Robustness of global asymptotic stability in indirect field-oriented control of induction motors

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Abstract

The influence of the rotor time constant mismatch on the stability of induction motors under indirect field oriented control is analyzed. The paper focuses on the global asymptotic stability property and extends the results of [3]. A global stability criterion based on the solution of an LMI problem is given. Robustness margins for global asymptotic stability with respect to rotor time constant mismatches are obtained using this criterion.

Keywords: Robustness margins, Global asymptotic stability, indirect field-oriented control, induction motors.

1 Introduction

Indirect Field Oriented Control (IFOC) is a well established and widely applied control technique when dealing with high performance induction motor drives [12, 10, 5]. The commissioning of an IFOC drive requires the knowledge of the rotor time constant, a parameter that can vary widely in practice [9, 11] and is known to cause performance and stability problems.

Most results in the literature address this problem from the application point of view focusing on the performance issue without providing guarantees of stability. Only the recent works [4, 2, 3, 14, 13, 7] have aimed at filling in this gap by providing IFOC with a firm theoretical foundation.

In [3, 7], a complete characterization of the equilibria with respect to rotor time constant mismatches has been given. Local stability properties of the equilibrium point have been investigated in [3, 2] where conditions for non-existence of both saddle-node and Hopf bifurcations were provided. Guidelines for setting the speed or position controller in order to guarantee a certain local stability margin with respect to rotor time constant mismatches for a practical loading range were also given (see also [1]).

Global stability of IFOC in speed regulation tasks has been studied in [7, 3, 8]. The robustness of this property in IFOC against mismatches in the rotor time constant has been established in [7] by means of a suited Lyapunov function. An improvement in this Lyapunov function has allowed to derive explicit formulae to conclude about robust global asymptotic stability in [3]. A passivity based analysis has been used in [8] to conclude about robust global asymptotic stability in the special case of zero load operation.

In this paper we characterize a class of Lyapunov functions for IFOC drives in speed regulation tasks. Section 2 formulates the problem and provides the complete model for the induction motor with IFOC; the model is valid both for proportional-integral rotor speed regulation and for rotor position regulation through proportional-derivative control. The model is parameterized in the rotor time constant mismatch, which allows for the robustness analysis. In section 3 an explicit condition to conclude about global asymptotic stability is provided, which also yields a test for robustness of the global stability property regarding rotor time constant mismatches. This tool is explored in section 4 to obtain robust global asymptotic stability margins with respect to rotor time constant mismatches in an example, showing that it is much less conservative than the ones previously presented [7, 3]. Finally, section 5 provides a discussion on the results obtained.

2 Problem statement

We consider the current fed induction motor model expressed in a reference frame rotating at synchronous speed. In terms of state variables, this model can be written as [12, 10, 5]

$$\dot{x}_1 = -c_1 x_1 - u_1 x_2 + c_2 u_3 \tag{1}$$
$$\dot{x}_2 = -c_1 x_2 + u_1 x_1 + c_2 u_2 \tag{2}$$
$$\dot{w} = -c_3 w + c_4[c_5(x_2 u_3 - x_1 u_2) - T_m] \tag{3}$$

where x_1 and x_2 represent the q-axis and d-axis rotor fluxes, respectively, w is the rotor speed, u_1, u_2 and u_3 stand for the inputs - the slipping frequency, the d-axis and q-axis stator current components, respectively; T_m is the load torque, which is assumed constant, and the "c" parameters are all positive. In particular, c_1 represents the inverse of the rotor time constant, which is a critical parameter for indirect field oriented control.

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In speed regulation applications the indirect field oriented control strategy is usually applied along with a PI speed loop as described by the following equations [12, 7]:

$$
\begin{aligned}
u_1 &= \hat{c}_1 \frac{u_3}{u_2} & (4)\\
u_2 &= u_2^0 & (5)\\
u_3 &= k_p(w_{ref} - w) + k_i \int_0^t (w_{ref} - w)(\zeta) d\zeta & (6)
\end{aligned}
$$

where $\hat{c}_1$ is an estimate for the inverse rotor time constant c_1, k_p and k_i are the gains of the PI speed controller, w_{ref} is the constant reference velocity and u_2^0 is a constant which defines the flux level. Equations (4) and (5) represent the field orientation control, while (6) is the proportional-integral speed controller.

