Hand over control of unstable object using manipulators
— An approach of continuously switching of controllers —

Napoleon
Department of Mechanical
and Environmental Informatics,
Tokyo Institute of Technology,
2-12-1 Oh-okayama Meguro
Tokyo 152-8552, Japan
leo@sc.ctrl.titech.ac.jp

Tasuku Hoshino
Department of Management
and System Science,
Science University of Tokyo,
Suwa College, 5000-1 Toyohira
Chino, Nagano 391-0292, Japan
hoshino@ss.suwa.sut.ac.jp

Katsuhisa Furuta
Department of Computers
and Systems Engineering,
Tokyo Denki University,
Hatoyama Hikigun
Saitama 350-0394, Japan
furuta@k.dendai.ac.jp

Abstract

This paper presents a new hand over method of unstable object using robot manipulators by an approach of continuously switching of controllers. Each of robot manipulator can be considered as a controller to stabilize the object. It is needed to switch from one controller to another so that the input of the system does not exceed the value which is required to stabilize the object. However, it is known that if the controller switches at unsuitable time, the system may become unstable. The idea of a new method that is presented here is to make a switching of the controllers based on Double Bracket Flow. It will be shown that even if switching time is done at any time, the system remains stable. Experimental results using two Puma-like 6 D.O.F. robot to hand over an inverted pendulum from one robot to another are included.

1 Introduction

Hand over control of an inverted pendulum as unstable object using robot manipulators has been successfully done using reliable design method [1, 2]. In this method, the unstable object corresponds to the plant which should be reliably stabilized, and the manipulators which are applying forces to the object correspond to the reliable controllers. The design of controller was reformulated as a reliable stabilization problem against controller failures. Thus, the controllers should be designed to be able to stabilize the plant together or individually. The manipulator which did not hold the object was regarded as in failure. Thus, there was a possibility that the system input exceeded the suitable one, especially when the object was held by two manipulators. While, force was thought as an input to the system which handed over inverted pendulum in this problem, the excess of applied forces will change into internal force. Even if theoretically, this internal force can be suppressed, the experimental result showed that the internal force was quite big. This internal force could damage the plant.

To minimize the internal force which occurred in using reliable design method to hand over unstable object, switching of the controllers which stabilizes the plant could be a possible way to carry out. However, switching of the controllers discontinuously at unsuitable time may enable the system to become unstable [3].

On the other hand, Double Bracket Flow that was introduced by Brockett [4, 5] can be used to sort real numbers continuously under some appropriate assumptions. The characteristics of Double Bracket Flow that sort real numbers continuously, can be used to parameterize the input which is applied into the plant. Thus, the input will be given from one controller to another as the change in sorting. And it will become one of tools to schedule the controllers in this problem. Thus, Double Bracket Flow is reformulated to switch the controllers continuously. It will be shown that the input of the system will not exceed the suitable one to stabilize the object. And then, handing over of unstable object can also be done while the object moving.

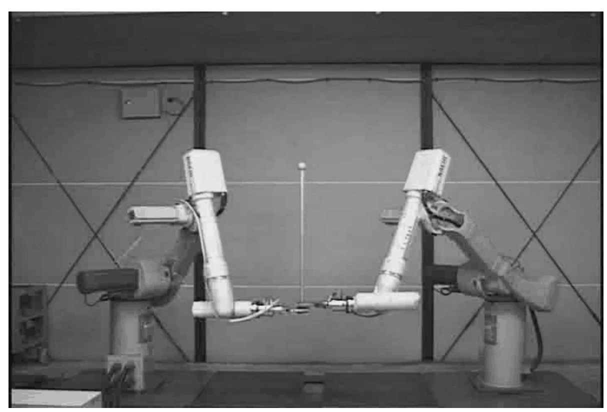


Figure 1: Hand over control of inverted pendulum

Hand over of unstable object using manipulators can be treated as a constrained robot manipulators problem, thus it is formulated using Differential Algebraic Equations (DAE) to make an easier analysis of the problem in the design of controllers.

