymptotically stable with disturbance attenuation γ and non-fragility. Here,

$$\begin{aligned} U = A^T(t,\alpha)P + PA(t,\alpha) + C^T(t,\alpha)C(t,\alpha) \\ + PB_2(t,\alpha)K(t,\beta) + K(t,\beta)^T B_2(t,\alpha)^T P. \end{aligned} \qquad (18)$$

Proof It is noticed that (18) implies (15). Therefore (18) ensures that asymptotic stability of the closed loop system. Under zero initial condition, let us introduce

$$J = \int_0^\infty [z(t)^T z(t) - \gamma^2 w(t)^T w(t)]\, dt \qquad (19)$$

Then performance measure (19) for any nonzero $w(t) \in L_2[0,\infty)$,

$$\begin{aligned} J &= \int_0^\infty \left[z(t)^T z(t) - \gamma^2 w(t)^T w(t) + \frac{d}{dt}\{x(t)^T Px(t)\}\right] dt \\ &\quad - x(\infty)^T Px(\infty) \\ &= \int_0^\infty x(t)^T [A(t,\alpha)^T P + PA(t,\alpha) + C(t,\alpha)^T C(t,\alpha) \\ &\quad + PB_2(t,\alpha)K(t,\beta) + K(t,\beta)^T B_2(t,\alpha)^T P]x(t) \\ &\quad + x(t)^T PB_1(t,\alpha)w(t) + w(t)^T B_1(t,\alpha)^T Px(t)\, dt \\ &\quad - x(\infty)^T Px(\infty), \end{aligned} \qquad (20)$$

then robust H^∞ condition

$$[x(t)^T \; w(t)^T]\begin{bmatrix} U & PB_1(t,\alpha) \\ B_1(t,\alpha)^T P & -\gamma^2 I \end{bmatrix}\begin{bmatrix} x(t) \\ w(t) \end{bmatrix} < 0. \qquad (21)$$

implies $\|z(t)\|_2 \le \gamma\|w(t)\|_2$ for all non-zero $w(t) \in L_2[0, \infty)$. ■

Theorem 1 Consider a linear time varying system with affine parameter uncertainties (8). If there exist matrix $Y_j(i=1,2,\cdots,L)$, positive definite matrix X, and positive constant ρ satisfying

$$\begin{bmatrix} W & B_1(t,\alpha) & XC^T(t,\alpha) \\ B_1^T(t,\alpha) & -\rho I & 0 \\ C(t,\alpha)X & 0 & I \end{bmatrix} < 0, \qquad (22)$$

then the closed loop system is asymptotic stable with disturbance attenuation γ and non-fragility. Here, some variables are defined as follow:

$$\begin{aligned} W &= A(t,\alpha)X + XA^T(t,\alpha) \\ &\quad + \sum_{j=1}^{L}\beta_j(t)[B_2(t)Y_j + Y_j^T B_2^T(t)], \qquad (23) \\ \rho &= \gamma^2, \quad Y_0 = K_0X, \quad Y_j = \tilde{K}_jX. \end{aligned}$$

Proof Using change of variable $X = P^{-1}$, (21) is equivalent to

$$[x(t)^T P \;\; w(t)^T]\begin{bmatrix} \tilde{U} & B_1(t) \\ B_1(t)^T & -\gamma^2 I \end{bmatrix}\begin{bmatrix} Px(t) \\ w(t) \end{bmatrix} < 0 \qquad (24)$$

where $\tilde{U}$ is described as

$$\begin{aligned} \tilde{U} &= XA^T(t,\alpha) + A(t,\alpha)X + XC^T(t,\alpha)C(t,\alpha)X \\ &\quad + B_2(t,\alpha)K(t,\beta)X + XK^T(t,\beta)B_2^T(t,\alpha). \end{aligned} \qquad (25)$$

