Regulation of a Nonholonomic Dynamic Wheeled Mobile Robot with Parametric Modeling Uncertainty using Lyapunov Functions

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Abstract

This paper addresses the problem of regulating the dynamic model of a nonholonomic wheeled robot of the unicycle type to a point with a desired orientation. A simple controller is derived that yields global convergence of the trajectories of the closed loop system in the presence of parametric modeling uncertainty. Controller design relies on a non smooth coordinate transformation in the original state space, followed by the derivation of a Lyapunov-based, adaptive, smooth control law in the new coordinates. Convergence to the origin is analyzed and simulation results are presented.

1 Introduction

The control of nonholonomic systems has been the subject of considerable research effort over the last few years. The reason for this trend is threefold: i) there are a large number of mechanical systems such as robot manipulators, mobile robots, wheeled vehicles, and space and underwater robots that have non integrable constraints; ii) there is considerable challenge in the synthesis of control laws for systems that are not transformable into linear control problems in any meaningful way and, iii) as pointed out in a famous paper of Brockett (Brockett, 1983), nonholonomic systems cannot be stabilized by continuously differentiable, time invariant, state feedback control laws. To overcome the limitations imposed by the celebrated Brockett's result, a number of approaches have been proposed for stabilization of nonholonomic control systems to equilibrium points. See (Kolmanovsky and McClamroch, 1995) and the references therein for a comprehensive survey of the field. Among the proposed solutions are smooth time-varying controllers (Samson, 1995; Godhavn and Egeland, 1997), discontinuous or piecewise smooth control laws (Canudas de Wit and Sørdalen, 1992; Bloch and Drakunov, 1994; Astolfi, 1999), and hybrid controllers (Hespanha, 1996; Aguiar and Pascoal, 2000). Specially attractive are discontinuous control laws, which in some cases can overcome the complexity and lack of good performance (e.g., low rates of convergence and oscillating trajectories) that are often associated with time-varying control strategies. The reader is referred to (Astolfi, 1999) for a discussion of this interesting circle of ideas.

Despite the vast amount of papers published on the stabilization of nonholonomic systems, the majority has concentrated on kinematic models of mechanical systems controlled directly by velocity inputs, while less attention has been paid to the control of nonholonomic dynamical mechanical systems where forces and torques are the true inputs. See for example (M'Closkey and Murray, 1994) where the authors extended time-varying exponential stabilizers to dynamic nonholonomic systems.

Another less studied problem is that of controlling uncertain nonholonomic systems. See (Jiang and Pomet, 1996) where a backstepping based time-varying adaptive control scheme for a special class of uncertain nonholonomic chained systems was proposed.

Motivated by the above considerations, this paper addresses the problem of regulating the dynamic model of a nonholonomic wheeled robot of the unicycle type to a point with a desired orientation. *A simple, discontinuous, adaptive state feedback controller is derived that yields global convergence of the trajectories of the closed loop system in the presence of parametric modeling uncertainty.* This is achieved by resorting to a polar representation of the kinematic model of the mobile robot that is a non smooth transformation in the original state space, followed by the derivation of a smooth, time-invariant control law in the new coordinates. The new control algorithm proposed as well as the analysis of its convergence build on Lyapunov stability theory and LaSalle's invariance principle. For an introduction to the polar representation and how it can be exploited to overcome the basic limitations imposed by Brockett's result, see (Aicardi *et al.*, 1995) and (Astolfi, 1999).

The paper is organized as follows: Section 2 describes

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the model of a wheeled mobile robot of the unicycle type and formulates the problem of regulating its motion to a point with a desired orientation in the presence of model uncertainties. A non-smooth polar representation of its kinematics is also presented. Section 3 introduces a simple Lyapunov-based adaptive strategy for vehicle control that builds on a series of candidate Lyapunov functions related to vehicle heading regulation, target distance regulation, and parameter adaptation. Section 4 offers a formal proof of convergence of the resulting adaptive regulation system. Section 5 contains simulation results that illustrate the performance of the proposed control strategy and show how it yields natural vehicle's behaviour. The paper concludes with a summary of results and recommendations for further research.