The knowledge of c_1 is the key issue in IFOC. If $\hat{c}_1 = c_1$, that is, if we have a perfect estimate of the rotor time constant, we say that the IFOC is tuned, otherwise it is said to be detuned. Accordingly, we define

$$\kappa \triangleq \frac{\hat{c}_1}{c_1} \qquad (7)$$

as the degree of tuning. It is clear that $\kappa > 0$ and the IFOC is tuned if and only if $\kappa = 1$.

We parameterize the closed-loop system (1)-(3) with the control (4)-(6) (see Figure 1) in terms of the degree of tuning κ, yielding a fourth-order system that can be described as:

$$
\begin{aligned}
\dot{x}_1 &= -c_1 x_1 + c_2 x_4 - \frac{\kappa c_1}{u_2^0} x_2 x_4 & (8)\\
\dot{x}_2 &= -c_1 x_2 + c_2 u_2^0 + \frac{\kappa c_1}{u_2^0} x_1 x_4 & (9)\\
\dot{x}_3 &= -c_3 x_3 - c_4[c_5(x_2 x_4 - u_2^0 x_1) - T_e] & (10)\\
\dot{x}_4 &= k_c x_3 - k_p c_4[c_5(x_2 x_4 - u_2^0 x_1) - T_e] & (11)
\end{aligned}
$$

where we have defined the new state variables $x_3 \triangleq w_{ref} - w$ and $x_4 \triangleq u_3$ and the new parameters

$$k_c \triangleq k_i - k_p c_3, \qquad T_e \triangleq T_m + \frac{c_3}{c_4} w_{ref} \qquad (12)$$

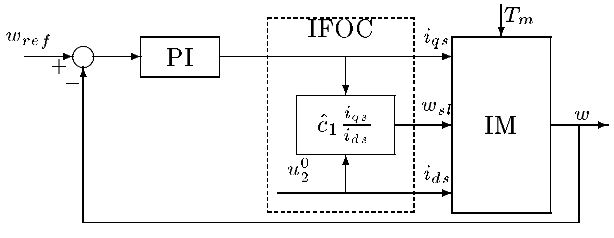

Figure 1: Block diagram of IFOC.

For position regulation equation (6) is substituted by a proportional-derivative controller:

$$u_3 = K_p(\delta_{ref} - \delta) + K_d \frac{d(\delta_{ref} - \delta)}{dt} \qquad (13)$$

where δ is the rotor position and δ_{ref} is the constant position reference. Defining the new state variables x_3 and x_4 as before (with $w_{ref} = 0$) and observing that $\dot{\delta}_{ref} = 0$ almost everywhere yields

$$
\begin{aligned}
\dot{x}_4 &= -K_p\dot{\delta} - K_d\ddot{\delta} = K_p x_3 - K_d \dot{w}\\
&= K_c x_3 - K_d c_4[c_5(x_2 x_4 - x_1 u_2^0) - T_m] \qquad (14)
\end{aligned}
$$

which is the same as (11) but with $K_c = K_p - K_d c_3$ in lieu of k_c and K_d in lieu of k_p. Since the resulting dynamic model for the position regulation is the same as for speed regulation with zero reference speed, all the results derived for speed regulation are also valid for position regulation, and we henceforth treat only the speed regulation case.

The first issue for the stability analysis of (8)-(11) is the characterization of equilibria. In [3, 2] we derived a convenient parameterization of the equilibria of the closed-loop system (8)-(11) for all degrees of tuning and loading conditions. Let us briefly review those results, which will be necessary for the developments in this paper. For further detail and proofs see [3].