To hand over the object from one robot to another one, the procedure consists of three phases, i.e. (i) stabilization of the object by one robot, (ii)stabilization of the

object by both of robots, and (iii) stabilization of the object by another robot.
The aim of this research is to propose a new method of switching of the controllers by using Double Bracket Flow as a tool. The stability of switching of the controllers in this form was firstly studied and proposed by Fukui [3]. The works of this research based on it and implement the method to an application of handing over of unstable object.
The plant which is treated in this paper is supposed to be linear system and does not have direct feedthrough term, that is the plant that can be described as follows

$$\Sigma : \begin{cases} \dot{x}(t) & = Ax(t) + Bu(t), \\ y(t) & = Cx(t), \end{cases}$$

and is referred to

$$\Sigma = \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array} \right].$$

And the dynamic controllers to be switched which can stabilize the plant above are supposed to be time-invariant and referred to

$$K_i = \left[\begin{array}{c|c} A_{ci} & B_{ci} \\ \hline C_{ci} & D_{ci} \end{array} \right].$$

K_i is regarded as the i-th controller for multiple controllers. These controllers consist of feedback and observer assembled together.

2 Preliminaries

In this section, some basic theories of Double Bracket Flow and Linear Parametrically Varying System used in this paper are described.

2.1 Double Bracket Flow

Double Bracket Flow was firstly introduced by Brockett in 1988 as a new class of isospectral flows on the set of real symmetric matrices. It is shown that it can be used to solved various combinatorial optimization tasks such as linear programming problems and the sorting of lists of real numbers [7].

Definition 1 (Double Bracket Flow)[4]
Double Bracket Flow is the ordinary differential equation which is defined as

$$\dot{H}(t) = [H(t), [H(t), N]], \qquad H(0) = H_o, \tag{1}$$

where $[A, B] := AB - BA$ denotes the Lie bracket for square matrices, H_o is a symmetric matrix and N is an arbitrary real symmetric matrix.

This equation, under suitable assumptions on N, diagonalizes any symmetric matrix $H(t)$ for $t \to \infty$ [7]. It has characteristics that:

i . The eigen values of H do not change as time evolves.

ii . The solution H is also symmetric matrix.

iii . The diagonal parts of H will be rearranged in the order of the eigen values of N.

iv . The solution H exists when N and H have unrepeated eigen values.

If matrix H has dimension n by n with distinct eigen values, Double Bracket Flow has $n!$ equilibrium points. Even if it reaches the equilibrium point it will not stay at the point. The addition of a linear term of the form $[H, \Omega]$ in the equation will leave this problem out. And the next lemma will be valid.

Lemma 1 *The addition of linear term into Double Bracket Flow to get* [6]

$$\dot{H}(t) = [H(t), [H(t), N]] + [H(t), \Omega], \qquad H(0) = H_o, \tag{2}$$

where $H(t)$ and N are symmetric and Ω is restricted to skew symmetric matrix. This equation has $n!$ equilibrium points exactly one of which is asymptotically stable.

The addition of the linear term in the equation does not change the characteristics of Double Bracket Flow as shown above, however it makes the Double Bracket Flow asymptotically converge into exactly one of the equilibrium point.

2.2 Linear Parametrically Varying System

Linear Parametrically Varying (LPV) System is a finite dimensional, linear system whose state-space entries depend continuously on a time-varying parameter vector $\rho(t) \in \mathbb{R}^s$. The complete formulation of an LPV is given by the following definition.

Definition 2 (Linear Parametrically Varying System) [8]
Assume that the followings are given: a compact set $\mathcal{P} \subset \mathbb{R}^s$, and continuous, bounded functions $A : \mathbb{R}^s \to \mathbb{R}^{n \times n}$, $B : \mathbb{R}^s \to \mathbb{R}^{n \times n_d}$, $C : \mathbb{R}^s \to \mathbb{R}^{n_e \times n}$. These represent an n-th order open-loop linear parametrically varying system (LPV) with scheduling parameter ρ whose dynamics evolve as

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(\rho(t)) & B(\rho(t)) \\ C(\rho(t)) & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}, \tag{3}$$

where $\rho \in \mathcal{F}_\rho$, and $\mathcal{F}_\rho$ is the Parameter Variation Set which is the set of all piecewise continuous functions mapping $\mathbb{R}$ into $\mathcal{P}$.