2 The wheeled mobile robot. Control problem formulation

This section describes the kinematic and dynamic equations of the wheeled mobile robot of the unicycle type shown in Figure 1 and formulates the problem of controlling it to a point with a desired orientation. The vehicle has two identical parallel, nondeformable rear wheels which are controlled by two independent motors, and a steering front wheel. It is assumed that the plane of each wheel is perpendicular to the ground and the contact between the wheels and the ground is pure rolling and nonslipping, i.e., the velocity of the center of mass of the robot is orthogonal to the rear wheels axis[1]. It is further assumed that the masses and inertias of the wheels are negligible and that the center of mass of the mobile robot is located in the middle of the axis connecting the rear wheels. Each rear wheel is powered by a motor which generates a control torque τ_i, $i = 1, 2$.

2.1 Robot Model. Problem Formulation

The following notation will be used in the sequel. The symbol $\{A\} := \{x_A, y_A\}$ denotes a reference frame with origin at O_A and unit vectors x_A, y_A. Let $\{U\}, \{G\}$, and $\{B\}$ be inertial, goal, and body reference frames, respectively. Assume, for simplicity of presentation, that $\{U\} = \{G\}$ and that the origin O_B of $\{B\}$ is coincident with the center of the rear wheels axle. Let $[x, y]^T$ specify the position of O_B in $\{U\}$ and let θ be the parameter that describes the orientation of $\{B\}$ with respect to $\{U\}$ (i.e., the robot orientation with respect to the inertial x-axis). The kinematics and dynamics of the mobile robot are modeled by the equations

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \omega \\ m\dot{v} &= F \\ I\dot{\omega} &= N \end{aligned} \tag{2.1}$$

[1]By assuming that the wheels do not slide, a nonholonomic constraint on the motion of the mobile robot of the form $\dot{x}\sin\theta - \dot{y}\cos\theta = 0$ is imposed.

where v and ω denote the linear and angular velocity of $\{B\}$ with respect to $\{U\}$, respectively. The control inputs are the force F along the vehicle axis x_B and the torque N about its vertical axes z_B. It is easy to see that

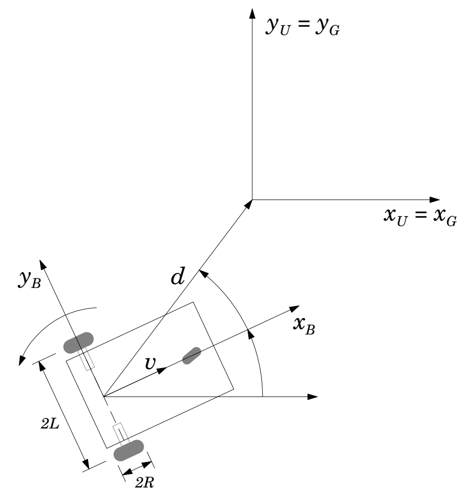

Figure 1: A wheeled mobile robot of the unicycle-type.

$$\begin{aligned} F &= \frac{1}{R}(\tau_1 + \tau_2) \\ N &= \frac{L}{R}(\tau_1 - \tau_2) \end{aligned} \tag{2.2}$$

where R is the radius of the rear wheels and $2L$ is the length of the axis between them. The symbols m and I are the mass and the moment of inertia of the mobile robot, respectively.

With the above notation, the problem considered in this paper can be formulated as follows:

Derive a feedback control for τ_1 and τ_2 to regulate $\{B\}$ to $\{G\} = \{U\}$ in the presence of uncertainty in the parameters m, I, R, and L.

2.2 Polar representation

This section introduces a change of coordinates that plays a crucial role in the development that follows. See (Astolfi, 1999) and the references therein for an introduction to this transformation, its rationale, and importance. Consider the coordinate transformation (see Figure 1)

$$e = \sqrt{x^2 + y^2} \tag{2.3a}$$
$$x = -e\cos(\theta + \beta) \tag{2.3b}$$
$$y = -e\sin(\theta + \beta) \tag{2.3c}$$
$$\theta + \beta = \tan^{-1}\left(\frac{-y}{-x}\right) \tag{2.3d}$$

where d is the vector from O_B to O_U, e is the length of d, and β denotes the angle measured from x_B to d. Notice that in equation (2.3d) care must be taken to select the proper quadrant for β. Differentiating (2.3) with respect to time, the dynamics of the wheeled robot in the new coordinate system can be written as

$$\begin{aligned}\dot{e} &= -v\cos\beta \\ \dot{\beta} &= \frac{\sin\beta}{e}v - w \\ \dot{\theta} &= w \\ m\dot{v} &= F \\ I\dot{\omega} &= N\end{aligned} \tag{2.4}$$