Also (24) can be transformed to (22) using Schur complements and (23). ■

Theorem 2 With the assumptions (A1) and (A2), the linear parameter uncertain system (8) is asymptotically stable with disturbance attenuation γ and non-fragility whenever there exist matrix $Y_j(i=1,2,\cdots,L)$, positive definite matrix X, and positive constant ρ such that

$$\begin{aligned} & x^TM_0(z)x + \sum_{i=1}^{L}\alpha_i x^TM_i(z)x + \sum_{j=1}^{L}\beta_j x^TN_j(z) \\ & + \sum_{i=1}^{L}\sum_{j=1}^{L}\max\Big\{-x^TM_{ij}(z)x\Big(\frac{\alpha_i^2+\beta_j^2}{2} - \frac{\alpha_i+\beta_j}{2} + 0.125\Big), \\ & x^TM_{ij}(z)x\frac{\alpha_i^2+\beta_j^2}{2}\Big\} < 0, \;\; \forall \|x\| = 1, \; (\alpha, \beta) \in \text{vert}\Gamma. \end{aligned} \qquad (26)$$

holds for z, $M_i(z)$, $N_j(z)$, and $M_{ij}(z)$ defined below:

$$M_0(z) = \begin{bmatrix} A_0X + XA_0^T + B_{20}Y_0 + Y_0^TB_{20}^T & B_{10} & XC_0^T \\ B_{10}^T & -\rho I & 0 \\ C_0X & 0 & I \end{bmatrix},$$

$$M_i(z) = \begin{bmatrix} A_iX + XA_i^T + B_{2i}Y_0 + Y_0^TB_{2i}^T & B_{1i} & XC_i^T \\ B_{1i}^T & 0 & 0 \\ C_iX & 0 & 0 \end{bmatrix}, \qquad (27)$$

$$N_j(z) = \begin{bmatrix} B_{20}Y_j + Y_j^TB_{20}^T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$M_{ij}(z) = \begin{bmatrix} B_{2i}Y_j + Y_j^TB_{2i}^T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Proof Using the modified PLMI form

$$\begin{aligned} & M_0(z) + \sum_{i=1}^{L}\alpha_i(t)M_i(z) + \sum_{j=1}^{L}\beta_j(t)N_j(z) \\ & \quad + \sum_{i=1}^{L}\sum_{j=1}^{L}\alpha_i(t)\beta_j(t)M_{ij}(z) < 0 \end{aligned} \qquad (28)$$

and applying lemma 1, proof is easily obtained. ■

Remark 2 (26) is converted finitely many LMI problems in terms of $Y_j(i=1,2,\cdots,L)$, X, ρ. Therefore the robust and non-fragile H^∞ state feedback controller K_0 and the region of controllers which satisfies non-fragility can be calculated from the $K_0 = Y_0X^{-1}$ and $\tilde{K}_j = Y_jX^{-1}$ after finding the LMI solutions from (26). Also the value of disturbance attenuation, γ, can be obtained by $\gamma = \sqrt{\rho}$ in (23).

Ⅳ. Numerical Example

Consider a linear time-varying system (8) with affine parameter uncertainty satisfying

$$\begin{aligned} A(t) &= \begin{bmatrix} 2 & 2 \\ 1 & -3 \end{bmatrix} + \alpha_1(t)\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \alpha_2(t)\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \\ B_1(t) &= \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \alpha_1(t)\begin{bmatrix} 0 \\ -0.5 \end{bmatrix} + \alpha_2(t)\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ B_2(t) &= \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} + \alpha_1(t)\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \alpha_2(t)\begin{bmatrix} 0.5 & 0 \\ 0 & -2 \end{bmatrix}, \\ C(t) &= [1 \;\; 1] + \alpha_2(t)[1 \;\; 0] \end{aligned} \qquad (29)$$

and parameter α_1, α_2 satisfying

$$\alpha_1(t) \in [1 \;\; 1.5], \qquad \alpha_2(t) \in [1 \;\; 2] \qquad (30)$$

In theorem 2, all solutions are obtained at the same time as follows:

$$\begin{aligned} Y_0 &= \begin{bmatrix} -107.0977 & -239.2440 \\ -109.5050 & 17.8642 \end{bmatrix}, \\ Y_1 &= \begin{bmatrix} -50.9456 & -109.5348 \\ -61.8098 & 9.2325 \end{bmatrix}, \\ Y_2 &= \begin{bmatrix} -25.1393 & -88.6227 \\ -38.4508 & 4.6056 \end{bmatrix}, \\ Y_3 &= \begin{bmatrix} 76.0850 & 198.1576 \\ 100.2607 & -13.8381 \end{bmatrix}, \\ X &= \begin{bmatrix} 7.8779 & -20.4586 \\ -20.4586 & 54.4882 \end{bmatrix}, \\ \rho &= 0.0722. \end{aligned} \qquad (31)$$

