Optimal transfer schemes for switching controllers

Jonathan Paxman Glenn Vinnicombe
Engineering Department, Cambridge University
Trumpington Street, Cambridge CB2 1PZ, United Kingdom
jpp27@eng.cam.ac.uk gv@eng.cam.ac.uk

Abstract

In this paper, we develop two general schemes for reducing the performance degradation caused by a signal substitution at the plant input. Such substitutions arise in modern control systems where we may transfer from manual to automatic, or between alternative controllers as operating points or performance criteria change. The problem reduces to the selection of the initial controller state for the controller which is to be switched in. The first scheme selects a controller state consistent with (hypothetical) signals at the plant input and output which are close (in the sense of a weighted 2-norm) to observed signals. The second scheme (for regulator and step reference problems) minimises directly the weighted plant input and output after the switch with respect to the controller initial state. We consider these schemes in the context of the regulator problem, and the reference tracking problem.

Keywords: bumpless transfer, Kalman filter, switching control, antiwindup.

1 Introduction

In the design of control systems it is a common practice to design a number of linear controllers for a single plant according to different criteria (different performance specifications, or plant operating points for instance). The controllers are often designed independently, and a straightforward means is employed to switch between them.

If no controller conditioning is employed, we may observe significant discontinuities and transient signals at the plant input and output when the control input is switched. Conditioning pre-designed controllers to minimise such transient signals is referred to as "bumpless transfer". The problem of input switching is closely related to the anti-windup problem, and much of the literature treats both problems together.

Existing solutions are generally equivalent to a coprime factor implementation of the controller (or to a controller implemented in observer form). See for example [1, 3–5].

In this paper, we consider the "bumpless transfer" problem to be that of finding the controller initial state which results in the least disruption to the plant input and output. Our results are derived in discrete time, though the principles are applicable to the continuous time case.

A preliminary version of some of these results was presented in [6].

2 Observing the off-line controller

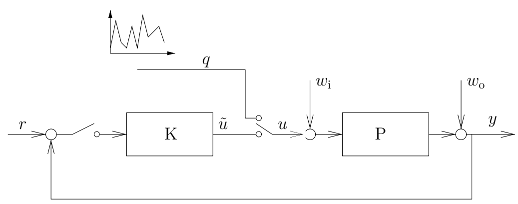

Figure 1: Generalised input substitution

Consider the generalised input substitution illustrated in figure 1, with:

$$u_k = \begin{cases} q_k & k < n \\ \tilde{u}_k & k \geq n \end{cases}. \tag{1}$$

We have a plant running open loop with input q_k initially, and switch the plant input at time $k = n$ so that the plant input is now generated by the known controller K. The initial input q_k could be generated by manual control, another automatic controller, or some other input source. w_i, and w_o are plant disturbances.

Let us initially consider the regulator problem $r_k = 0$.

2.1 Selection of controller state

We wish to determine a controller state at time $k = n$ which will result in a smooth transfer from the input q_k to the input from the controller $\tilde{u}_k$.

One method of controller state selection is to measure the plant input and output in the time leading up to the switch, and find an input/output signal compatible with the off-line controller, which is *close* in some sense to the observed signal. The controller state corresponding to the "nearest" input/output sequence is then selected as the controller is switched on.

In other words, we recast the setup of figure 2 into that of figure 3. We then find sequences $\hat{u}_k$ and $\hat{y}_k$ for $k < n$ such that $\left\| \begin{matrix} \hat{w} \\ \hat{v} \end{matrix} \right\|$ is minimised.

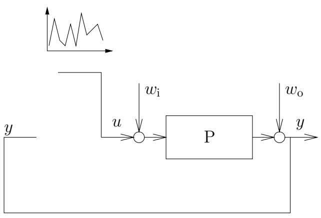

Figure 2: System prior to switch

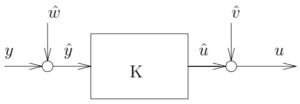

Figure 3: Recasting of figure 2

The equations for figure 3 are as follows:

$$\begin{aligned} x_{k+1} &= Ax_k + B\hat{y}_k, \\ \hat{u}_k &= Cx_k + D\hat{y}_k. \end{aligned} \tag{2}$$