Remark 1 Notice that the coordinate transformation (2.3) is only valid for non zero values of the distance error e, since for $e = 0$ the angle β is undefined. This will introduce a discontinuity in the control law that will be derived later, which will obviate the basic limitations imposed by the celebrated result of Brockett. The creation of this singular point is at the core of the design methodology adopted in this paper, which was inspired by the work of (Astolfi, 1999).

3 Nonlinear Controller Design

This section proposes a nonlinear adaptive control law to regulate the motion of the mobile robot described by equations (2.1) and (2.2) to a point with a desired orientation, in the presence of parametric modeling uncertainty. Only the rationale for the control law proposed is introduced, a formal proof of convergence being deferred to Section 4. For the sake of clarity, candidate Lyapunov functions are introduced recursively in a sequence of logical steps directly related to vehicle heading regulation, target distance regulation, and parameter adaptation. This methodology borrows heavily from the techniques of backstepping (Krstić *et al.*, 1995). A switching term is introduced in the control law at the last stage in order to solve the indeterminacy at $e = 0$ caused by the polar representation adopted.

Step 1. (Heading regulation) Define the variables

$$\rho = \frac{v}{e}, \quad \sigma = \beta + \theta,$$

and rewrite the equations of motion (2.4) as

$$\dot{\sigma} = \rho\sin\beta \tag{3.1a}$$

$$\dot{\beta} = \rho\sin\beta - \omega \tag{3.1b}$$

$$\dot{\omega} = \frac{N}{I} \tag{3.1c}$$

and

$$\dot{e} = -\rho\cos\beta e \tag{3.2a}$$

$$\dot{\rho} = \frac{F}{me} + \rho^2\cos\beta \tag{3.2b}$$

where system (2.4) has been divided in two subsystems that will henceforth be referred to as the heading and distance subsystems, respectively. Consider the heading subsystem (3.1) and suppose (only at this stage) that $\rho = k_1 > 0$. Define the control Lyapunov function

$$V_1 = \frac{1}{2}k_\sigma\sigma^2 + \frac{1}{2}\beta^2,$$

and compute its time derivative along trajectories of (3.1) to obtain

$$\dot{V}_1 = \beta\left[k_1 k_\sigma \sigma\frac{\sin\beta}{\beta} + k_1\sin\beta - \omega\right].$$

Following the nomenclature in (Krstić *et al.*, 1995) let ω be a virtual control input and

$$\alpha_1(\sigma,\beta) = k_1 k_\sigma \sigma\frac{\sin\beta}{\beta} + k_1\sin\beta + k_2\beta, \tag{3.3}$$

$k_2 > 0$, a virtual control law. Introduce the error variable

$$z_1 = \omega - \alpha_1, \tag{3.4}$$

and compute $\dot{V}_1$ to obtain

$$\dot{V}_1 = -k_2\beta^2 - \beta z_1.$$

Step 2. (Backstepping) The function V_1 is now augmented with a quadratic term in z_1 to obtain the new candidate Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_1^2.$$

The time derivative of V_2 can be written as

$$\dot{V}_2 = -k_2\beta^2 + z_1\left[\frac{N}{I} - f_1(\cdot)\right],$$

where

$$\begin{aligned} f_1(\sigma,\beta,z_1,\rho) &= \frac{\partial\alpha}{\partial\sigma}\dot{\sigma} + \frac{\partial\alpha}{\partial\beta}\dot{\beta} - \beta \\ &= k_1 k_\sigma\dot{\sigma}\frac{\sin\beta}{\beta} + k_1 k_\sigma\dot{\beta}\frac{\beta\cos\beta - \sin\beta}{\beta^2} \\ &\quad + k_1\dot{\beta}\cos\beta + k_2\dot{\beta} + \beta. \end{aligned}$$

Notice that the terms $\frac{\sin\beta}{\beta}$ and $\frac{\beta\cos\beta - \sin\beta}{\beta^2}$ are well defined and continuous at zero. Using L'Hopital's rule it is easy to see that when $\beta = 0$ the first and second terms are equal to 1 and 0, respectively.

