STABILITY ANALYSIS FOR A TELEOPERATION SYSTEM WITH TIME DELAY AND FORCE FEEDBACK

Cecilia E. Garcia, Beatriz Morales, Ricardo Carelli y Jose Postigo

Instituto de Automatica, Universidad Nacional de San Juan
Av. San Martin 1109 (oeste). 5400. San Juan. ARGENTINA
{cgarcia,bmorales,rcarelli,jpostigo}@inaut.unsj.edu.ar

Abstract: The stability analysis of a two-degree-of freedom teleoperation system, considering the remote station as a non-linear system, is presented. The non-linearity arises in the remote robot's model. Force and position data are backfed from the local to the remote station. The time delay between both stations is fixed and known. A model of the human operator is incorporated into the dynamics of the local station, while a model of the environment is incorporated into the dynamics of the remote station. The analysis, based on operators theory, ensures that system signals remain within a small bounded region. The validity of the theoretical results are verified by simulation.

1. Introduction

In the last decade, teleoperation systems have been extensively developed and their use has reached different areas, such as space and submarine explorations, military and public services (e.g., maintenance and repairing of electric lines), agriculture and livestock raising, medicine and others.

Several authors have dealt with stability analysis for teleoperation systems [7], [9], [11]. However, their analysis suppose a linear model for the system. In [7] and [9], the models for the human operator and of the environments are considered in the system, and the stability is guaranteed with these components included in the teleoperation system. Authors in [12] proposed a control scheme where the non-linear master and slave are treated separately and are subjected to independent adaptive motion/force control. Stability is analysed for this control scheme.

In this paper, the theoretical analysis of a teleoperation system, considering a non-linear model of the remote robot and time delay between both stations, is presented. The system presented in this paper does not use the two-port model technique. The analysis is based on the theory of operators and was inspired in [2], allowing to find a bounded region within which the position error signal remains confined. As a consequence of this, it will be shown as well, that the force signal is also bounded.

The work is organised as follows. Section 2 presents the model for the teleoperation system and a brief description of a structure for compensating the time delay based on that model. In Section 3, the theoretical analysis of the robotic teleoperation system is detailed. A numeric example for computing the boundaries is presented in Section 4. Finally, some conclusions are given.

2. Description of the system

This section presents a brief description of the teleoperation system used in the analysis. The model is detailed in references [4].

2.1. Local Station

Besides the standard modules comprised in a local station, we regard the human operator as an integrating member of the teleoperation system and he/she is included in the modelling, as well as a local manipulator and a linearized replica of the remote station. This latter model is part of the structure proposed for compensating the time delay.

Eqs. (1) and (2) represent respectively the human operator's linear model G_h [5] and the local manipulator's model along with the local controller.

$$G_h(s) = \frac{K_h e^{-T_e s}(1+T_L s)}{(1+T_N s)(1+T_I s)} \tag{1}$$

$$x_j = \frac{k}{m_l s^2 + (b_l + k_v)s + k_p} f_{ef} \tag{2}$$

In eqs. (1) and (2), K_h is the constant operator's gain; T_e is the reaction delay of the operator (operator's time delay); T_L is the operator lead time constant, T_N is the neuromuscular lag time constant whereas T_I is the operator lag time constant. The lead and lag constants are related to adaptability of the human operator to different dynamical models; k is the gain of the local manipulator–controller closed loop, f_{ef} is the total force exerted by the human operator on the local manipulator, k_v and k_p are PD controller gains; m_l and b_l are the local manipulator's inertia and friction coefficients, respectively.

Figure 1 shows the proposed compensation control structure based on Smith's predictor design. The structure is model-based; therefore, the system performance will depend on how precisely the model is known. In Fig. 1, $\boldsymbol{T_i}$ with $\boldsymbol{i} = 1,2$ is the estimated time delay. Delays are assumed to be known and constant.

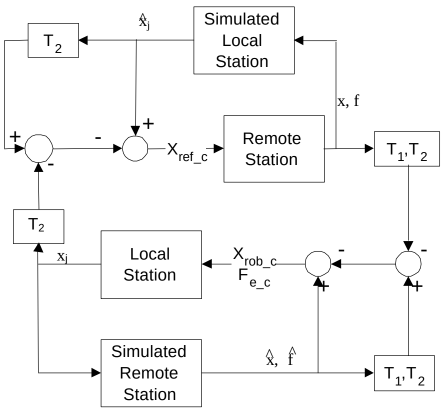

Figure 1: Compensation scheme.