With $x_k \in \mathbb{R}^s$, $\{u_k, \hat{u}_k, v_k\} \in \mathbb{R}^p$, and $\{y_k, \hat{y}_k, w_k\} \in \mathbb{R}^q$ By iterating these equations, we can write x_{n-i} and y_{n-i} (for any i with $0 \leq k \leq n$), in terms of the state at the switching instant x_n. Thus we can write the (hypothetical) controller output vector $\hat{U} = \begin{bmatrix} \hat{u}_{n-1}^T & \hat{u}_{n-2}^T & \dots & \hat{u}_{n-r}^T \end{bmatrix}^T$ in terms of the state at time $k = n$ and the (hypothetical) controller input $\hat{Y} = \begin{bmatrix} \hat{y}_{n-1}^T & \hat{y}_{n-2}^T & \dots & \hat{y}_{n-r}^T \end{bmatrix}^T$. r $(0 \leq r \leq n)$ is an arbitrary optimisation horizon.

$$\hat{U} = \Gamma x_n - T\hat{Y}, \tag{3}$$

where

$$\Gamma = \begin{bmatrix} CA^{-1} \\ CA^{-2} \\ \vdots \\ CA^{-r} \end{bmatrix},$$

$$T = \begin{bmatrix} CA^{-1}B - D & \dots & & 0 \\ CA^{-2}B & & & 0 \\ \vdots & & \ddots & \vdots \\ CA^{-r}B & \dots & CA^{-2}B & CA^{-1}B - D \end{bmatrix}.$$

Let the actual plant input be $U = \begin{bmatrix} u_{n-1}^T & u_{n-2}^T & \dots & u_{n-r}^T \end{bmatrix}^T$, and the plant output $Y = \begin{bmatrix} y_{n-1}^T & y_{n-2}^T & \dots & y_{n-r}^T \end{bmatrix}^T$. Then we can define a weighted least squares cost function

$$J = \left\| W \begin{bmatrix} U - \hat{U} \\ Y - \hat{Y} \end{bmatrix} \right\|^2. \tag{4}$$

We seek to find the values of $\hat{U}$, $\hat{Y}$, and x_n which minimise this cost function. The weighting matrix W may be used for instance to introduce a "forgetting factor" such that the plant input and output are most closely matched immediately prior to the switch at time $k = n$. A similar cost function is considered in [8].

Theorem 2.1 *For the discrete time dynamical system defined in* (2) *and given some signals U and Y, the optimal choice of x_n with respect to the cost function* (4) *and corresponding $\hat{Y}$ is given by:*

$$\begin{bmatrix} \hat{Y} \\ x_n \end{bmatrix} = \left(W \begin{bmatrix} -T & \Gamma \\ I & 0 \end{bmatrix} \right)^\dagger W \begin{bmatrix} U \\ Y \end{bmatrix}. \tag{5}$$

($\dagger$ denotes the left pseudo inverse)

Proof:

$$\begin{aligned} J &= \left\| W \begin{bmatrix} U - \hat{U} \\ Y - \hat{Y} \end{bmatrix} \right\|^2 \\ &= \left\| W \begin{bmatrix} U \\ Y \end{bmatrix} - W \begin{bmatrix} -T & \Gamma \\ I & 0 \end{bmatrix} \begin{bmatrix} \hat{Y} \\ x_n \end{bmatrix} \right\|^2, \end{aligned}$$

so by a least squares optimisation procedure (see for example, [7])

$$\underset{\left[\begin{smallmatrix} \hat{Y} \\ x_n \end{smallmatrix}\right]}{\text{argmin}} J = \left(W \begin{bmatrix} -T & \Gamma \\ I & 0 \end{bmatrix} \right)^\dagger W \begin{bmatrix} U \\ Y \end{bmatrix},$$

and

$$\min_{\left[\begin{smallmatrix} \hat{Y} \\ x_n \end{smallmatrix}\right]} J = \left\| \left(I - W \begin{bmatrix} -T & \Gamma \\ I & 0 \end{bmatrix} \left(W \begin{bmatrix} -T & \Gamma \\ I & 0 \end{bmatrix} \right)^\dagger \right) W \begin{bmatrix} U \\ Y \end{bmatrix} \right\|^2.$$

■

If the weighting matrix is block diagonal, i.e

$$W = \text{diag} \begin{bmatrix} W_{n-1} & W_{n-2} & \dots & W_{n-r} & V_{n-1} & \dots & V_{n-r} \end{bmatrix}, \tag{6}$$

where each $W_j \in \mathbb{R}^{p \times p}$ and $V_j \in \mathbb{R}^{q \times q}$ is symmetric and positive definite, then the optimal controller state can be found recursively using a Kalman filter.