Suppose that there exist a dynamic controller, whose state-space entries depend continuously on $\rho \in \mathcal{P}$. Specifically, there exist a nonnegative integer m, and continuous, bounded functions $A_K : \mathbb{R}^s \to \mathbb{R}^{m \times m}$, $B_K : \mathbb{R}^s \to \mathbb{R}^{m \times n_y}$, $C_K : \mathbb{R}^s \to \mathbb{R}^{n_u \times m}$, and $D_K : \mathbb{R}^s \to \mathbb{R}^{n_u \times n_y}$, such that the dynamical, ρ-dependent feedback can be described as follows:

$$\begin{bmatrix} u(t) \\ \dot{x}_c(t) \end{bmatrix} = \begin{bmatrix} D_K(\rho(t)) & C_K(\rho(t)) \\ B_K(\rho(t)) & A_K(\rho(t)) \end{bmatrix} \begin{bmatrix} y(t) \\ x_c(t) \end{bmatrix}.$$

Using $x_s = \begin{bmatrix} x^T & x_c^T \end{bmatrix}^T$ as natural coordinates of the closed-loop system, time-varying dynamics are governed by

$$\dot{x}_s = A_s x_s, \tag{4}$$

where

$$A_s = \begin{bmatrix} A(\rho) + B(\rho)D_K(\rho)C(\rho) & B(\rho)C_K(\rho) \\ B_K(\rho)C(\rho) & A_K(\rho) \end{bmatrix}.$$

The controller synthesis related to quadratic stability can be stated as follows.

Definition 3 (Quadratic Stability) [8]
Suppose that the set of scheduling parameter $\mathcal{P} \in \mathbb{R}^s$ is a compact set , and the LPV system consists of continuous functions. The LPV system is quadratically stable over scheduling parameter $\rho \in \mathcal{P}$ with the dynamic controller defined above, if for all $\rho \in \mathcal{P}$

$$A_s^T P + P A_s < 0, \tag{5}$$

there exist a $P \in \mathbb{R}^{n \times n}$, $P = P^T > 0$.

3 Problem formulation

In this section, the switching problem of controllers and modeling of a pendulum and manipulators are described.

3.1 Double Bracket Flow as a switching tool

The switching of the controllers that is meant in this problem can be described as the switching between two controllers continuously, so that the plant remains stable, and as much two controllers give the input to the plant.

It is recommended to design identical controllers to stabilize the object, since the system can be reformed into the simple one. Thus, it is easier to find Lyapunov function P that fulfill the Quadratic Stability shown in the previous section. From this merit, the controllers that are treated here are supposed to have identical structure.

Definition 4 (Switching of controllers)
Assume that there are n identical controllers. Switching of the controllers is defined as to switch active controller K_i into K_j so that K_j is activated and K_i becomes inactive. Here, $i \neq j$ and switching parameters ρ_i of the controllers have an affine form relation that

$$\sum_{i=1}^{n} \rho_i = 1, \tag{6}$$

for all $\rho_i \in \mathcal{P}$ and $\rho_i \in [0, 1]$.

The switching of the controllers can be conducted in two ways, i.e. discontinuously and continuously. However, switching the controllers discontinuously will cause discontinuous change in controllers, and since controllers correspond to manipulators in our problem, such discontinuity occurs in the joint acceleration of manipulators. And the stability in this case also depends on the switching time. Thus, continuously switching of the controllers is recommended.

This problem can be thought as the sorting problem of two real numbers, that is zero and one. Using one of the benefits of Double Bracket Flow which has a feature to sort real numbers dynamically and change continuously, the switching problem of the controllers is reformulated. By taking diagonal parts of H in Double Bracket Flow as the switching parameters of the controllers , the system forms Linear Parametrically Varying system (LPV). Thus, the stability of the system with switching parameters ρ is treated as Quadratic Stability in LPV system. Since switching parameters are used to parameterized the input to the plant so that the input does not exceed the required one, these switching parameters do not depend on state variable of the system.

Switching time can be adjusted by changing anti-diagonal parts of Ω. The bigger absolute value of anti-diagonal parts of Ω, the less switching time will be occurred.