The compensated reference for the remote manipulator is given by;

$$\boldsymbol{x}_{ref\_c} = \hat{\boldsymbol{x}}_j - \left(\hat{\boldsymbol{x}}_j\, exp^{-sT_2} - \boldsymbol{x}_j\, exp^{-sT_2}\right)$$

The signals, which are fed back to the local station, are also compensated by;

$$\boldsymbol{x}_{rob\_c} = \hat{\boldsymbol{x}} - \left(\hat{\boldsymbol{x}}\, exp^{-sT_1} - \boldsymbol{x}\, exp^{-sT_1}\right)$$

and

$$\boldsymbol{f}_{e\_c} = \hat{\boldsymbol{f}} - \left(\hat{\boldsymbol{f}}\, exp^{-sT_2} - \boldsymbol{f}\, exp^{-sT_2}\right)$$

Hat-signed parameters are estimations of the real ones.

The simulated remote station is modelled as a linear system, because we assume that the remote robot model is well-known and it is governed by an inverse dynamics controller. Therefore, the entire local station of the teleoperation system becomes linear. Then, by applying the Superposition Principle, we obtained the output signal as a function of the input signals. In eq. (3), x_j is regarded a function of all input signals to the local station. Time delays were approximated by Pade 's polynomials.

$$x_j = P\,F_{int} + Q\,R_m + U\,x\,e^{-sT_1} + V\,f\,e^{-sT_2} + R\,\hat{K}_m \hat{x}_m \quad (3)$$

where

$$Q = \frac{K(T_L s+1)[Zs^2 + (K_v s + K_p)(\hat{K}_m + Z)]}{D}$$

$$P = \frac{(T_N s+1)(T_I s+1)[Zs^2 + (K_v s + K_p)(\hat{K}_m + Z)]}{D} + \frac{\alpha K(T_L s+1)[Zs^2 + (K_v s + K_p)(\hat{K}_m + Z)]}{D}$$

$$U = \frac{K(T_L s+1)[Zs^2 + (K_v s + K_p)(\hat{K}_m + Z)]}{D}$$

$$R = \frac{K(T_L s+1)K_1(e^{-s\hat{T}_2} - 1) + MK_2(e^{-s\hat{T}_1} - 1)}{D}$$

$$K_1 = K_v s + K_p$$

$$K_2 = K_1 K_m - Zs^2 + (K_v s + K_p)(\hat{K}_m + Z)$$

$$H = Zs^2 + (K_v s + K_p)(\hat{K}_m + Z)$$

$$V = \frac{\left(Zs^2 + (K_v s + K_p)(\hat{K}_m + Z)\right)\left(K(T_L s+1)(1-\alpha)\right)}{D} - \frac{K_f(T_N s+1)(T_I s+1) + K_f \alpha K(T_L s+1)}{D}$$

$$D = G\,H + K(T_L s+1)Z(K_v s + K_p)(e^{-sT_2} - 1) + M\,Z\,K_m(K_v s + K_p)(e^{-sT_1} - 1)$$

$$M = (1-\alpha)K(T_L s+1) - K_f(T_N s+1)(T_I s+1) + K_f \alpha K(T_L s+1)$$

$$G = \left(m_l s^2 + (b_l + k_v)s + k_p\right)\left((T_N s+1)(T_I s+1) + \alpha K(T_L s+1)\right) - m_h(T_N s+1)(T_I s+1)s^2$$

$\boldsymbol{K}_v$ and $\boldsymbol{K}_p$ are design parameters of the PD controller at the simulated remote station; m_h is the muscular inertia of the human operator's arm; $\hat{K}_m$ and $\hat{x}_m$ are the elasticity constant and the position of the simulated environment, respectively; Z is a linear impedance used in the simulated remote station. Constant α is used to represent the human operator's sensorial integration. F_{int} is an internal reference signal of the local controller, R_m are the reference signals of the human operator; x and f are the position and force signals backfed to the local station; $\hat{K}_m \hat{x}_m$ is an additional input signal which arises when the simulated remote station is included in the modelling.

