Inhibitor Arc Based State Avoidance Controller for Non-convex Forbidden State Problems in Petri nets [1]

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Abstract

In this paper, a state avoidance controller with inhibitor arcs called "IA-controller" is proposed for non-convex forbidden state problems in Petri nets. The forbidden condition is transformed into non-convex marking constraints in controlled PNs. Simple formulas for IA-controllers are developed to avoid the non-convex marking condition. By extending enabling rule of controllable transitions, the IA-controller can realize OR-logics, which is impossible in ordinary PN structure-type controllers. The significance of IA-controller is that the controller can be easily implemented in the plant model. In addition, IA-controllers are synthesized in a modular way in order to solve multiple forbidden conditions.

keywords : Petri nets, forbidden state problem, non-convex constraint.

1 Introduction

In recent years, much research has focused on the synthesis of controllers in discrete event systems (DESs). To achieve their desired behaviors, it is essential to guarantee that the system never enter forbidden states. This is referred to as the *forbidden state problem* in DESs and the controller to solve these problems is referred to as *state avoidance controller* [1].

Petri net is an appropriate model to describe the behavior of DESs because of their flexibility and visualization [2]. Many researchers have used Petri nets as a tool to solve the forbidden state problems in DESs [3, 4, 5, 6]. In Petri nets, forbidden state problems are generally expressed by a number of linear marking constraints called *a forbidden condition*. The plant modeled by a Petri net can avoid the forbidden condition by synthesizing PN structure-type controller or arbitrary logic-type controller into the plant.

It is well known that the marking region to avoid this forbidden condition forms a convex region under the assumption that all transitions are controllable. To handle the convex marking region, PN structure-type controllers have been proposed in [7, 8] with P-invariant structures, which can be easily computed. However, a forbidden condition often incurs non-convex marking regions under the existence of uncontrollable transitions. It was proved that non-convex marking regions cannot be avoided by PN structure-type controllers, because reachable state space in PN structure-type controllers has the convex nature [9].

To solve forbidden state problems of non-convex marking regions called *non-convex forbidden state problems*, arbitrary logic-type controllers based on path-based algorithms have been developed for several classes of Petri nets such as controlled marked graphs (CMGs) [3], controlled state machines (CtlSMs) [4], generalized controlled Petri net [5, 6, 10]. These methods have shown how to synthesize arbitrary logic-type controllers guaranteeing to avoid forbidden states without an exhaustive search of the whole state space of the system. Especially, an important contribution in the area of arbitrary logic-type controllers has been made in 'vector discrete event systems' (VDESs) [11]. VDESs showed that a forbidden condition can be transformed into a kind of non-convex linear marking constraints for a special classe of loop-free Petri nets and proposed an arbitrary logic-type state feedback controller which can be accomplished by the linear integer programming (LIP). However, there are some limitations as follows: i) LIP may not be suitable to online control applications due to the computational complexity and ii) arbitrary logic-type controllers are more difficult to analyze and implement than PN structure-type controllers in many cases. Thus, it is necessary to find an efficient and easy method in order to solve non-convex forbidden state problems.

In this paper, a PN structure-type controller with inhibitor arcs called *IA-controller* is proposed for non-convex forbidden state problems. Owing to the step-evaluation method developed in [10], a forbidden condition can be transformed efficiently into non-convex marking constraints for a general class of PNs under the existence of uncontrollable transitions. In addi-

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tion, simple formulas are proposed to calculate the IA-controller. By extending enabling rule of controllable transitions, the IA-controller can realize OR-logics, which is impossible in the other PN structure-type controllers. In addition, IA controllers are synthesized in a modular way to solve multiple forbidden conditions.

2 Forbidden State Problems

A plant is modeled by a Petri net defined as $N = (\mathcal{P}, \mathcal{T}, A, m_0)$ where $\mathcal{P}$ is a place set, $\mathcal{T}$ is transition set, A is arc function, m_0 is an initial marking. The only difference between the plant and a general model of Petri net is that $\mathcal{T}$ is divided into two classes of controllable transitions $\mathcal{T}_c$ and uncontrollable transitions $\mathcal{T}_{uc}$.