Let us define the dimensionless variables $r \triangleq \frac{x_4^e}{u_2^0}$ and $r^* \triangleq \frac{T_e c_1}{c_5 c_2 (u_2^0)^2}$, where T_e has been defined in (12). The constant r^* represents the normalized load, since it is proportional to the electrical torque developed in steady-state. It also coincides with the steady-state ratio between the quadrature-axis component and the direct-axis component of stator currents in the tuned condition, so that it coincides with r in that operating condition. The parameter r can be shown to satisfy the third-order polynomial equation

$$\kappa r^3 - r^* \kappa^2 r^2 + \kappa r - r^* = 0 \qquad (15)$$

and the equilibria $x^e = [x_1^e \; x_2^e \; x_3^e \; x_4^e]^T$ can be written as

$$
\begin{bmatrix} x_1^e \\ x_2^e \\ x_3^e \\ x_4^e \end{bmatrix} = \begin{bmatrix} \frac{c_2 u_2^0}{c_1} \frac{1-\kappa}{1+\kappa^2 r^2} r \\ \frac{c_2 u_2^0}{c_1} \frac{1+\kappa r^2}{1+\kappa^2 r^2} \\ 0 \\ u_2^0 \, r \end{bmatrix} \qquad (16)
$$

The equilibria are parameterized in terms of a single dimensionless quantity r, which satisfies equation (15). This is a third order polynomial equation whose coefficients are also dimensionless and depend only on the degree of tuning κ and the motor load as denoted by r^*.

The real solutions of equation (15) give the equilibrium values of r for any given degree of tuning - κ - and any given load - r^*. Equation (15) has at least one and at most three real solutions, depending on the particular values of κ and r^*. The complete characterization of the equilibria was given in [3], where it was shown that equation (15) has a unique real solution for any load if and only if $\kappa \leq 3$. Hence, global asymptotic stability can be obtained for all load only within this range of parameter mismacth. This is also the practical range of this parameter in most drives, as practical variations of the rotor time constant due to temperature and load variations are usually within 200% [9]. Accordingly, since in this paper we are mostly concerned with global asymptotic stability, we consider only the parameter range $\kappa \times r^* \in \{(0, 3) \times \Re\}$.

We look for establishing allowable margins for the degree of tuning κ which preserve global asymptotic stability for given PI settings and loading conditions.

3 Stability by quadratic Lyapunov functions

Define the change of coordinates $z \stackrel{\Delta}{=} x - x^e$. Writing the sytem (8)-(11) in these new coordinates and using (16) yields

$$\dot{z} = [A_0(\kappa, r) + z_4 A_1(\kappa)]z \tag{17}$$

with

$$A_0(\kappa, r) = \begin{bmatrix} -c_1 & -c_1 r\kappa & 0 & c_2(1-\kappa)\frac{1+\kappa r^2}{1+\kappa^2 r^2} \\ r\kappa c_1 & -c_1 & 0 & \kappa c_2 \frac{1-\kappa}{1+\kappa^2 r^2} r \\ c_4 c_5 u_2^0 & -c_4 c_5 u_2^0 r & -c_3 & -\frac{c_4 c_5 c_2 u_2^0}{c_1}\frac{1+\kappa r^2}{1+\kappa^2 r^2} \\ k_p c_4 c_5 u_2^0 & -k_p c_4 c_5 u_2^0 r & k_c & -\frac{k_p c_4 c_5 c_2 u_2^0}{c_1}\frac{1+\kappa r^2}{1+\kappa^2 r^2} \end{bmatrix} \tag{18}$$

$$A_1(\kappa) = \begin{bmatrix} 0 & -\frac{\kappa c_1}{u_2^0} & 0 & 0 \\ \frac{\kappa c_1}{u_2^0} & 0 & 0 & 0 \\ 0 & -c_4 c_5 & 0 & 0 \\ 0 & -k_p c_4 c_5 & 0 & 0 \end{bmatrix} \tag{19}$$

For a given operating condition (given load r^* and parameter mismatch κ) A_0 and A_1 are constant. Hence, for each operating condition the system equation presents two terms: a linear term due to A_0 and a bilinear term given by A_1.

Let us take a quadratic Lyapunov candidate:

$$V(z) = z^T P z \; ; \quad P = P^T > 0 \tag{20}$$

Then the Lyapunov derivative is

$$\dot{V}(z) = z^T [A_0^T(\kappa, r^*)P + PA_0(\kappa, r^*)]z + z_4 z^T [A_1^T(\kappa)P + PA_1(\kappa)]z \tag{21}$$

and we have the following result.