2.2 Remote Station

The remote station is also constituted by two parts: the real robotic manipulator and the environment where the task is being carried out. It also includes a local station model that performs as a predictor for reference signals that are commanded from the local station. In order to make this possible, it is necessary to pre-program the task. The remote station is supervised by a deterministic automaton [5], so that the system is considered as a hybrid one [1]. In the analysis, however, only the continuous linear system is considered.

In eq.(4) the dynamical robot manipulator model in Cartesian coordinates is presented. In this model, the robot is interacting with the environment;

$$\boldsymbol{F} = \boldsymbol{M}(x)\ddot{x} + \boldsymbol{C}(\boldsymbol{x}, \dot{x})\dot{x} + \boldsymbol{g}(x) + \varphi(\boldsymbol{x}, \dot{\boldsymbol{x}}) + \boldsymbol{f}_e \quad (4)$$

where $\boldsymbol{M}(\boldsymbol{x}) \in \Re^{n\times n}$ is the inertia manipulator matrix, with $\boldsymbol{M}(\boldsymbol{x}) = \boldsymbol{M}^T(\boldsymbol{x})$ and $\boldsymbol{M}(\boldsymbol{x}) > 0$; $\boldsymbol{C}(\boldsymbol{x}, \dot{\boldsymbol{x}}) \in \Re^{n\times n}$ is the centrifugal-and-Coriolis-forces matrix; $\boldsymbol{g}(\boldsymbol{x}) \in \Re^n$ is the vector of gravitational force or torques, $\varphi(\dot{\boldsymbol{x}})$ is the friction force, $\boldsymbol{x} \in \Re^n$ is the vector of Cartesian positions and $\boldsymbol{f_e}$ is the interaction force between the manipulator's end-effector and the environment. The control law that was implemented is an impedance robust control given by [8]:

$$\mathbf{F} = \hat{\mathbf{M}}\mathbf{a} + \hat{\mathbf{C}}[\dot{\mathbf{x}} + \nu] + \hat{\mathbf{g}} + \hat{\varphi} + \mathbf{f_e} + \mathbf{K}\,\mathbf{sgn}\left(\nu^{\boldsymbol{T}}\left(\hat{\mathbf{M}}\mathbf{a} + \hat{\mathbf{C}}\nu + \hat{\mathbf{g}} + \hat{\varphi}\right)\right) \quad (5)$$

where $\mathbf{sgn}(\bullet)$ is the signal function and;

$$\boldsymbol{a} = \ddot{\boldsymbol{x}}_0 + \boldsymbol{K}_v(\dot{\boldsymbol{x}}_0 - \dot{\boldsymbol{x}}) + \boldsymbol{K}_p(\boldsymbol{x}_0 - \boldsymbol{x}) \quad (6)$$

$$\boldsymbol{x}_0 = \boldsymbol{x}_{ref} - \boldsymbol{x}_a \quad (7)$$

$$\dot{n} + \mathrm{l}n = (\ddot{\mathbf{x}}_0 - \ddot{\mathbf{x}}) + \mathbf{K}_v(\dot{\mathbf{x}}_0 - \dot{\mathbf{x}}) + \mathbf{K}_p(\mathbf{x}_0 - \mathbf{x}) \quad (8)$$

Here, $\hat{\boldsymbol{M}}$, $\hat{\boldsymbol{C}}$, $\hat{\boldsymbol{g}}$ and $\hat{j}$ have the same structure as $\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{g}$ and j respectively, though with parameters estimates. $\boldsymbol{K}_v$ and $\boldsymbol{K}_p$ are positive definite gain matrices. $\boldsymbol{K}$ is a diagonal gain matrix. $\boldsymbol{x}_a$ is the correction to be exerted on the position commanded by the human operator whenever needed in the interaction with the environment. The correction is based on the impedance concept [6]. $\boldsymbol{x}_{ref}$ is the reference position sent by the operator to the remote robot, $\boldsymbol{x}$ is the robot's actual position.

The simulated local station is linear and the command estimation given by the simulated operator is presented in eq. (9);

$$\hat{x}_j = Y\,\hat{F}_{int} - W\,\hat{R}_m + W\,x + N\,F \quad (9)$$

with

$$Y = \frac{T_N s + aK + 1}{\hat{D}},$$

$$W = \frac{K}{\hat{D}},$$

$$N = \frac{K - K_f(T_N s + aK + 1)}{\hat{D}}$$

$$\hat{D} = \left(\hat{m}_l s^2 + (\hat{b}_l + k_v)s + k_p\right)(T_N s + aK + 1) - \hat{m}_h(T_N s + 1).$$

Finally, the command received by the remote robot is,

$$x_{ref} = \hat{x}_j\left(1 - e^{-sT_1}\right) + x_j e^{-sT_1} \quad (10)$$

3. Stability Analysis

According to eqs. (2), (9) and (10), the teleoperation system can be represented by a block scheme as in Figure (2). However, this scheme is not practical to analyse the system; therefore, we will reduce it to a more convenient form.