Theorem 2.2 *The solution* (5) *to the weighted optimisation with weighting matrix* (6) *is equivalent to the solution $\hat{x}_n$ given by a Kalman filter observing the (hypothetical) noisy controller of figure 3 with "input" U and "output" Y. The filter has output noise covariance $R_k = (V_k^2)^{-1}$, input noise covariance $Q_k' = (W_k^2)^{-1}$ and initial state error Ψ such that $\Psi^{-1} = 0$.*

This result is obtained directly from the deterministic interpretation of the Kalman filter presented by Bertsekas and Rhodes [2].

Bersekas and Rhodes present an alternative interpretation of the Kalman filter equations, where stochastic estimates of the input and output noise are replaced by a deterministic energy bound

$$[x_0 - \mu_0]^T \Psi^{-1} [x_0 - \mu_0] + \sum_{j=0}^{N-1} \left(w_j^T (Q_j^{'})^{-1} w_j + v_j^T R_j^{-1} v_j \right) \leq 1.$$

where w_j and v_j are the input and output noise respectively, μ_0 is the initial estimate of the state. Ψ, $Q_j^{'}$ and R_j are bounding matrices on the initial error and noise determined by the nature of the problem (these replace the initial state error and noise covariance matrices in the stochastic Kalman filter formulation).

The problem is solved by forming the cost function

$$J = [x_0 - \mu_0]^T \Psi^{-1} [x_0 - \mu_0] + \sum_{j=0}^{k-1} \left(w_j^T (Q_j^{'})^{-1} w_j + v_j^T R_j^{-1} v_j \right), \tag{7}$$

and solving a tracking control problem in reverse time. The result yields the standard (time varying) Kalman filter recursion equations.

The optimisation cost function (4) is equal to the cost (7) precisely when the weighting matrix is as given in (6), and $\Psi^{-1} = 0$. Thus the optimisation yields the same solution as the time varying Kalman filter.

In the uniformly weighted infinite horizon case, the equations reduce to the time invariant Kalman filter, and may thus be implemented in observer form.

The Kalman filter riccati equation is in fact identical to the equation which corresponds to some left coprime factorisation [9] of the controller, and so we may implement the controller as shown in figure 4 where $K = V^{-1}U$ is a coprime factorisation of the controller. Note also that U and V may be implemented with a shared state space.

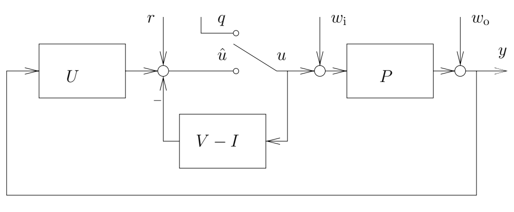

Figure 4: Coprime factorisation form

2.2 The reference problem

Let us now consider the general reference tracking problem ($r_k \neq 0$). In many applications the actual reference signal for the current online controller may not be available, or may be incompatible with the off-line controller we are considering. For example, if we switch from manual to automatic pilot in an aircraft control system, the reference signal in the past may be difficult to define and impossible to measure.

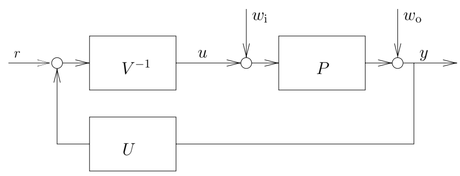

Figure 5: reference framework

Suppose we rewrite the system prior to the switch in the form shown in figure 5, instead of 3. We have introduced a hypothetical reference signal r. Then we may define a bumpless transfer problem as follows:

Given signals u and $y \in \ l_2(-\infty, n)$, find a signal $r \in \ l_2(-\infty, n)$ and a controller state x_n compatible with such signals in terms of figure 5.