Theorem 1 *Consider a given parameter mismacth κ and normalized load r^*. If there exists a matrix P satisfying the following conditions:*

$$A_1^T(\kappa)P + PA_1(\kappa) = 0 \tag{22}$$
$$A_0^T(\kappa, r^*)P + PA_0(\kappa, r^*) < 0 \tag{23}$$
$$P = P^T > 0 \tag{24}$$

then the system (17) is globally asymptotically stable.

Proof: *It is clear that (22) and (23) imply that the Lyapunov derivative is negative definite globally and (24) implies that the Lyapunov candidate is positive definite globally.*

Notice that these conditions are necessary and sufficient for a quadratic Lyapunov function to guarantee global asymptotic stability (g.a.s.).

Theorem 2 *A quadratic Lyapunov function ensures global asymptotic stability of the system (17) for given parameter mismatch κ and load r^* if and only if there exists a matrix P such that the matrix relations (23), (22) and (24) are satisfied.*

Proof: *The 'if' part is given in the previous theorem. On the other hand, if (22) is violated there will always exist a z_4 which will make the second term in the derivative positive and larger in modulus than the first term, so that the derivative can not be globally negative in this case.*

The conditions (23), (22) and (24) define a class of Lyapunov functions for the system (17). Other Lyapunov functions proposed in earlier works [7, 3] are also quadratic and can be easily shown to satisfy these conditions (which has been proven necessary in the above theorem), therefore being particular cases belonging to this class.

These conditions are in the standard form of linear matrix inequalities and equalities (LMI's and LME's) and as such can be solved with standard software [6]. They provide a simple verification procedure to conclude about global asymptotic stability for any particular IFOC induction motor drive in any particular operating condition. They also provide a way to characterize robust global asymptotic stability margins with respect to κ for such systems, i.e., for a given PI setting and loading condition find, if possible, a range of κ for which global asymptotic stability of system (17) is guaranteed. That such a range does exist around $\kappa = 1$ for any PI setting and load condition has been proven elsewhere [3, 7]. Its determination can be acomplished through the following steps:

1. define the range of interest for variation of the parameters $\mathcal{P} \triangleq \{[\kappa_{min}, \kappa_{max}] \times [r^*_{min}, r^* max]\}$ and a mesh of points inside this range;
2. for each point in the mesh above run the LMI/LME problem (22), (23), (24);
3. all the points in the mesh for which the problem is feasible represent a globally asymptotically stable operating condition.

When (22) is not satisfied, local asymptotic stability is guaranteed as the Lyapunov derivative is negative in the set

$$\mathcal{D} = \{z \ : \ \mid z_4 \mid < \frac{\min \mid \lambda(A_0^T P + PA_0^T) \mid}{\max \mid \lambda(A_1^T P + PA_1^T) \mid}\} \qquad (25)$$

An estimate of the region of attraction in this case is given by the largest level curve of $V(z)$ which fits within the set $\mathcal{D}$.

4 An example

Theorem 1 provides a test for global stability which can be applied for any motor in any operating condition. The matrix equations in the test depend on all the parameters of the driving, which can be divided into four sets:

- the physical parameters of the motor (the 'c' parameters);
- the setting of the PI (k_p and k_i);
- the load (r^*);
- the mismatch in the rotor time constant (κ).

In order to get a better insight to the problem and establish typical robustness margins, we apply the global stability test to data taken from a real induction motor. We aim to study, for a given motor and a given setting of the PI speed loop, what is the region of the parameter plane $\kappa \times r^*$ for which global stability is achieved. To this end, we apply the test with the c parameters, k_p and k_i fixed, varying κ and r^* in the set of practical significance $(\kappa, r^*) \in (0,3) \times [0,2]$.

Furthermore, we perform this procedure for different settings of the PI speed loop, in order to verify its influence on the robustness of the global stability. The PI parameters are usually set in order to provide a desired performance to the system under the assumption of perfect tuning ($\kappa = 1$). Accordingly, we refer to the PI settings with regard to the transient performance they provide to the tuned system, normalized to the rotor time constant. We assume that the PI is set so that the tuned system's transient response is over damped and is η times faster than the rotor time constant c_1. Then the parameter η is used to represent the PI setting. That this is a one to one parameter mapping has been shown in [2]. It was also shown in [2] that the equilibrium point is locally asymptotically stable for all $\kappa \in (0,3)$ and $r^* \in [0,2]$ provided that $\eta \leq 18$.