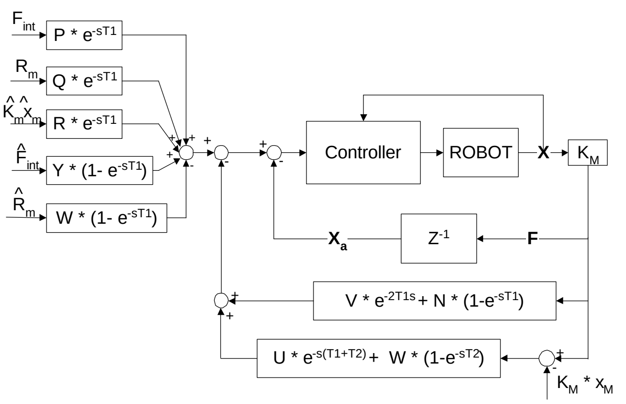

Figure 2: Teleoperation system block scheme.

The interaction force can be computed as: $F_e = K_m(x - x_m)$. We consider $K_m x_m$ as an input step signal that starts acting when the manipulator's end-effector interacts with the environment. Then, the proposed reduced scheme will be as in Figure (3).

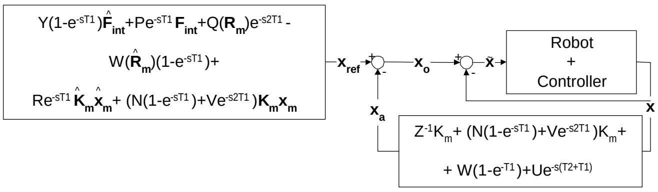

Figure 3: Reduced scheme for system analysis.

The transfer functions that generate the input signal $\boldsymbol{x}_{ref}$, have all their poles in the left half-plane and they are strictly proper [4]; thus, this signal is bounded. The final outline of the analysis is sketched in Fig.(4) where **R** is a non-linear operator that represents the remote robot along with its non-linear controller. **L** is a linear operator that takes into account the rest of the modelled teleoperation system.

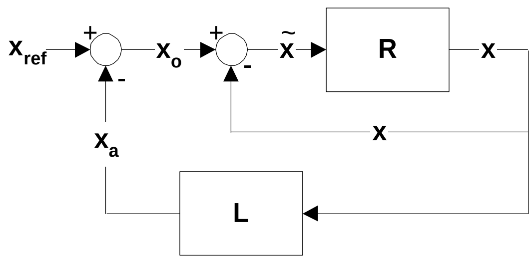

Figure 4: Control scheme for system analysis.

From Fig. (4):

$$\boldsymbol{x}_0 = \boldsymbol{x}_{ref} - \boldsymbol{L} * \boldsymbol{x} \quad (11)$$

The convolution product is denoted by "*". The closed-loop system (robot-robust impedance controller) is obtained by equating eqs. (4) and (5) and by substituting eqs. (6) to (8) into the resulting equality. The equation for the closed-loop system becomes,

$$(\ddot{\boldsymbol{x}}_0 - \ddot{\boldsymbol{x}}) + \boldsymbol{K}_v(\dot{\boldsymbol{x}}_0 - \dot{\boldsymbol{x}}) + \boldsymbol{K}_p(\boldsymbol{x}_0 - \boldsymbol{x}) = \hat{\boldsymbol{M}}^{-1}\Upsilon \quad (12)$$

where

$$\Psi = \left(\tilde{\mathbf{M}}\ddot{\mathbf{x}} + \tilde{\mathbf{Q}} + \mathbf{T}_d - \hat{\mathbf{C}}n - \mathbf{K}\,\mathbf{sgn}\left(n^T\left(\hat{\mathbf{M}}\boldsymbol{a} + \hat{\mathbf{C}}n + \hat{\mathbf{g}} + \hat{j}\right)\right)\right)$$
$$\tilde{\boldsymbol{Q}} = \tilde{\mathbf{C}}\dot{\mathbf{x}} + \tilde{\mathbf{g}} + \tilde{j} \quad (13)$$