Let the control law for N be chosen as

$$N = If_1(\cdot) - k_3 z_1, \quad k_3 > 0.$$

Then

$$\dot{V}_2 = -k_2\beta^2 - \frac{k_3}{I}z_1^2 \leq 0,$$

that is, $\dot{V}_2$ is negative semidefinite.

Step 3. (Distance regulation) Consider now the distance subsystem (3.2). A new error variable $z_2 = \rho - k_1$ is defined and a third candidate Lyapunov function is introduced as

$$V_3 = V_2 + \frac{1}{2}z_2^2.$$

Computing its time derivative gives

$$\dot{V}_3 = -k_2\beta^2 - \frac{k_3}{I}z_1^2 + z_2\left[\frac{F}{me} + f_2(\cdot)\right],$$

where

$$f_2(\sigma,\beta,\rho) = \rho^2\cos\beta + k_\sigma\sigma\sin\beta + \beta\sin\beta.$$

The last two terms of f_2 are due to the fact that ρ is not constant, but $\rho = k_1 + z_2$ instead. They are simply computed by replacing ρ by $k_1 + z_1$ in the expression for V_1 and propagating the corresponding terms down to $\dot{V}_3$. Now, by choosing the control input

$$F = -mf_2(\cdot)e - k_4 z_2 e,$$

the time derivative of V_3 becomes

$$\dot{V}_3 = -k_2\beta^2 - \frac{k_3}{I}z_1^2 - \frac{k_4}{m}z_2^2,$$

that is, $\dot{V}_3$ is negative semidefinite.

Step 4. (Parameter adaptation) Suppose that the values of the physical constants m, I, R, and L are not known precisely. Define the control inputs u_i, $i = 1,2$ as $u_1 = \tau_1 - \tau_2$ and $u_2 = \tau_1 + \tau_2$. Then, from (2.2) the dynamic equations for the mobile robot can be written as

$$\dot{\omega} = \frac{u_1}{c_1}, \quad \dot{v} = \frac{u_2}{c_2},$$

where $c_1 = \frac{IR}{L}$ and $c_2 = mR$ are positive unknown parameters.
Consider the augmented candidate Lyapunov function

$$V_4 = V_3 + \frac{1}{2c_1\gamma_1}\Delta c_1^2 + \frac{1}{2c_2\gamma_2}\Delta c_2^2,$$

where $\hat{c}_i$; $i = 1,2$ are nominal value of the parameters c_i, $\Delta c_i = c_i - \hat{c}_i$ are parameter estimation errors, and $\gamma_i > 0$; $i = 1,2$ are adaptation gains. The time derivative of V_4 can be computed to yield

$$\dot{V}_4 = -k_2\beta^2 + z_1\left[\frac{u_1}{c_1} - f_1(\cdot)\right] + z_2\left[\frac{u_2}{c_2 e} + f_2(\cdot)\right] - \frac{\Delta c_1}{c_1\gamma_1}\dot{\hat{c}}_1 - \frac{\Delta c_2}{c_2\gamma_2}\dot{\hat{c}}_2.$$

Motivated by the choices in steps 2 and 3, choose the control laws

$$\begin{aligned} u_1 &= \hat{c}_1 f_1(\cdot) - k_3 z_1, \\ u_2 &= -\hat{c}_2 f_2(\cdot)e - k_4 z_2 e, \end{aligned} \tag{3.5}$$

to obtain

$$\dot{V}_4 = -k_2\beta^2 - \frac{k_3}{c_1}z_1^2 - \frac{k_4}{c_2}z_2^2 - \frac{\Delta c_1}{c_1}\left[z_1 f_1(\cdot) + \frac{\dot{\hat{c}}_1}{\gamma_1}\right] + \frac{\Delta c_2}{c_2}\left[z_2 f_2(\cdot) - \frac{\dot{\hat{c}}_2}{\gamma_2}\right].$$