The weight of an arc from a place p_i to a transition t_j is denoted by $a(p_i, t_j)$ and the weight of an arc from t_j to p_i is denoted by $a(t_j, p_i)$. If $a(p_i, t_j) > 0$ (or $w(t_j, p_i) > 0$), p_i (or t_j) is called an *input* place (or transition) of t_j (or p_i) and also t_j (or p_i) is called an *output* transition (or place) of p_i (or t_j). $^\bullet t$ ($t^\bullet$) denotes the set of input (output) places of t and $^\bullet p$ ($p^\bullet$) denotes the set of input (output) transitions of p.

A state in PNs is described by a marking $m : \mathcal{P} \to \mathcal{N}$ indicating the current distribution of tokens in places. $m(p)$ is the number of tokens in the place p. If $m(p) \geq w(p, t)$ for all places in $^\bullet t$, the transition t is said to be *state-enabled*. By the firing of a transition set $T \subseteq \mathcal{T}$, a marking m is reached to a new marking m' by a state transition rule as well known, which is denoted by $m[T > m'$. Given G, all the reachable marking set is denoted by $R(G)$.

The supervisory control goal in Petri net model is to restrict the reachable markings of a plant, which is specified by a forbidden condition. A *single forbidden condition* is specified as follows:

$$F : l^T m \leq b \tag{1}$$

where l^T is non-negative integer weight vector and b is scalar. The constraint of (1) is not a general form as the language based behavioral constraints [1]. However, linear state constraints can be widely used because of the ease with which they can be enforced on Petri net modeled systems. They are also useful for realizing "generalized mutual exclusions constraints", which includes both serial and parallel mutual exclusions [7].

The i-th place such that $l_i > 0, l_i \in l^T$ is called a *forbidden place*. If p is a forbidden place of F, we denote $p \in F$. The forbidden marking set for a forbidden condition F is defined as

$$M(F) := \{m \in R(G) \mid l^T m > b\}. \tag{2}$$

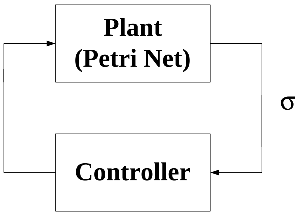


Figure 1: plant and controller

The controller should restrict the behavior of plant by disabling or enabling the controllable transitions according the marking state at each evolution time. The basic scheme of feedback control in Petri net model is shown in Figure 1. The controller monitors the sequence of the firing transition sets, $\sigma = T_1 T_2 T_3 \cdots$, at each evolution time and update the internal states of the controller and then decide the control input, γ, to avoid the forbidden states in controlled plant, where γ is a mapping such that

$$\gamma : \mathcal{T}_c \to \{0, 1\}. \tag{3}$$

γ_{one}, *one control*, denotes the control such that $\gamma_{one}(t) = 1$ for all $t \in \mathcal{T}_c$ and γ_{zero}, *zero control*, denotes the control such that $u_{zero}(t) = 0$ for all $t \in \mathcal{T}_c$. To decide the control input, the controller should consider the reachable markings by firing sequences of uncontrollable transitions.

3 Weakly Forbidden Conditions

Given a forbidden marking set $M(F)$, it is the fact that the marking set to be avoided is larger than the forbidden marking set $M(F)$ due to uncontrollable firing sequences. This larger marking set is said to be the *weakly forbidden marking set* and defined in [3] as

$$W(F) = \{m \mid R_\infty(\gamma_{zero}, m) \cap M(F) \neq \emptyset\}.$$

This means that any marking in $W(F)$ is reachable to a forbidden marking in $M(F)$ under even the most restrictive control γ_{zero}. The set $A(M) \overset{\triangle}{=} R(G) - W(F)$ is said to be the *admissible marking set* w.r.t. F.