Eq. (12) is not suitable to find the system's error equation because it depends on $\boldsymbol{x}_0$. Besides, it is not possible to ensure that this signal is bounded. Considering eq.(11) and based on Leibnitz's rule for derivation under the integral sign, the first and second derivatives will be:

$$\dot{\boldsymbol{x}}_0 = \dot{\boldsymbol{x}}_{ref} - \boldsymbol{L} * \dot{\boldsymbol{x}} - \boldsymbol{x}(0)\boldsymbol{L}$$
$$\ddot{\boldsymbol{x}}_0 = \ddot{\boldsymbol{x}}_{ref} - \boldsymbol{L} * \ddot{\boldsymbol{x}} - \dot{\boldsymbol{x}}(0)\boldsymbol{L} - \boldsymbol{x}(0)\dot{\boldsymbol{L}} \quad (14)$$

By substituting eq. (14) into eq. (12),

$$(\ddot{x}_{ref}-\ddot{x}) - L*\ddot{x} - \dot{x}(0)L - x(0)\dot{L} + K_v(\dot{x}_{ref}-\dot{x}) + K_v(-L*\dot{x}-x(0)L) + K_p(x_{ref}-x) + K_p(-L*x) = \hat{M}^{-1}\Psi \quad (15)$$

The terms $\dot{x}(0)L$, $x(0)\dot{L}$ and $x(0)L$ are bounded because L and $\dot{L}$ are L_∞. Then, by shifting these terms to the right member of eq. (12), the left member of eq. (15) can be expressed as the following;

$$(\ddot{x}_{ref}-\ddot{x}) + K_v(\dot{x}_{ref}-\dot{x}) + K_p(x_{ref}-x) - L*(\ddot{x}+K_v\dot{x}+K_p x) \quad (16)$$

By adding and subtracting $L*(\ddot{x}_{ref}+K_v\dot{x}_{ref}+K_p x_{ref})$, it yields:

$$(\ddot{x}_{ref}-\ddot{x}) + K_v(\dot{x}_{ref}-\dot{x}) + K_p(x_{ref}-x) + L*(\ddot{x}_{ref}-\ddot{x}+K_v(\dot{x}_{ref}-\dot{x})) + K_p L*(x_{ref}-x) - L*(\ddot{x}_{ref}+K_v\dot{x}_{ref}+K_p x_{ref}) \quad (17)$$

The term $L*(\ddot{x}_{ref}+K_v\dot{x}_{ref}+K_p x_{ref})$ is bounded, because we assumed that a smooth and bounded desired trajectory is specified so that $x_{ref}, \dot{x}_{ref}$ and $\ddot{x}_{ref} \in L_\infty$. Thus, we move it to right member of eq. (12).

We define the error as $E = x_{ref} - x$. Therefore eq. (12) can be written as:

$$\ddot{E} + K_v\dot{E} + K_pE + L*(\ddot{E}+K_v\dot{E}+K_pE) = \hat{M}^{-1}\Psi + x(0)L + \dot{x}(0)L + x(0)\dot{L} - L*(\ddot{x}_{ref}+K_v\dot{x}_{ref}+K_p x_{ref}) \quad (18)$$

In the right member of eq. (18), the terms: $x(0)L + \dot{x}(0)L + x(0)\dot{L} - L*(\ddot{x}_{ref}+K_v\dot{x}_{ref}+K_p x_{ref})$ do not depend neither on $\dot{x}(t)$, nor on $\ddot{x}(t)$; but $\hat{M}^{-1}\Psi$, is not independent from these signals. In [2] this problem is solved with more detail. Here, we follow the same procedure.

The force created by the mechanism's inertia can be expressed in terms of above defined error E;

$$\tilde{M}\ddot{x} = \tilde{M}(\ddot{x}_{ref} - \ddot{E}) \quad (19)$$

Likewise, the i[th] term corresponding to the centrifugal and Coriolis forces can be written as $(\dot{x}_{ref}-\dot{E})^T\tilde{C}_i(\dot{x}_{ref}-\dot{E})$. Expanding this product and taking advantage of the fact that $\tilde{C}_i(x)$ are symmetrical, leads to,

$$\tilde{C}(x,\dot{x}) = \tilde{C}(x,\dot{x}_{ref}) - 2\tilde{C}_i(x,\dot{x}_{ref})\dot{E} + \tilde{C}(x,\dot{E}) \quad (20)$$

where $\tilde{C}_i(x,\dot{x}_{ref}) = \dot{x}_{ref}^T\tilde{C}_i(x)$.