Notice in this equation how the terms containing Δc_i have been grouped together. To eliminate them, choose the parameter adaptation law as

$$\begin{aligned} \dot{\hat{c}}_1 &= -\gamma_1 z_1 f_1(\cdot), \\ \dot{\hat{c}}_2 &= \gamma_2 z_2 f_2(\cdot), \end{aligned} \tag{3.6}$$

to yield

$$\dot{V}_4 = -k_2\beta^2 - \frac{k_3}{c_1}z_1^2 - \frac{k_4}{c_2}z_2^2 \leq 0. \tag{3.7}$$

Step 5. (Switching control law) So far, it has been assumed that the mobile robot will never start at or reach[2] the position $x = y = 0$ in finite time, because the polar representation (2.3) and consequently the control law described above are not defined at $e = 0$. To deal with this situation, a switching control law must be introduced at this stage. A possible solution is to make

$$\begin{aligned} u_1 &= -k_{\dot{\theta}}\dot{\theta} - k_\theta\theta \\ u_2 &= 0 \end{aligned} \tag{3.8}$$

when $e = 0$, where $k_{\dot{\theta}}$ and k_θ are positive constants. The motivation for this control law can be simply understood by noticing that it aims at rotating the vehicle in place under the action of the proportional and derivative terms $k_\theta\theta$ and $k_{\dot{\theta}}\dot{\theta}$, respectively.

The complete control law is thus given by

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{cases} (3.5), (3.6) & e \neq 0 \\ (3.8) & e = 0 \end{cases} \tag{3.9}$$

4 Convergence Analysis

This section proves convergence to zero of the trajectories of the closed-loop system consisting of equations (2.1) and (3.9). The following theorem establishes the main result.

Theorem 1 *Consider the closed-loop nonlinear invariant system Σ described by (2.1) and (3.9). Let $\mathcal{X} : [t_0, \infty) \to \mathbb{R}^7$, $\mathcal{X}(t) = (x, y, \theta, v, w, \Delta c_1, \Delta c_2)'$; $t_0 \geq 0$ denote any solution of Σ. The following properties hold.*

1. *$\mathcal{X}(t)$ exists, is unique and defined for all $t \geq t_0$ and all $\mathcal{X}(t_0) = \mathcal{X}_0$.*
2. *$\mathcal{X}(t)$ is bounded for any $\mathcal{X}_0$.*
3. *For any initial condition $\mathcal{X}_0$, the state variables $q = (x, y, \theta, v, \omega)'$ converge to zero as $t \to \infty$.*

Proof: The existence and uniqueness of $\mathcal{X}(t)$ is proven by showing first that for the closed-loop system Σ the

[2] In fact, it will be shown later that if the initial condition is $e(t_0) \neq 0$ than this situation will never arise.

only switching scenario is when $e = 0$ and $v \neq 0$. It can be easily checked that the manifold $F = \{\mathcal{X} : x = 0 \wedge y = 0 \wedge v = 0\}$ is positively invariant. Also for $e \neq 0$, if solutions to Σ exist then (3.7) and LaSalle's invariance principle (La Salle and Lefschetz, 1961) show that $\rho(t) \to k_1$ which means that e and v must be either zero or non-zero at the same time. The only case where this is violated is when $e(t_0) = 0$ and $v(t_0) \neq 0$. In this case, from (2.1) it can be seen that for $t = t_0 + \delta$ with $\delta > 0$, $e(t_0 + \delta) \neq 0$, i.e., the control system will switch to the case $e \neq 0$. It can thus be concluded that for any initial condition $\mathcal{X}_0$ there occurs at most one switching, and the closed-loop system Σ has a finite number of discontinuities. Using (Hale, 1980, Theorem 5.3, Section I.5), a unique solution to Σ exists over a maximal interval $[0, t_f)$. Now, it remains to prove that the maximum interval of existence is infinite. For $e \neq 0$, solutions of (3.1) and (3.2) exist. Also, from (3.7) one can conclude that $\sigma(t)$, $\beta(t)$, $z_1(t)$, $\rho(t)$, $z_2(t)$, $\hat{c}_1(t)$, and $\hat{c}_2(t)$ are bounded when $\mathcal{X}_0 \notin F$. Thus, the above variables are well defined on the infinite interval. Notice that $\beta(t) \to 0$ and $\rho(t) \to k_1$ as $t \to \infty$. Thus, there exists a finite time $T \geq t_0 \geq 0$ such that for all $t \geq T$, $\rho \cos\beta > 0$. From (3.2a)