A string of places and transitions $\pi = p_0 t_1 p_1 t_2 \cdots p_{n-1} t_n p_n$ such that $t_i \in p_{i-1}^\bullet \cap {}^\bullet p_i$, $1 \leq i \leq n$, $t_i \neq t_j$ for $1 \leq i \neq j \leq n$ is said to be a *path*. The length of π is defined as the number of places lying in π and denoted by $|\pi|$ and 't (or p) $\in \pi$' means that t (or p) lies in π. The set of input transitions of $p_0 \in \pi$ is denoted by $^\bullet\pi$ and the set of output transitions of $p_n \in \pi$ by $\pi^\bullet$.

Definition **1** *A path π is said to be uncontrollable path of F if following conditions are hold*
1) any $t \in \pi$ is not controllable,
2) $p_n \in F$,
2) $^\bullet\pi \cap \mathcal{T}_c \neq \emptyset$ if $^\bullet\pi \neq \emptyset$.

The uncontrollable path of F has the same meaning of precedence path in [3] or influence path in [4]. The set of uncontrollable paths of F is denoted by Π_F.

Given a plant model $G = (P, T, A, m_0)$, an *uncontrollable subnet* w.r.t. F is defined as a tuple $G_F = (P_F, T_F, A_F, m_F^0)$, where $P_F = \{p | p \in \pi$, for each $\pi \in \Pi_F\}$, $T_F = \{t | t \in \pi$, for each $\pi \in \Pi_F\}$, $A_F \subseteq W$ is arc function defined on $(P_F \times T_F) \cup (T_F \times P_F)$. m_F^0 is a sub-marking of m_0 w.r.t. P_F. In other words, the uncontrollable subnet G_F w.r.t. F is constructed by the paths which lead to forbidden places of F and does not contain any controllable transitions. If there is no loop in G_F, G_F is called *loop free* (LF). In this sequel, we assume that the considered uncontrollable subnet is LF.

Definition **2** *Given a forbidden condition F, A LF uncontrollable subnet G_F is said to be step-aligned if $|\pi| = |\pi'|$, for any pairs $\pi, \pi' \in \Pi_F$ such that $\pi \neq \pi'$.*

A step-aligned subnet has some useful properties which are efficinet to analyze its marking behavior. Let P_i be the set of i-th places lying in $\pi \in \Pi_F$ and T_i be the set of i-th transitions lying in $\pi \in \Pi_F$. The incidence matrix of a step-aligned subnet G_F can be a special form as follows:

$$M_{inc} = \begin{bmatrix} -E_{11} & 0 & 0 & 0 \\ E_{21} & -E_{22} & 0 & 0 \\ 0 & E_{32} & -E_{33} & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & -E_{(L-1)(L-1)} \\ 0 & 0 & 0 & E_{L(L-1)} \end{bmatrix} \tag{4}$$

where $-E_{ii} \in Z^{|P_i| \times |T_i|}$ is the incidence matrix between P_i and T_i and $E_{(i+1)i} \in Z^{|P_{i+1}| \times |T_i|}$ is the incidence matrix between P_{i+1} and T_i. Note that P_L, the last place set of $\mathcal{G}_F$, is equal to the set of forbidden places in F.

Thanks to the step-alignment algorithm proposed in [10], we can transform any kinds of LF uncontrollable subnets into step-aligned subnets. The step-aligned subnet transformed from G_F by the step-alignment algorithm is denoted by $\tilde{G}_F = (\tilde{P}_F, \tilde{T}_F, \tilde{A}_F)$. Step-aligned subnet $\tilde{G}_F$ has a property that the reachable markings of $P_i \in P_F$ are dependent only on the reachable markings of preceding places.