The same procedure is applied with the criterion provided in [3] for comparison. Recall that the class of Lyapunov functions from which this criterion is derived is a subset of the class defined in this paper and that the Lyapunov function given in [7] is a particular case in this subset.

We take data from a three phase induction motor, with 1 HP nominal power output and 220 V nominal line voltage. The parameters of the motor are given in the Appendix. Let the parameters of the PI be chosen such that the transient response of the tuned system is over damped and dominated by a time constant which is half the rotor time constant, that is, $\eta = 2$. Then we apply the global stability test for κ and r^* varying in steps of 0.1. Figure 2 shows the region of the parameter space $\kappa \times r^*$ for which the test gives a positive answer: the lower plot shows the results obtained with the LMI/LME criterion in this paper; the results obtained with the criterion in [3] are shown in the upper plot. The operating point is guaranteed to be globally asymptotically stable for all load and parameter mismatch in the dotted region.

One can see that the range of parameters for which g.a.s. is guaranteed is larger with the proposed LMI/LME criterion. As faster response is assigned through the PI settings, this difference becomes larger, as shown in Figures 3, 4 and 5, which show the cases for $\eta = 5$, $\eta = 10$ and $\eta = 18$. For faster system's response the new LMI/LME is far less conservative.

For moderate values of η, that is, when the system is not made too fast by the PI settings, global asymptotic stability is guaranteed for most practical values of κ and r^*. As the system is made faster, the range of parameter values for which g.a.s is guaranteed gets smaller, particularly for $\kappa < 1$. For $\eta = 18$ this range is much smaller than the practical range of interest. Recall that for $\eta > 18$ not even local stability is guaranteed in general.

5 Concluding remarks

We have provided a test for robust global asymptotic stability which can be easily implemented provided the physical parameters of the motor are known. This test provides allowable margins of errors in the rotor time constant for any given IFOC drive.

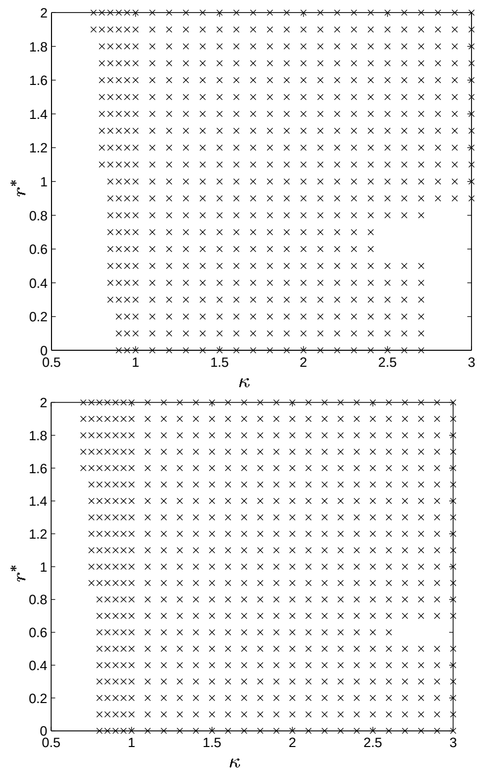

Figure 2: Parameter range of g.a.s. for the 1HP motor, $\eta = 2$. Upper: previous method; Lower: LMI/LME test.