Clearly, we can express y and u in terms of r, w_i, and w_o as follows:

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} P \\ I \end{bmatrix} (I - KP)^{-1} V^{-1} r + \begin{bmatrix} I \\ K \end{bmatrix} (I - KP)^{-1} \begin{bmatrix} P & I \end{bmatrix} \begin{bmatrix} w_\mathrm{i} \\ w_\mathrm{o} \end{bmatrix}$$

then

$$\begin{bmatrix} -U & V \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} = (-UP + V) V^{-1} (I - UPV^{-1})^{-1} r$$

$$+ (-U + U)(I - KP)^{-1} \begin{bmatrix} P & I \end{bmatrix} \begin{bmatrix} w_\mathrm{i} \\ w_\mathrm{o} \end{bmatrix} = r$$

Now comparing this expression with figure 4 prior to the switch, we can see that $u - \hat{u} = Vu - Uy$, which is precisely the hypothetical r required to achieve bumpless transfer according to our definition above. That is, if the switch occurs at time $k = n$, and the reference input $\tilde{r}_k$ is to be applied after the switch, then the closed loop behaviour of the switched system with controller implemented as shown in figure 4 is identical to what would be observed if the new controller was in operation for *all* time with a reference

$$r_k = \begin{cases} Vu_k - Uy_k & k < n \\ \tilde{r}_k & k \geq n \end{cases}. \tag{8}$$

Thus the implementation of figure 4 is valid for reference problems, where the reference of the online controller is unknown to the off-line controllers. To find a weighted solution, we may consider alternative factorisations of the controller.

3 Optimal transfer

3.1 Finite horizon

A sensible measure of the "size" of the bump, used in simulation examples is the weighted norm of the input output vector

$$J = \left\| W' \begin{bmatrix} U \\ Y \end{bmatrix} \right\|^2, \qquad (9)$$

where $U = \begin{bmatrix} u_n^T & u_{n+1}^T & \dots & u_{n+N}^T \end{bmatrix}^T$ and $Y = \begin{bmatrix} y_n^T & y_{n+1}^T & \dots & y_{n+N}^T \end{bmatrix}^T$, with N some finite time (usually the time to the end of the simulation), and W' a general weighting matrix.

It is sensible to now consider optimising this cost directly with respect to the controller state at time n, x_n. Assuming zero noise, we have after switching on the controller the system illustrated in figure 6. This is essentially an initial value problem with no exogenous inputs. Let us assume that the plant (with state

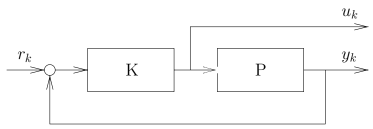

Figure 6: System after switch

$x_k \in \mathbb{R}^p$) and controller (with state $x_k^c \in \mathbb{R}^q$) are linear and time invariant, defined as follows:

$$K = \left[\begin{array}{c|c} A^c & B^c \\ \hline C^c & 0 \end{array}\right], \qquad P = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]. \qquad (10)$$

Then we can rewrite this as a single system $\tilde{P} = \left[\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline \tilde{C} & 0 \end{array}\right]$ with input r, state $\tilde{x}_k = \begin{bmatrix} x_k^c \\ x_k \end{bmatrix}$, and output $\tilde{y}_k = \begin{bmatrix} u_k \\ y_k \end{bmatrix}$, where

$$\tilde{A} = \begin{bmatrix} \tilde{A}_1 & \tilde{A}_2 \end{bmatrix} = \begin{bmatrix} A^c + B^c D C^c & B^c C \\ B C^c & A \end{bmatrix},$$

$$\tilde{C} = \begin{bmatrix} \tilde{C}_1 & \tilde{C}_2 \end{bmatrix} = \begin{bmatrix} C^c & 0 \\ D C^c & C \end{bmatrix}, \quad \text{and} \quad \tilde{B} = \begin{bmatrix} B^c \\ 0 \end{bmatrix}.$$