Consider that there are n time-invariant controllers that can stabilize the plant individually and are described as

$$K_i = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right] \qquad i = 1 \cdots n.$$

Define all controllers with switching parameters as

$$K(\rho) = \left[\begin{array}{c|c} A_K(\rho) & B_K(\rho) \\ \hline C_K(\rho) & D_K(\rho) \end{array}\right],$$

where

$$A_K(\rho) = \sum_{i=1}^{n} \rho_i A_K, \qquad B_K(\rho) = \sum_{i=1}^{n} \rho_i B_K,$$
$$C_K(\rho) = \sum_{i=1}^{n} \rho_i C_K, \qquad D_K(\rho) = \sum_{i=1}^{n} \rho_i D_K.$$

As the problem is restricted to the controllers which have same dynamics, the closed-loop system of A_s becomes

$$A_s = \begin{bmatrix} A + \sum_{i=1}^{n} \rho_i B D_K C & \sum_{i=1}^{n} \rho_i B C_K \\ \sum_{i=1}^{n} \rho_i B_K C & \sum_{i=1}^{n} \rho_i A_K \end{bmatrix}. \tag{7}$$

Since the switching parameters of the controllers fulfill an affine form relation, all controllers with switching parameters will form the same dynamics as a controller. Thus, the system input will equal as input that is generated by a controller.

Lemma 2 *The closed-loop system which is defined in equation (7) is stable in the sense of Quadratic Stability.*

Proof:
Using the definition of Quadratic Stability in equation

(5) causes the sufficient condition of the closed-loop system becomes

$$A_s^T P + P A_s < 0,$$

$$\begin{bmatrix} A + \sum_{i=1}^n \rho_i B D_K C & \sum_{i=1}^n \rho_i B C_K \\ \sum_{i=1}^n \rho_i B_K C & \sum_{i=1}^n \rho_i A_K \end{bmatrix}^T P + P \begin{bmatrix} A + \sum_{i=1}^n \rho_i B D_K C & \sum_{i=1}^n \rho_i B C_K \\ \sum_{i=1}^n \rho_i B_K C & \sum_{i=1}^n \rho_i A_K \end{bmatrix} < 0.$$

Since $\sum_{i=1}^n \rho_i = 1$, positive and continuous for all $\rho \in \mathbb{R}^s$, and matrix functions in the closed-loop system A_s fulfill time-invariant Quadratic Stability, the closed-loop system is stable in the sense of Quadratic Stability. □

3.2 Modeling of a pendulum and manipulators

Based on Differential Algebraic Equations (DAE) which was firstly proposed by McClamroch [9], modeling of a pendulum and manipulators are carried out [10]. The modeling in this form will cause the controller design become easier.

Pendulum is overlaid on the base and to stabilize pendulum, the robot hold the base and applies appropriate force. Let q_o be the state of position and posture of the pendulum, and the equation of motion is

$$M(q_o)\ddot{q}_o + F(q_o, \dot{q}_o) = \begin{bmatrix} I_p \\ 0 \end{bmatrix} f. \quad (8)$$

The equation of motion of two robot manipulators which hold the base of pendulum can be described as follows

$$M_1(q_1)\ddot{q}_1 + F_1(q_1, \dot{q}_1) = \tau_1 + \tau_{o1}, \quad (9)$$

$$M_2(q_2)\ddot{q}_2 + F_2(q_2, \dot{q}_2) = \tau_2 + \tau_{o2}, \quad (10)$$

where, $M(q_o)$, $M_1(q_1)$ and $M_2(q_2)$ are inertial parts which are positive definite matrices, $F(q_o, \dot{q}_o)$, $F_1(q_1, \dot{q}_1)$ and $F_2(q_2, \dot{q}_2)$ are Colioris, Centrifugal, gravity and friction parts which are brought together. f is generalized force applied to the base of pendulum. I_p is identity matrix whose size depends on the degree of freedom of the base motion and for simplicity let $I_{po} = \begin{bmatrix} I_p & 0 \end{bmatrix}^T$. τ_1 and τ_2 are torques which applied to each of joint manipulator. τ_{01} and τ_{02} are additional joint torques due to the constraint.