$$
\begin{aligned}
e(t) &= e(t_0) \exp\left(- \int_{t_0}^{t} \rho(\tau) \cos\beta(\tau) d\tau \right) \\
&= e(t_0) e^{-\int_{t_0}^{T} \rho(\tau)\cos\beta(\tau) d\tau} e^{-\int_{T}^{t} \rho(\tau)\cos\beta(\tau) d\tau},
\end{aligned}
$$

for all $t \geq t_0$. Therefore, it can be immediately seen that $e(t)$ is bounded and $e(t) \to 0$ as $t \to \infty$. Since $\sigma(t)$ and $\beta(t)$ are bounded, then $\theta(t)$ is bounded. Thus, the trajectory $\mathcal{X}(t)$ is bounded for all $\mathcal{X}_0 \notin F$. When $\mathcal{X}_0 \in F$ the same conclusions can be immediately drawn. Since the trajectory $\mathcal{X}(t)$ is bounded, it exists over the infinite interval, that is, $t_f = \infty$.

The rest of the proof shows that q converge to zero. If $\mathcal{X}_0 \in F$ then $\theta \to 0$ and $\omega \to 0$ as $t \to \infty$. If $\mathcal{X}_0 \notin F$, $e(t) \to 0$ which implies that $x(t) \to 0$ and $y(t) \to 0$ as $t \to \infty$. Moreover, since $\rho(t) \to k_1$, then $v(t) \to 0$ as $t \to \infty$. It remains to prove convergence of θ and ω. This is done by resorting to the LaSalle's invariance principle. Define $\Omega \triangleq \{\mathcal{X} : V_4(\mathcal{X}) \leq V_4(\mathcal{X}_0) = c\}$ which is a positively invariant set since $\dot{V}_4 \leq 0$. Let E be the set of all points in Ω such that $\dot{V}_4(\mathcal{X}) = 0$, that is, $E = \{\mathcal{X} \in \Omega : \beta = 0 \wedge z_1 = 0 \wedge z_2 = 0\}$. Let M be the largest invariant set contained in E. LaSalle's theorem assures that every bounded solution starting in Ω converges to M as $t \to \infty$. To characterize the set M, observe that in the set E the variables β and $\dot{\beta}$ are zero. Therefore, from (3.1b) $\omega = 0$. Notice also from (3.3) and (3.4), that if $\mathcal{X} \in E$ then $z_1 = \alpha_1 = k_1 k_\sigma \sigma$, and since $z_1 = 0$ in E it follows that $\sigma = 0$. Consequently, one can conclude that $\omega(t) \to 0$, $\sigma(t) \to 0$ and therefore $\theta(t) \to 0$ as $t \to \infty$. Thus, $\mathcal{X}(t) \to 0$ as $t \to \infty$. This concludes the proof of Theorem 1. ■

5 Simulation results

This section illustrates the performance of the proposed control scheme (in the presence of parametric uncertainty) using computer simulations. The objective is to regulate the position and attitude of the robot to zero. The following parameters were adopted: $m = 10.0\,Kg$, $I = 1.25\,Kgm^2$, $L = 0.5\,m$, and $R = 0.1\,m$. The control parameters were selected as $k_1 = 0.47$, $k_2 = 0.1$, $k_3 = 1.0$, $k_4 = 0.5$, $\gamma_1 = 0.1$, $\gamma_2 = 5.0$, $k_\sigma = 2.0$, $k_{\dot\theta} = 0.8$, and $k_\theta = 0.32$. The initial estimates for the vehicle parameters were $\hat{m} = 20.0\,Kg$, $\hat{I} = 1.6\,Kgm^2$, $\hat{L} = 0.4\,m$, and $\hat{R} = 0.15\,m$.
Figure 2 shows the vehicle trajectories in the xy-plane for different initial conditions in θ. Figures 3-5 display the time responses of the relevant state space variables for the initial condition $q(t_0) = (x_0, y_0, \theta_0, v_0, \omega_0) = (-10, -2, 0, 0, 0)$. Notice how, in spite of parameter uncertainty, the mobile robot converges asymptotically to the origin with a "natural", smooth trajectory. Notice also that the estimated parameters $c_i, i = 1, 2$ are bounded, as expected. However, as Figure 5 shows, the estimation error Δc_1 does not converge to zero. This is due to the particular structure of the adaptive control system adopted that allows for asymptotic convergence of q to zero with values of the estimated parameter $\hat{c}_1$ different from the "true" one.