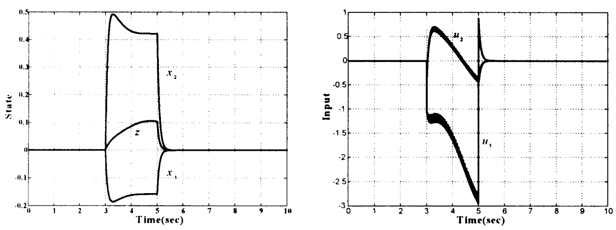
Therefore the robust and non-fragile H^∞ state feedback gain, vertex of perturbation satisfying non-fragility, and the value of disturbance attenuation in closed loop system are represented from the changes of variables (23) as follows:

$$
\begin{aligned}
K_0 &= 10^3 \times \begin{bmatrix} -1.0030 & -0.3810 \\ -0.5236 & -0.1963 \end{bmatrix}, \\
\widetilde{K}_1 &= \begin{bmatrix} -468.9418 & -178.0836 \\ -297.1525 & -111.4023 \end{bmatrix}, \\
\widetilde{K}_2 &= \begin{bmatrix} -297.5151 & -113.3343 \\ -187.0294 & -70.1393 \end{bmatrix}, \\
\widetilde{K}_3 &= \begin{bmatrix} 766.4569 & 291.4179 \\ 484.1819 & 181.5416 \end{bmatrix}, \\
\gamma &= 0.2686.
\end{aligned} \tag{32}
$$

For computer simulation, the value of $w(t)$ is defined by

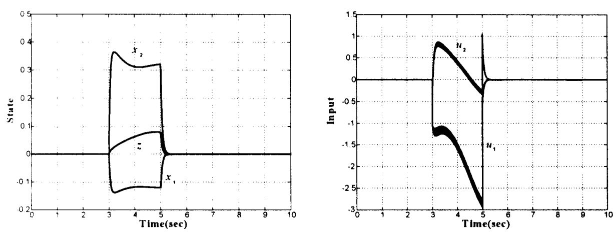
$$
w(t) = \begin{cases} 5, & 3\text{sec} \le t \le 5\text{sec} \\ 0, & \text{otherwise} \end{cases} \tag{33}
$$

When norminal control K_0 is applied, the trajectories of states, controlled output, and control input are given in Fig. 1. And when the vertices of controller polytope K_1, K_2, and K_3 are applied, the responses are given in Fig. 2, Fig. 3, and Fig. 4, respectively. This example shows that the vertices of controller polytope guarantees the asymptotic stability and disturbance attenuation γ of closed loop system. Therefore we conclude that the obtained robust and non-fragile H^∞ controller guarantees the asymptotic stability and disturbance attenuation $\|z(t)\|_2 \le 0.2686 \|w(t)\|_2$ for any $w(t) = L_2[0, \infty)$, in spite of the controller gain variations with the resulted polytopic region.



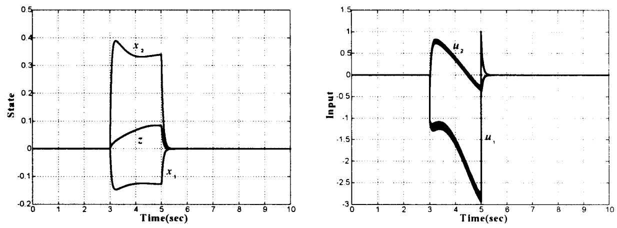
(a) The trajectories of states & controlled output (b) The trajectories of control input

Fig. 1. The case of nominal conroller K_0.