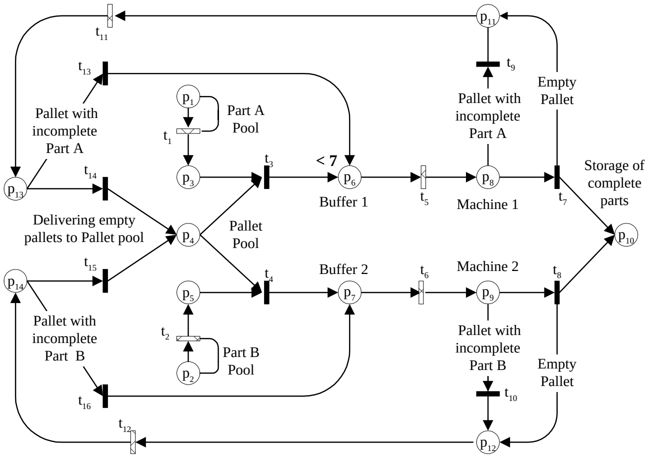
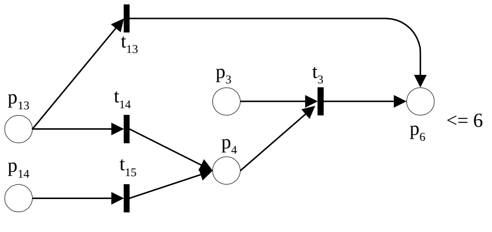
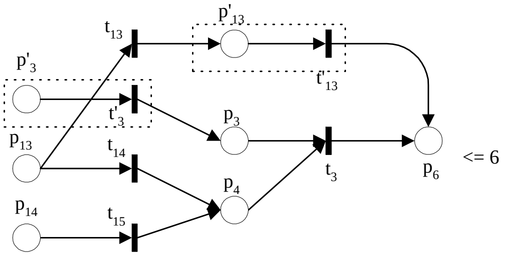


Figure 2: A Petri net model for a manufacturing system



(a) An uncontrollable subnet



(b) A Step-alinged uncontrollable subnet

Figure 3: An example of non-step-aligned subnet and step-aligned subnet.

Example **1** Figure 2 shows a Petri net model of a manufacturing system producing two kinds of products from raw parts of A and B. Each part of A (or B) is loaded on one empty pallet by the firing of the transition t_3 (or t_4). After machine processing, finished parts are released to the final storage and empty pallets are delivered into its pool. Otherwise, pallets with incomplete parts are returned to the corresponding buffers. it is assumed that entry points of raw parts, returning points of pallets and the machining points of Machine 1 and Machine 2 are only controllable (i.e. $T_c = \{t_1, t_2, t_5, t_6, t_{11}, t_{12}\}$). The constraint in the system is specified such that Buffer 1 should have finite capacity of six pallets. Then, the single forbidden condition of the system is $F : m(p_6) \leq 6$. The uncontrollable subnet G_F is shown in Figure 3.(a). Since all the paths in G_F do not have same length, G_F is not step-aligned. Figure 3.(b) shows a step-aligned subnet $\tilde{G}_F$ transformed from G_F by the refinement of $t_{13}p_6$ and p_3t_3.

It was already proved that the weakly forbidden marking set can be specified by the disjunction of linear constraints for a general tree structure (GTS) in

[11]. In this paper, this statement is extended for a LF uncontrollable subnet which is a general set of GTS. Then, the non-convex linear constraints to express the weakly forbidden markings is called a *weakly forbidden condition.*

Lemma **1** *Given a forbidden condition and the LF uncontrollable subnet* G_F*, the weakly forbidden condition can be specified as*

$$\tilde{F} : \bigvee_{i=1}^{n_F} (l_i^T m \le b_i) \tag{5}$$

where, l_i^T *is non-negative integer weight vector,* b_i *is scalar.*

Proof : The proof is omitted, but may be found in [10].