When robot manipulator holding base of pendulum, the following constrains will be comprised.

$$\begin{cases} 0 &= \phi_1(q_1) - q_{o1} \\ 0 &= \phi_2(q_2) - q_{o1} \\ \tau_{o1} &= J_1(q_1)^T f_1 \\ \tau_{o2} &= J_2(q_2)^T f_2 \\ f &= -(f_1 + f_2) \end{cases} \quad (11)$$

where, ϕ_1 and ϕ_2 are smooth functions which denote position of robot hands and $J_1(q_1) = (\partial\phi_1(q_1)/\partial q_1)$ and $J_2(q_2) = (\partial\phi_2(q_2)/\partial q_2)$. f_1 and f_2 are applied forces at robot hands. Describe the system in DAE form and reduce it into Ordinary Differential Equations (ODE) will result state equation that only consists of pendulum parameters [10]

$$\frac{d}{dt}\begin{bmatrix} q_o \\ \dot{q}_o \end{bmatrix} = \begin{bmatrix} \dot{q}_o \\ -M^{-1}F \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -M^{-1}I_{po} & -M^{-1}I_{po} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}.$$

The forces which are the inputs of the system are generated by robot manipulators by applying torque to each of joint of robot manipulators by relation

$$\tau_i = M_i J_i^{-1} \left\{ -I_{po}^T M^{-1} \left(F + I_{po}(f_1 + f_2) \right) - \dot{J}_i \dot{q}_i \right\} + F_i - J_i^T f_i, \qquad i = 1, 2 \quad (12)$$

4 An application to hand over of an inverted pendulum

In this section, a concrete design of switching of the controllers to hand over an inverted pendulum based on Double Bracket Flow is described. In the actual experiments, the inverted pendulum is controlled in three dimensions of motion. However, for simplicity the explanation in this section is restricted to one dimension of motion as others have same characteristics.

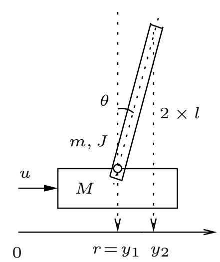


Figure 2: Pendulum on the base

4.1 Pendulum model

Pendulum is overlaid on the base as shown in Figure 2. The equations of motion can be described as

$$\begin{aligned} (M+m)\ddot{r} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta &= u, \\ ml\ddot{r}\cos\theta + (J + ml^2)\ddot{\theta} - mlg\sin\theta &= 0, \end{aligned}$$

where M is the mass of the base of pendulum, and m, $2 \times l$, J are the mass, length and moment inertia of pendulum, respectively. r is the horizontal position of the base and θ is the deviation of the pendulum from upright position. u is the input of the system which is force in this case. The parameter value of pendulum and base are: $M = 0.70[kg]$, $m = 0.66[kg]$, $l = 0.40[m]$, and $J = 0.033[kg.m^2]$.

The state vector is chosen as $x = [r, \theta, \dot{r}, \dot{\theta}]$ and the output of the system is chosen as the positions of the base and the top of pendulum, that is $y = [r, r + 2l\theta]$.

4.2 Controller design

Since the motion of pendulum is restricted in one dimension, linearization of equation of motion around the

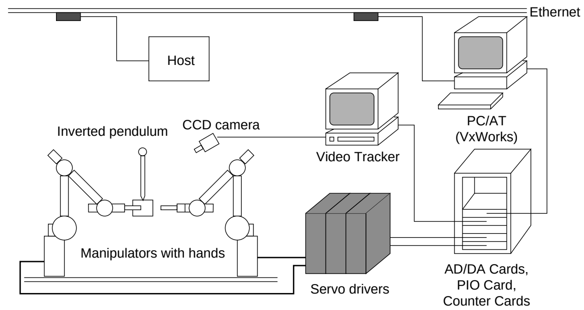


Figure 3: System configuration

equilibrium point will result the plant as follows

$$\Sigma = \left[\begin{array}{cccc|cc} 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & -7.51 & 0.0 & 0.0 & -1.52 & -1.52 \\ 0.0 & 32.98 & 0.0 & 0.0 & 2.90 & 2.90 \\ \hline 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.8 & 0.0 & 0.0 & 0.0 & 0.0 \end{array}\right].$$