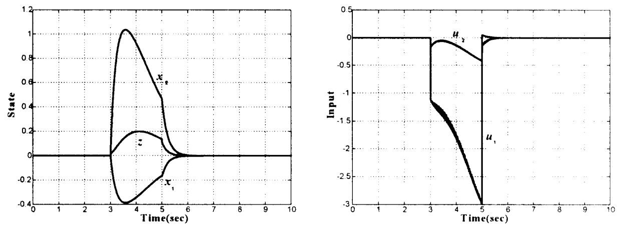
(a) The trajectories of states & controlled output (b) The trajectories of control input

Fig. 2. The case of vertex K_1.



(a) The trajectories of states & controlled output (b) The trajectories of control input

Fig. 3. The case of vertex K_2.



(a) The trajectories of states & controlled output (b) The trajectories of control input

Fig. 4. The case of vertex K_3.

V. Conclusion

In this paper, we presented the robust and non-fragile H^∞ controller design method for linear systems with affine parameter uncertainties and state feedback controller with polytopic uncertainty. Also, the robust and non-fragile controller, the level of disturbance attenuation, and the region of controllers which satisfies non-fragility were calculated at a time using PLMI approach. In spite of the controller gain variations within the resulted polytopic region, the obtained robust and non-fragile H^∞ controller guaranteed the asymptotic stability and disturbance attenuation γ of the closed loop system. The area of future research is extension of output feedback case and considering of time delay in states.

References

[1] P. Apkarian and H. D. Tuan, "Parameterized LMIs in control theory," *Proc. IEEE Conf. Dec. Contr. in Florida*, pp. 152-157, 1998.

[2] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, 1994.

[3] J. R. Corrado and W. M. Haddad, "Static output feedback controllers for systems with parametric uncertainty and controller gain variation," *Proc. Amer. Contr. Conf. in San Diego, California*, pp. 915-919, 1999.

[4] P. Dorato, "Non-fragile controller design: An overview," *Proc. Amer. Contr. Conf. in Philadelphia, Pennsylvania*, pp. 2829-2831, 1998.

[5] P. Dorato, C. T. Abdallah, and D. Famularo, "On the design of non-fragile compensators via symbolic quantifier elimination," *World Automation Congress in Anchorage, Alaska*, pp. 9-14, 1998.

[6] D. Famularo, C. T. Abdallah, A. Jadbabaie, P. Dorato, and W. M. Haddad, "Robust non-fragile LQ controllers: The static state feedback case," *Proc. Amer. Contr. Conf. in Philadelphia, Pennsylvania*, pp. 1109-1113, 1998.

[7] P. Gahinet and P. Apkarian, "A linear matrix inequality approach to H^{∞} control," *Int. J. of Robust Nonlinear Control*, vol. 4, pp. 421-448, 1994.

[8] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox*, The Math Works Inc., 1995.

[9] W. M. Haddad and J. R. Corrado, "Robust resilient dynamic controller for systems with parametric uncertainty and controller gain variations," *Proc. Amer. Contr. Conf. in Philadelphia, Pennsylvania*, pp. 2837-2841, 1998.

[10] A. Jadbabie, C. T. Abdallah, D. Famularo, and P. Dorato, "Robust, non-fragile and optimal controller via linear matrix inequalities," *Proc. Amer. Contr. Conf. in Philadelphia, Pennsylvania*, pp. 2842-2846, 1998.

[11] L. H. Keel and S. P. Bhattacharyya, "Digital implementation of fragile controllers," *Proc. Amer. Contr. Conf. in Philadelphia, Pennsylvania*, pp. 2852-2856, 1998.

[12] L. H. Keel and S. P. Bhattacharyya, "Robust, fragile, or optimal," *IEEE Trans. Automat. Contr.*, vol. 42, no. 8, pp. 1098-1105, 1997.

[13] J. H. Kim, S. K. Lee, and H. B. Park, "Robust and non-fragile H^{∞} control of parameter uncertain Time-varying delay systems," *SICE in Morioka*, pp. 927-932, July 1999.

[14] H. D. Tuan and P. Apkarian, "Relaxations of parameterized LMIs with control applications," *Int. J. of Robust Nonlinear Control*, vol. 9, pp. 59-84, 1999.