In addition, algorithms to transform given a single forbidden condition into a weakly forbidden condition is suggested in [10] called *Step-Evaluation Method.*

4 Synthesis of Inhibitor Arc Controller

Given an uncontrollable subnet $G_F = (P_F, T_F, A_F, m_F^0)$ and a weakly forbidden condition (5), IA-controller is defined as a tuple of $S = (C, T_c, A_c, I_c, m_c^0)$ where C is the set of control places $C = \{c_1, c_2, \cdots, c_{n_F}\}$, $T_c = {}^\bullet P_F \cup P_F{}^\bullet$, A_c is the incidence matrix between C and T_c and obtained by the following equation:

$$A_c = [l_1 l_2 \cdots l_{n_F}]^T A_F. \tag{6}$$

$I_c : C \to \mathcal{T}$ is an inhibitor arc function from a control place to an input transition of G_F and defined as

$$I(c_i, t) = \begin{cases} b_i & \text{if } t \in {}^\bullet P_F - P_F{}^\bullet \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

and m_c^0 is the initial marking of C and obtained by

$$m_c^0 = [l_1 l_2 \cdots l_{n_F}]^T m_F^0. \tag{8}$$

According to the definition of IA-controller, we can mention the following remarks:

Remark 1 : $T_c \subseteq \mathcal{T}$. This means that no new transitions are created by IA-controller . The control places are linked to the transitions in the plant by some arcs.

Remark 2 : By the definition of I, $I(c_i, t) = I(c_i, t') = b_i$ for any $t, t' \in {}^\bullet c_i$.

Remark 3 : The tokens reserved in the control place $c_i, i = 1, \cdots, n_F$, in each evolution time, represents the left side equation of each linear constraint (i.e. $m(c_i) = l_i^T m$).

Remark 4 : The controller does not create a new marking $m \notin R(G, m_0)$ except the markings of control places, which means that the IA-controller is a kind of restrictive controller (not enforcing controller).

Given a plant model G, a weakly forbidden condition $\tilde{F}$ and the IA-controller S, the controlled plant is expressed as $G||S = (\bar{P}, T, \bar{A}, \bar{I}, \bar{m}_0)$ where $\bar{P} = \mathcal{P} \cup C$ and $\bar{A} = [A; A_c]$, $\bar{I} = [\mathbf{0}; I_c]$ and $\bar{m}_0 = (m_0 \; c_0)$.

Lemma **2** *Let* $R_{|\mathcal{P}}(G||S, \bar{m}_0)$ *be the reachable marking set projected on* $\mathcal{P}$ *for* $G||S$*. Then,* $R_{|\mathcal{P}}(G||S, \bar{m}_0) = R(G, m_0)$ *if* $\bar{I} = \mathbf{0}$*.*

Proof : By Remark 4, $R_{|\mathcal{P}}(G||S, \bar{m}_0) \subseteq R(G, m_0)$. It is sufficient to prove $R(G, m_0) \subseteq R_{|\mathcal{P}}(G||S, \bar{m}_0)$. $\bar{I} = 0$ means that there is no inhibition arcs. The controlled plant behaves like standard PNs. Since the state transition equation is different within the uncontrollable subnet G_F. we consider that the state transition equation is

$$\begin{bmatrix} m \\ m_c \end{bmatrix} = \begin{bmatrix} m_0 \\ m_c^0 \end{bmatrix} + \begin{bmatrix} A_F \\ A_c \end{bmatrix} x.$$

Let x be a feasible firing vector in the plant model G. Then, x should satisfies $m = m_0 + A_F x \ge 0$ because the subnet is loop-free [11]. Hence, $m \in R(G, m_0)$. By the definition of A_c, $m_c^0 + A_c x = m_0 + [l_1 \cdots l_{n_F}]^T A_F x \ge 0$, because each element in $l_i, i = 1, \cdots, n_F$ is non-negative. Then, x is also feasible in $G||S$ and $[m; m_c] \in R_{|\mathcal{P}}(G||S, \bar{m}_0)$. Hence, $R(G, m_0) \subseteq R_{|\mathcal{P}}(G||S, \bar{m}_0)$. ■

$\bar{I} = 0$ means that the control places does not restrict the controllable transitions at any marking conditions. Lemma 2 guarantees that the controlled system within admissible marking states behave the same as the original plant.