The controller was designed based on Linear Quadratic Control. The elements of cost function of the state were used 1000 to weight the position of the base, 1 to weight the posture of pendulum and 1 to weight the velocity. The input weighting was set to be 1. The velocity of the state was estimated using Observer with poles at 0.8. Both of the controllers have same dynamics, and can be described as

$$K_1 = K_2 = \left[\begin{array}{cc|cc} 0.887 & 0.073 & -13.957 & 6.521 \\ -0.165 & 0.660 & 17.798 & -29.031 \\ \hline -21.070 & -17.870 & 157.324 & -1592.658 \end{array}\right].$$

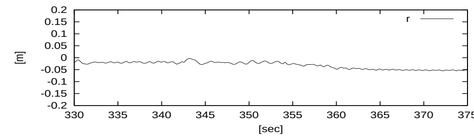
The initial value of Double Bracket Flow to switch the controllers was given by

$$H_o = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix} \quad N = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} \quad \Omega = \begin{bmatrix} 0.0 & 0.02 \\ -0.02 & 0.0 \end{bmatrix}.$$

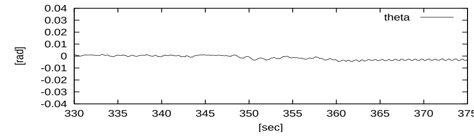
In this case, switching time that was occurred is about 5 sec.

4.3 Experiments

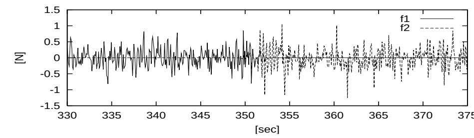
Experiments have been done using two 6 D.O.F Puma-like robot manipulators which have mechanical hands(Figure 1). The experiment was done in two ways, that is hand over of inverted pendulum when the base of pendulum stopped and the base of pendulum was moving at constant velocity 3 [cm/sec]. Both of experiments have been successfully done. The control laws were computed using a 400 MHz Intel Processor with real-time OS VxWorks which executed the computations of inverse dynamics and coordinate transformations every 3 msec. The top position of pendulum was measured by a CCD camera and then the posture of pendulum was computed based on measurement results and position of the base. Since the CCD camera can only measure in two dimensions, the measurement was made by assuming that the pendulum stay in the same



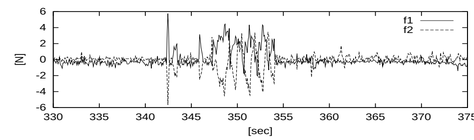
(a) position of pendulum (r)



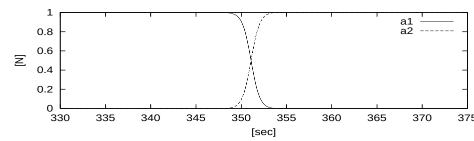
(b) posture of pendulum(θ)



(c) applied force (f)



(d) internal force



(e) switching of the controllers

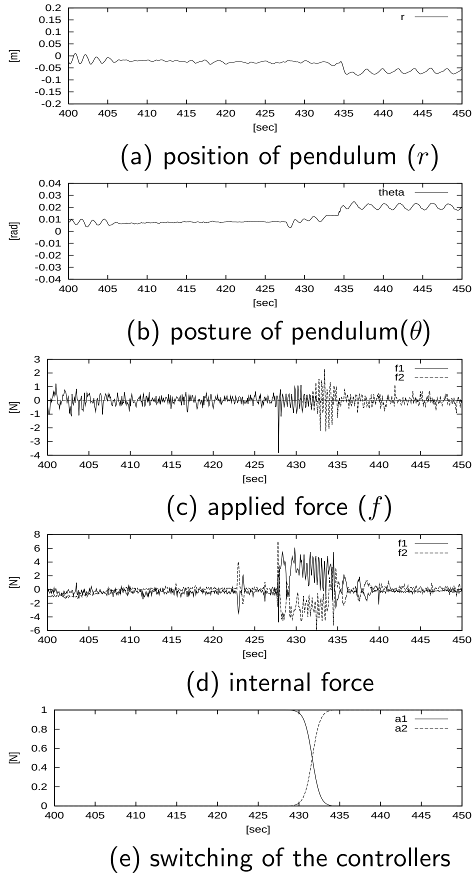
Figure 4: Experimental results when the base of pendulum stopped

height during the experiment was being done. System configuration of all system is shown in Figure 3. Experimental results of handing over when the base of pendulum stopped are shown in Figure 4. And those when the base of pendulum was moving are shown in Figure 5.