As in [2], we consider only the viscous friction term. The terms of Coulomb friction and static friction are included in the perturbation torque T_d. Therefore;

$$\varphi(\dot{x}) = \varphi_i\dot{x} = \varphi_i(\dot{x}_{ref} - \dot{E}) \quad (21)$$

By substituting eqs. (19) to (21) into eq. (18),

$$\begin{aligned}&(I+\hat{M}^{-1}\tilde{M})\ddot{E} + K_v\dot{E} + K_pE + L*(\ddot{E}+K_v\dot{E}+K_pE) =\\ &\hat{M}^{-1}(\tilde{M}\ddot{x}_{ref} + \varphi\dot{x}_{ref} + \tilde{C}(x,\dot{x}_{ref}) + \tilde{C}(x,\dot{E}) + T_d)\\ &-\hat{M}^{-1}(2\tilde{C}_i(x,\dot{x}_{ref}) + \varphi)\dot{E} - \hat{M}^{-1}(\hat{C}\nu + K sng(\nu^T\phi))\\ &x(0)L + \dot{x}(0)L + x(0)\dot{L} - L*(\ddot{x}_{ref}+K_v\dot{x}_{ref}+K_p x_{ref})\end{aligned} \quad (22)$$

Let us consider again the left member of eq. (22). Note that $I + \hat{M}^{-1}\tilde{M} = \hat{M}^{-1}M$. Therefore, by pre-multiplying eq. (22) by $M^{-1}\hat{M}$,

$$\ddot{E} + M^{-1}\hat{M}K_v\dot{E} + M^{-1}\hat{M}K_pE + M^{-1}\hat{M}L*(\ddot{E}+K_v\dot{E}+K_pE) = \Sigma \quad (23)$$

where

$$\begin{aligned}\Sigma =\ & M^{-1}(\tilde{M}\ddot{x}_{ref} + \varphi\dot{x}_{ref} + \tilde{C}(x,\dot{x}_{ref}) + \tilde{C}(x,\dot{E})) -\\ & M^{-1}(2\tilde{C}_i(x,\dot{x}_{ref}) + \varphi)\dot{E} + M^{-1}(\hat{g} + T_d - \hat{C}\nu - K sng(\phi)) +\\ & M^{-1}\hat{M}(x(0)L + \dot{x}(0)L + x(0)\dot{L} - L*(\ddot{x}_{ref}+K_v\dot{x}_{ref}+K_p x_{ref}))\\ \phi =\ & \nu^T(\hat{M}a + \hat{C}\nu + \hat{g} + \hat{\varphi})\end{aligned} \quad (24)$$

By applying the definition of convolution on the term $M^{-1}\hat{M}L*(\ddot{E}+K_v\dot{E}+K_pE)$, the term $L*\ddot{E}$ can be expressed,

$$L*\ddot{E} = \int_0^t L(\tau)\ddot{E}(t-\tau)d\tau = -L(0)\dot{E}(t) + L(t)\dot{E}(0) + \dot{L}*\dot{E} \quad (25)$$

Therefore, the term $M^{-1}\hat{M}L*(\ddot{E}+K_v\dot{E}+K_pE)$ can be written as;

$$M^{-1}\hat{M}L*(\ddot{E}+K_v\dot{E}+K_pE) = M^{-1}\hat{M}(\dot{L}+K_vL)*\dot{E} + M^{-1}\hat{M}K_pL*E + M^{-1}\hat{M}(-L(0)\dot{E}(t) + L(t)\dot{E}(0)) \quad (26)$$

Finally, eq. (23) can be expressed as:

$$\ddot{E} + M^{-1}\hat{M}K_v\dot{E} + M^{-1}\hat{M}K_pE = \xi' + \xi_0 \quad (27)$$

with

$$\begin{aligned}\xi' &= \Sigma - M^{-1}\hat{M}(\dot{L}+K_vL)*\dot{E} - M^{-1}\hat{M}K_pL*E + M^{-1}\hat{M}L(0)\dot{E}(t)\\ \xi_0 &= -M^{-1}\hat{M}L(t)\dot{E}(0)\end{aligned} \quad (28)$$