Then we have

$$\tilde{y}_{n+k} = \tilde{C}\tilde{A}^k \tilde{x}_n + \tilde{C}\tilde{B} r_{n+k-1} + \dots + \tilde{C}\tilde{A}^{k-1}\tilde{B} r_n \quad (11)$$

so we can write:

$$\tilde{Y} = \Gamma \tilde{x}_n + TR = \Gamma_1 x_n^c + \Gamma_2 x_n + TR, \qquad (12)$$

where

$$\Gamma = \begin{bmatrix} \tilde{C}_1 & \tilde{C}_2 \\ \tilde{C}\tilde{A}_1 & \tilde{C}\tilde{A}_2 \\ \tilde{C}\tilde{A}\tilde{A}_1 & \tilde{C}\tilde{A}\tilde{A}_2 \\ \vdots & \\ \tilde{C}\tilde{A}^{N-1}\tilde{A}_1 & \tilde{C}\tilde{A}^{N-1}\tilde{A}_1 \end{bmatrix} = \begin{bmatrix} \Gamma_1 & \Gamma_2 \end{bmatrix}, \quad \tilde{Y} = \begin{bmatrix} \tilde{y}_n \\ \tilde{y}_{n+1} \\ \vdots \\ \tilde{y}_{n+N} \end{bmatrix},$$

$$R = \begin{bmatrix} r_n \\ r_{n+1} \\ \vdots \\ r_{n+N} \end{bmatrix}, \quad T = \begin{bmatrix} 0 & \dots & 0 \\ \tilde{C}\tilde{B} & & \\ \vdots & \ddots & \vdots \\ \tilde{C}\tilde{A}^{N-1}\tilde{B} & \dots \quad \tilde{C}\tilde{B} & 0 \end{bmatrix}.$$

Let us now specialise to the regulator problem $R = 0$, and also assume that we have the full plant state x_n available to us. We can now optimise the cost function (9), or equivalently

$$J = \left\| W\tilde{Y} \right\|^2 \qquad (13)$$

with respect to the controller state at the switching time x_n^c.

Theorem 3.1 *Consider the system illustrated in figure 6, and represented by equations* (10). *Assume* $r_k = 0 \quad \forall k$. *The optimal controller state* x_n^c *with respect to the cost function* (13) *is given by:*

$$x_n^c = -\left(W\Gamma_1\right)^{\dagger} W\Gamma_2 x_n. \qquad (14)$$

Proof: from equation (12) we obtain the following:

$$W\tilde{Y} = W\Gamma_1 x_n^c + W\Gamma_2 x_n,$$

and by proceeding as for other least squares optimisation problems:

$$\underset{x_n^c}{\operatorname{argmin}}\, J = -\left(W\Gamma_1\right)^{\dagger} W\Gamma_2 x_n.$$

■

3.2 Infinite horizon

We now consider the case of optimising the unweighted response in the infinite horizon, as $N \to \infty$. That is, we wish to minimise the cost function:

$$J = \left\| \tilde{Y} \right\|^2, \qquad (15)$$

where $\tilde{Y} = \begin{bmatrix} u_n^T & y_n^T & u_{n+1}^T & y_{n+1}^T & \dots \end{bmatrix}^T$.

Theorem 3.2 *Consider the system illustrated in figure 6, and represented by equations* (10). *Assume that the controller* K *stabilises the plant* P, *and* $r_k = 0 \quad \forall k$. *Then the optimal controller state* x_n^c *with respect to the cost function* (15) *is given by:*

$$x_n^c = \left(\tilde{C}_1^T \tilde{C}_1 + \tilde{A}_1^T M \tilde{A}_1\right)^{-1} . \left(\tilde{C}_1^T \tilde{C}_2 + \tilde{A}_1^T M \tilde{A}_2\right) x_n, \qquad (16)$$

where M is the solution to the discrete time Lyapunov equation

$$M - \tilde{A}^T M \tilde{A} - \tilde{C}^T \tilde{C} = 0 \tag{17}$$

Proof: Essentially, we must show that the expression for the optimal x_k^c in the finite horizon case:

$$x_n^c = -\left(\Gamma_1\right)^{\dagger} \Gamma_2 x_n, \tag{18}$$

converges to the equation (16) in the limit as $N \to \infty$.

$$\lim_{N\to\infty} \Gamma_1^{\dagger}\Gamma_2 x_n = \lim_{N\to\infty} \left(\Gamma_1^T\Gamma_1\right)^{-1} \Gamma_1^T \Gamma_2 x_n$$
$$= \left(\tilde{C}_1^T \tilde{C}_1 + \tilde{A}_1^T \sum_{n=0}^{\infty}\left(\left(\tilde{A}^T\right)^n \tilde{C}^T \tilde{C}\tilde{A}^n\right)\tilde{A}_1\right)^{-1}$$
$$\cdot \left(\tilde{C}_1^T \tilde{C}_2 + \tilde{A}_1^T \sum_{n=0}^{\infty}\left(\left(\tilde{A}^T\right)^n \tilde{C}^T \tilde{C}\tilde{A}^n\right)\tilde{A}_2\right).$$