At first, the pendulum was stabilized by the first manipulator and second manipulator tracked the motion of the first manipulator. If the tracking error was getting small, the base of pendulum was gripped by both of manipulators. And here, the switching from the first manipulator to the second manipulator was carried out. Finally, when switching was completed the first manipulator releases the base and hand over of pendulum was completed. The curves of switching of the controllers are shown in Figure 4(e) and 5(e).

Figure 4(c) and 5(c) show the force which was applied to the base of pendulum to stabilize pendulum. The sharing force changed continuously from the first manipulator to the second one. The internal force that occurred during hand over operation is less than 4 [N] and is shown in Figure 4(d) and 5(d). Thus, compared to reliable design method, this force is small. So, the objective to reduce internal force which occurred can be achieved.

There are offsets in positions of base and posture of pen-



(a) position of pendulum (r)

(b) posture of pendulum(θ)

(c) applied force (f)

(d) internal force

(e) switching of the controllers

Figure 5: Experimental results when the base of pendulum was moving at constant velocity

dulum which are caused by the position measurement errors by CCD camera.

5 Conclusions

Hand over control of unstable object using manipulators by an approach of continuously switching of controllers has been conducted. Experiments have been carried out successfully, not just limited to an inverted pendulum which was stopped, but also successfully done while the inverted pendulum was moving at constant velocity. The method of switching of the controllers could become a new way to design coordinated controllers of cooperating manipulators, since reliable design method has some limits. For example, the handing over of an object by either/both of the manipulators remains stable if and only if a pair of the controllers achieves reliable stabilization of the plant. In other words, it is possible if and only if the object is strongly stabilizable. Thus, the design of the controllers become very complicated.
Since the experimental results depend on the control rate, it is preferable to try an experiment with shorter control rate to see the internal force that may occur in the object. At this moment there is no the best way to find a suitable value for initial of Double Bracket Flow. The design of Double Bracket Flow to recognize the environment and to decide switching of the controllers will become the future work of this research. The analysis of switching of the controllers that have different dynamics is also important to think about.

Acknowledgment
The first author would like to express gratitude to Mr. Yasuhito Fukui for his works on the early stage of this research.

References

[1] T. Hoshino, M. Hara and K. Furuta, "A Reliable design approach to coordinated control of manipulators -application to hand-over control of an inverted pendulum," *Proceedings of IEEE/RSJ Int. Conf. of Intelligent Robots and Systems*, pp. 455-460, 1998.

[2] M. Hara, "Hand over Control of Inverted Pendulum with Manipulators," Master thesis, Department of Mechanical and Environmental Informatics, Tokyo Institute of Technology, 1998.

[3] Y. Fukui, "A study on Global Stability of Switching Controller," Master thesis, Department of Mechanical and Environmental Informatics, Tokyo Institute of Technology, 1999.

[4] R. W. Brockett, "Dynamical systems that sort lists, diagonalize matrices and solve linear programming problems," *Proceedings of the 27th IEEE Conference on Decision and Control*, pp. 779-803, 1988.

[5] R. W. Brockett, "Dynamical systems that sort lists, diagonalize matrices and solve linear programming problems," *Linear Algebra and its Applications*, Vol. 146, pp. 79-91, 1991.

[6] R. W. Brockett, "Smooth dynamical systems which realize arithmetical and logical operations," *Three decades of Mathematical System Theory*, 135, Lecture notes in Control and Information Science, Springer-Verlag, pp. 19-30, 1989.

[7] U. Helmke and J. B. Moore, "Optimization and Dynamical Systems", Springer-Verlag, 1993.

[8] G. Becker, A. Packard, D. Philbrick and G. Balas, "Control of Parametrically-Dependent Linear Systems: A Single Quadratic Lyapunov Approach", *Proceedings of the American Control Conference*, San Fransisco, pp.2795-2799, 1993.

[9] N. H. McClamroch, "Singular systems of differential equations as dynamic models for constrained robot systems,"Proc. IEEE Conf. Robotics Automation, San Fransisco, CA, pp. 21-28, 1986.

[10] A. Kumar and P. Daoutidis, "Control of nonlinear differential algebraic equation systems", Chapman & Hall /CRC research notes in mathematics series, 1999.