Enabling and firing rules of the controllable transitions connected from inhibition arcs in $G||S$ is defined as follows:

Definition **3** *Transition* $t \in T$ *is said to be inhibitor arc-enabled (IA-enabled) if there exist at least one control place* $c \in {}^\bullet t \cap C$ *such that* $I(p, t) \le m(c)$*.*

If a transition is both state-enabled and IA-enabled, the transition is said to be control-enabled. But, all the control-enabled transitions can not be firable at the same time. There can exist structural conflicts by the affect of inhibition arcs and control places. A transition set ${}^\bullet c$ w.r.t. a control place $c \in C$ is said to be structurally IA-conflicted if $|{}^\bullet c| > 1$. The IA-conflicted transition set w.r.t. $c \in C$ is denoted by $T_{cft}(c)$. It means

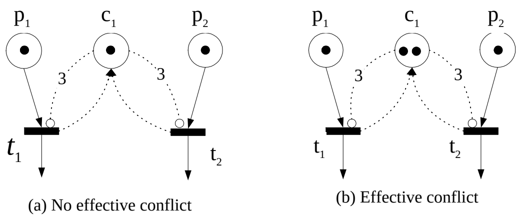


Figure 4: No effective and effective conflict

that the firing of a transition $t \in T_{cft}(c)$ affect the enabling condition of other transitions in $T_{cft}(c)$. So, some transition in IA-conflicted transition set should not be fired simultaneously according to a marking condition as like conflicts in ordinary Petri nets.

Definition **4** *Given a marking m, A transition set $T \subseteq T_{cft}(c)$ w.r.t. a control place $c \in C$ is said to be effective IA-conflicted if all the transitions of T are control-enabled and*

$$m(c) + \sum_{t \in T} A(t,c) > b. \tag{9}$$

where $I(c,t) = I(c,t') = b$ for any $t, t' \in T$ which is possible as mentioned at Remark 2.

When a transition set T is effective IA-conflicted, the simultaneous firing of effective IA-conflicted transitions should be avoided, which is called *conflict resolving.* By resolving the IA-conflicts, we can guarantee that the controlled plant does not enter weakly forbidden states by the following lemma.

Lemma **3** *Given IA-controller S and $m_0 \notin M(\tilde{F})$, conflict resolving avoiding the condition (9) guarantees that $R(G||S, \bar{m}_0) \cap M(\tilde{F}) = \emptyset$.*

Proof : The proof is omitted, but may be found in [10].

Example **2** We consider uncontrollable subnet G_F of Figure 3 with $F : m(p_6) \leq 6$. The controllable transition set is $\{t_1, t_2, t_{11}, t_{12}\}$. By the step evaluation method [10], we can find the weakly forbidden condition $\tilde{F}$ as follows:

$$\begin{aligned} &[m(p_{13}) + m(p_{14}) + m(p_4) + m(p_6) \leq 6] \quad \bigvee \\ &\qquad [m(p_{13}) + m(p_3) + m(p_6) \leq 6]. \end{aligned}$$

Then, IA-controller $S = (C, T_c, A_c, I, c_0)$ is obtained, where $C = \{c_1, c_2\}$, $T_c = \{t_{11}, t_{12}, t_1, t_{13}, t_{14}, t_{15}, t_3, t_5\}$,

$$\begin{aligned} A_c &= [l_1 \ l_2]^T A_F \\ &= \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}. \end{aligned}$$

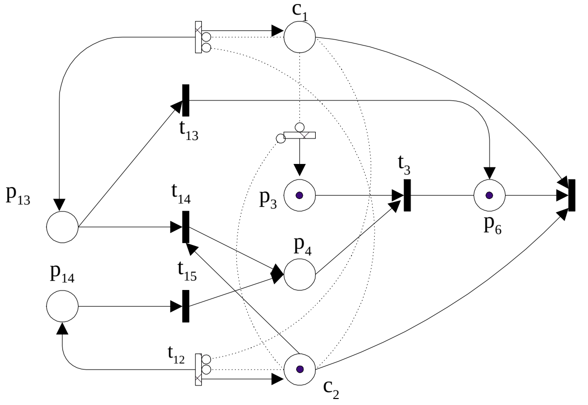


Figure 5: An example of IA-controller

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & -1 & 0 & 0 & -1 \end{bmatrix} \tag{10}$$

and

$$I(c_i, t) = \begin{cases} 6 & \text{for } t \in \{t_{11}, t_{12}, t_1\} \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where $i = 1, 2$. The initial marking is $m_c^0 = [4 \ 5]$. The controlled plant by the IA-controller synthesized as shown in Figure 5. Each control pattern γ is determined by the structure of the IA-controller and the conflict resolving algorithm.