Let M be defined as follows:

$$M := \sum_{n=0}^{\infty}\left(\left(\tilde{A}^T\right)^n \tilde{C}^T \tilde{C}\tilde{A}^n\right). \tag{19}$$

Pre multiplying by $\tilde{A}^T$, and post multiplying by $\tilde{A}$ we obtain

$$\tilde{A}^T M \tilde{A} = \sum_{n=1}^{\infty}\left(\left(\tilde{A}^T\right)^n \tilde{C}^T \tilde{C}\tilde{A}^n\right) = M - \tilde{C}^T\tilde{C}, \tag{20}$$

and M exists precisely when the discrete time Lyapunov equation 17 has a solution. That is, when $\tilde{A}$ is stable (see for example [9]) or equivalently, when K stabilises P. ∎

In the case where the plant state is unavailable we use an optimal plant state estimate, and minimise a norm of expectation values, yielding the same results with x_n replaced by an optimal estimate $\hat{x}_n$.

3.3 Reference problem

The general tracking problem cannot be dealt with so easily using this method, unless we know the reference signal r_k *a priori*. It is possible to solve the problem assuming a constant reference after the switch. We first calculate the steady state values of the plant input and output, and to then optimise the cost function

$$J = \left\| W\left[\tilde{Y} - \tilde{Y}_{\mathrm{s.s}}\right]\right\|^2, \tag{21}$$

yielding the (finite horizon) solution

$$x_n^c = -\left(W\Gamma_1\right)^{\dagger}\left(W\Gamma_2 x_n + WTR - W\tilde{Y}_{s.s}\right) \tag{22}$$

where $\tilde{Y}_{\mathrm{s.s}}$ is the steady state value of the plant input and output (constant vector with same dimension as $\tilde{Y}$).

4 Examples

Consider the following plant transfer function

$$P = \frac{1}{s(s^2 + 0.2s + 1)}. \tag{23}$$

The controller is the 10% suboptimal $\mathscr{H}_\infty$ loopshaping controller.

We consider the regulator problem. That is, we have reference input $r = 0$.

We use a discretised version of the above plant and controller, with a sampling time $T = 0.05$. We drive the plant open loop by a random noise signal q_k, and switch to the controller K at time $k = 160$. The system is set up as illustrated in figure 1. No conditioning is applied to the controller prior to the switch, so the controller state remains zero until the switching instant. Figure 7 shows the plant input and output for the system as described.

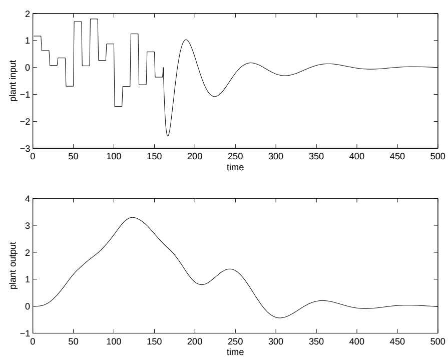

Figure 7: Unconditioned response to switch at k=160

We can clearly observe the bump at the switch. The transient signal caused by the bump has a significant effect, taking a further 240 time steps to settle. In this example the unweighted two-norm of the input/output after the bump is 17.46.

4.1 First method

Applying the least squares optimisation procedure described in theorem 2.1 (uniform weighting), we obtain the result shown in figure 8.

The solid lines show the actual plant input and output for the simulation. The dotted lines show the hypothetical controller input and output which lie closest to the observed signals in the sense of the cost function (4) with $W = I$. We can clearly see an improvement in the transient response of the plant following the switch. The norm of the signal following the switch is 13.15, a significant improvement over the unconditioned case.

4.2 Second method

First consider the unweighted infinite horizon case.