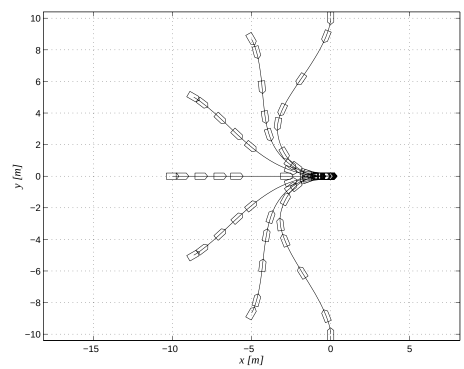

Figure 2: Trajectories in the xy-plane. Initial conditions: $e = 10$, $\beta = v = \omega = 0$, and $\theta = -\frac{\pi}{2}, -\frac{\pi}{3}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$.

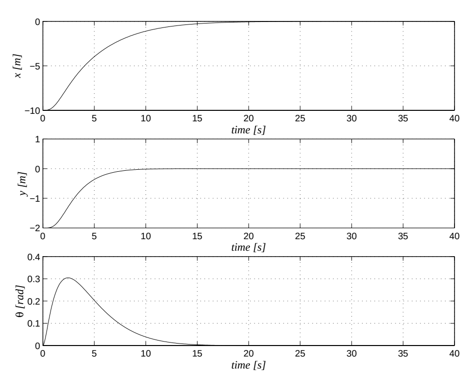

Figure 3: Time evolution of position variables $x(t)$, $y(t)$, and orientation variable $\theta(t)$.

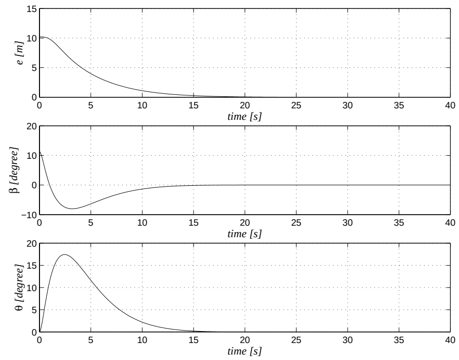

Figure 4: Time evolution of variables $e(t)$, $\beta(t)$, and $\theta(t)$.

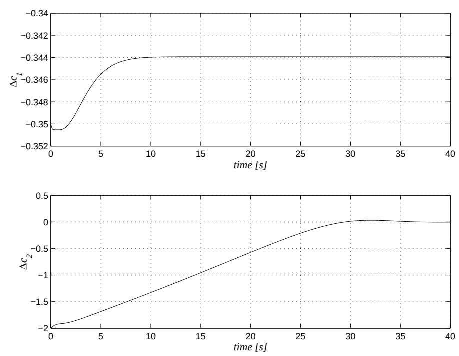

Figure 5: Time evolution of parameter estimation errors Δc_1 and Δc_2.

6 Conclusions

This paper proposed a new solution to the problem of regulating the dynamic model of a nonholonomic wheeled robot of the unicycle type to a point with a desired orientation. A discontinuous, bounded, time invariant, nonlinear adaptive control law that yields global convergence of the trajectories of the closed loop system in the presence of parametric modeling uncertainty was derived. Controller design relied on a non smooth coordinate transformation in the original state space, followed by the derivation of a Lyapunov-based, smooth control law in the new coordinates. Convergence to the origin was analyzed and simulations were performed to illustrate the behaviour of the proposed control scheme. Simulation results show that the control objectives were achieved successfully. Future research will address the extension of the method proposed to underwater vehicles. This poses considerable challenges to control system analysis and design, since the models of those vehicles typically include a drift vector field that is not in the span of the input vector fields, thus precluding the use of input transformations to bring them to driftless form. Another open problem that warrants further research is the control and analysis of mechanical nonholonomic systems in the presence of noisy measurements, actuator saturation constraints and observer dynamics.

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