5 Modular IA-Controllers

In this section, we consider a conjunction of some single forbidden conditions as follows:

$$\mathcal{F} : \bigwedge_{i=1}^{K} F$$

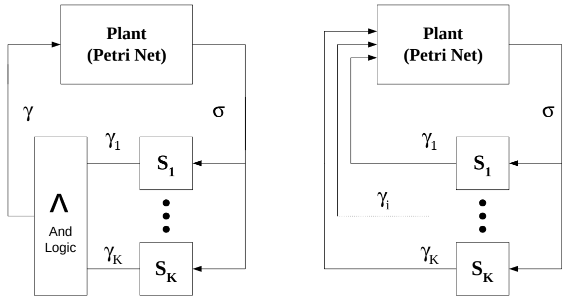
which is called multiple forbidden condition $\mathcal{F}$. The weakly forbidden condition for $\mathcal{F}$ can be represented by

$$\tilde{\mathcal{F}} : \bigwedge_{i=1}^{K} \left[\bigvee_{j=1}^{n_i} l_{ij}^T m \leq b_{ij} \right] \tag{12}$$

For each $F_i, i = 1, \cdots, K$, IA-controller $S_i = (C_i, T_{c_i}, A_{c_i}, I_i, m_{c_i}^0)$ is obtained according to its definition. The control-enabled transition set of S_i is corresponding to the control pattern γ_i.

The final control input for $\mathcal{F}$ is

$$\gamma = \bigwedge_{i=1}^{K} \gamma_i. \tag{13}$$



(a) A structure of general modular IA controller

(b) A structure of Independent modular IA controller

Figure 6: modular IA-controllers

The control structure of the modular IA-controller is shown in Figure 5.(a) and denoted by $\mathcal{S} = S_1 \bigwedge \cdots \bigwedge S_K$. Based on Lemma 3, the behavior of the controlled plant with modular IA controller S guarantees the following statement.

Proposition **1** *Given a plant G and a multiple forbidden condition $\mathcal{F}$, The controlled plant by modular IA controller $\mathcal{S}$ satisfies $L(G||\mathcal{S}) \cap M(\tilde{\mathcal{F}}) = \emptyset$.*

If $({}^\bullet P_{F_i} - P^\bullet_{F_i}) \cap ({}^\bullet P_{F_j} - P^\bullet_{F_j}) = \emptyset$, for any pair of F_i and F_j in $\mathcal{F}$, there is no dependence between the control patterns γ_i and γ_j. In this special conditions of multiple forbidden conditions, we can find a simple structure of modular IA-controllers by removing the AND logic in controller part as shown in Figure 5.(b).

6 Conclusion

A PN structure-type controller with inhibitor arcs called *IA-controller* is presented to solve non-convex forbidden state problems in a class of PNs. The non-convex marking constraint was computed efficiently by the step-evaluation method proposed in [10]. In addition, simple formulas of IA-controllers are proposed to avoid the non-convex marking condition. By extending enabling rule of inhibitor arcs, IA-controller can realize OR-logics which is impossible in the P-invariant structure. The significance of IA-controller is that the controller can be easily implemented in the plant model and the synthesis procedure is simple, because the IA-controller can be computed by a single matrix multiplication. Another advantage of the proposed controller is that it can be used to design Petri net controllers in a modular way. If the specifications of the system can be decomposed into a collection of a single forbidden condition, it may be possible to realize the modular IA-controllers.

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