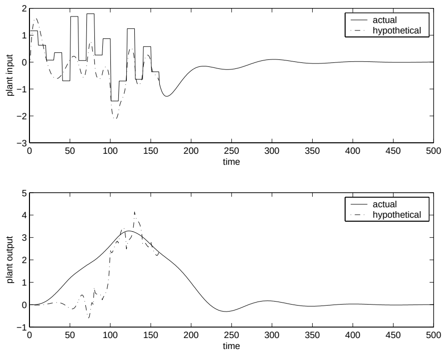

Figure 8: Least squares controller state selection

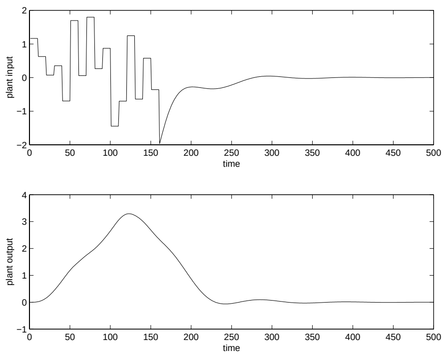

Figure 9: Optimal bumpless transfer, finite horizon

The result, shown in figure 9 is a slight improvement over the first method, with a signal norm of 12.79 following the switch.

Note that there is still a discontinuity immediately following the switch, decaying very rapidly. If we require a smoother transition at the expense of longer decay time, we can manipulate the weighting matrix to achieve the desired results. Figure 10 shows the finite horizon weighted case, with a diagonal exponentially weighted cost, from 1 immediately following the switch, to 10^{-4} at the end of the simulation. The result is a reduced discontinuity at the time of the switch, but slightly greater settling time and more oscillatory response.

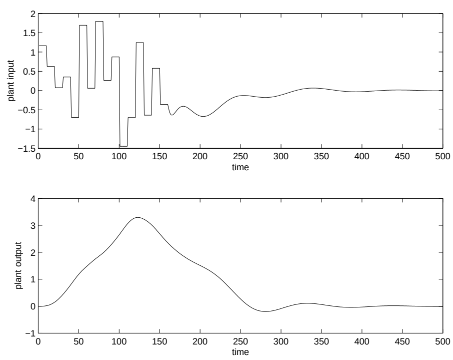

Figure 10: Weighted case

5 Conclusions

Our two approaches to the problem both initially require direct manipulation of the states of the controller. In digital control systems this is usually not a problem, as controller manipulation is a straightforward matter. In some analogue control systems the controller states may not be directly accessible.

We showed that the optimisation of theorem 2.1 is equivalent to a Kalman filter implementation of the controller, and in the infinite horizon (for certain weightings) to a coprime factor implementation. Thus we may use this method without directly manipulating the states of the controller. We also showed a sensible interpretation of this method in the context of reference tracking control.

The first method has computational advantages over the second, and is valid in the general reference tracking problem, and in problems with no explicit linear plant model available. The second method produces a truly optimal result in well defined cases with zero or step reference inputs.

References

[1] K. J. Åström and L. Rundqwist. Integrator windup and how to avoid it. In *Proceedings American Control Conference*, pages 1693–1698, Pittsburgh, 1989.

[2] D. P. Bertsekas and I. B. Rhodes. Recursive state estimation for a set-membership description of uncertainty. *IEEE Transactions on Automatic Control*, AC-16(2):117–128, 1971.

[3] R. Hanus, M. Kinnaert, and J. L. Henrotte. Conditioning technique, a general anti-windup and bumpless transfer method. *Automatica*, 23(6):729–739, 1987.

[4] M. V. Kothare, P. J. Campo, M. Morari, and C. N. Nett. A unified framework for the study of anti-windup designs. *Automatica*, 30(12):1869–1883, 1994.

[5] S. Miyamoto and G. Vinnicombe. Robust control of plants with saturation nonlinearities based on coprime factor representations. In *Proceedings IEEE Conference on Decision and Control*, pages 2838–2840, Kobe, 1996.

[6] J. Paxman and G. Vinnicombe. Optimal and bumpless transfer for switching controllers. In *UK Automatic Control Conference*, Cambridge, UK, 2000.

[7] R. F. Stengel. *Optimal control and estimation.* Dover, 1994.

[8] M. Turner and D. Walker. Modified linear quadratic bumpless transfer. In *Proceedings American Control Conference*, pages 2285–2289, San Diego, 1999.

[9] K. Zhou, J. C. Doyle, and K. Glover. *Robust and optimal control.* Prentice-Hall, Inc., 1996.