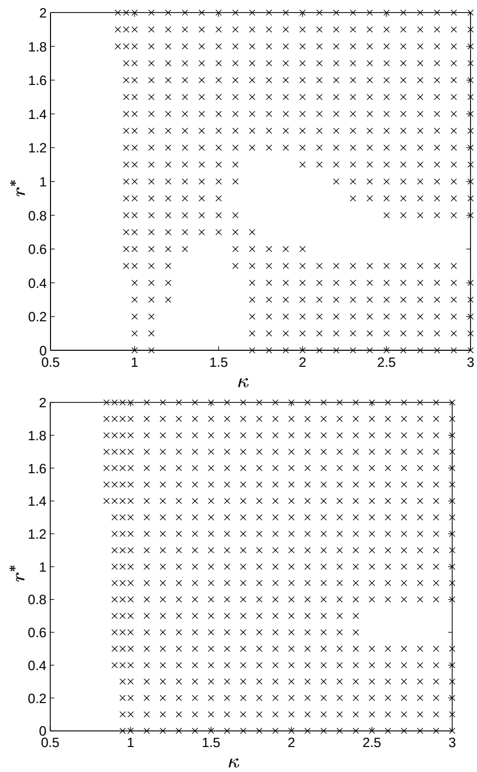

Figure 3: Parameter range of g.a.s. for the 1HP motor, $\eta = 5$. Upper: previous method; Lower: LMI/LME test.

Rules of thumb for tuning the PI speed loop can also be derived from these results. If global asymptotic stability is required (as in drives subject to large disturbances and/or large set-point variations) then the speed loop should not be made too fast, as the robustness margins would become too small. These results and rules reinforce the results on local asymptotic stability provided in previous works [2, 3, 8, 7].

A Data for the 1 HP motor

c_1	13.7 s^{-1}	c_2	1.56 Ω
c_3	0.59 s^{-1}	c_4	1.18 $kg^{-1}m^{-2}$
c_5	2.86	u_2^0	4 A

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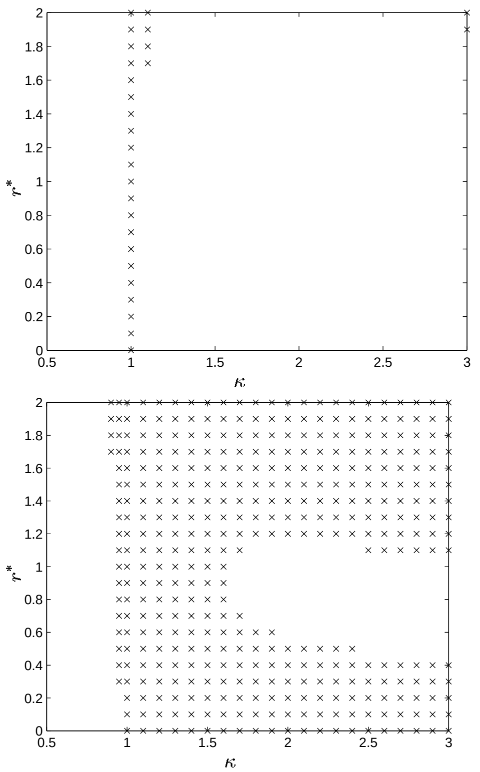

Figure 4: Parameter range of g.a.s. for the 1HP motor, $\eta = 10$. Upper: previous method; Lower: LMI/LME test.

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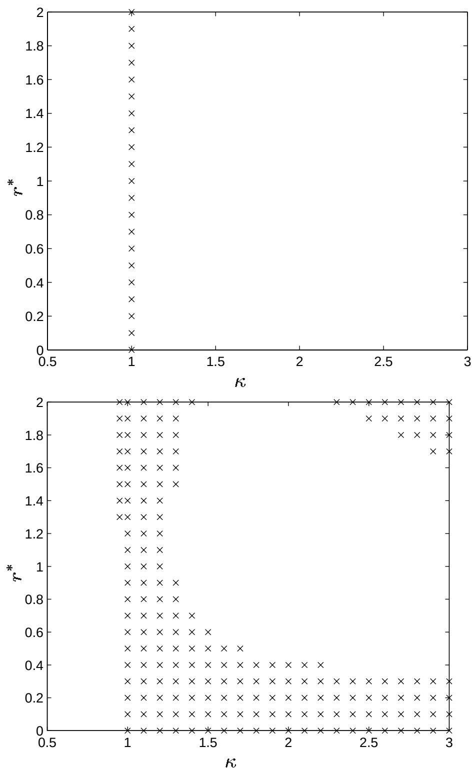

Figure 5: Parameter range of g.a.s. for the 1HP motor, $\eta = 18$. Upper: previous method; Lower: LMI/LME test.