Eq. (27) will become a linear and decoupled differential equation. This is done following the procedure [2]:

$$M^{-1}\hat{M}K_v\dot{E} = (M^{-1}\hat{M}K_v + K_v - K_v)\dot{E} = K_v\dot{E} - (I - M^{-1}\hat{M})K_v\dot{E} \quad (29)$$

Operating likewise with K_p and substituting into eq. (28),

$$\begin{aligned}\ddot{E} + K_v\dot{E} + K_pE &= \xi' + (I - M^{-1}\hat{M})K_v\dot{E} + (I - M^{-1}\hat{M})K_pE + \xi_0\\ &= \xi + \xi_0 = \Omega\end{aligned} \quad (30)$$

Left side of eq. (30) is a linear and decoupled differential expression in terms of the error E. Right side of eq. (30) can be interpreted as a forcing input Ω that should also be bounded. The L_∞ gain of operators $\mathrm{N} : \mathrm{W} \to \boldsymbol{E}$ and $\mathrm{M} : \mathrm{W} \to \dot{\boldsymbol{E}}$ is given by [3],

$$\|\mathrm{N}\|_\infty = \frac{1}{k_p} = \mathrm{b}_1 \qquad \|\mathrm{M}\|_\infty = \frac{4e^{-1}}{k_v} = \mathrm{b}_2 \tag{31}$$

Then;

$$\|\boldsymbol{E}\|_{T\infty} \le \mathrm{b}_1 \|\mathrm{x}\|_{T\infty} + \mathrm{b}_{01} \quad \text{and} \quad \|\dot{\boldsymbol{E}}\|_{T\infty} \le \mathrm{b}_2 \|\mathrm{x}\|_{T\infty} + \mathrm{b}_{02} \tag{32}$$

where $\|\mathrm{x}\|_{T\infty}$ is the L_∞^n norm truncated at T. From eq. (32), $\mathrm{x} \in \boldsymbol{L}_{\infty e}^n$ implies $\boldsymbol{E}$, $\dot{\boldsymbol{E}} \in L_{\infty e}^n$ and furthermore, if $\mathrm{x} \in \boldsymbol{L}_\infty^n$ then $\boldsymbol{E}$ and $\dot{\boldsymbol{E}} \in L_\infty^n$.

A bound for the signal $\|\mathrm{x}\|_{T\infty}$ will be found as a function of $\|\boldsymbol{E}\|_{T\infty}$ and $\|\dot{\boldsymbol{E}}\|_{T\infty}$. Constants $\mathrm{b}_{01} = \mathrm{b}_1 \|\mathrm{x}_0\|_\infty$ and $\mathrm{b}_{02} = \mathrm{b}_2 \|\mathrm{x}_0\|_\infty$ are finite and non-zero.

Signal n, defined in eq. (8), depends on x_0. Therefore it is necessary to find a bound that is independent from x_0. Signal n is a filter used to avoid the direct measure of the acceleration and it is strictly proper. By applying Laplace Transform to eq. (8),

$$\frac{(x_0 - x)(s)}{\mathrm{n}(s)} = f = \frac{s + \mathrm{l}}{s^2 + k_v s + k_p} \tag{33}$$

Let us assume that the filter's L_∞ gain is g. Then by considering the equality in gain definition, $(x_0 - x)(t) \le g\mathrm{n}(t)$, and substituting into eq. (11),

$$\mathrm{n}(t) = \tfrac{1}{g}\left(x_{ref} - L * x - x = E - L * x + L * x_{ref} - L * x_{ref}\right)$$
$$= \tfrac{1}{g}\left(E + L * E - L * x_{ref}\right) \tag{34}$$

Finally, by substituting eq. (34) into eq. (24), and applying the norm function to input x and substituting this result into eq. (32), the following expressions will be obtained for $\|\boldsymbol{E}\|_{T\infty}$ and $\|\dot{\boldsymbol{E}}\|_{T\infty}$;

$$\|\mathbf{E}\|_{T\infty} \le \mathrm{b}_{01} + \mathrm{b}_1\mathrm{a}_0 + \mathrm{b}_1\mathrm{a}_1 + \mathrm{b}_1\mathrm{a}_2 \|\mathbf{E}\|_{T\infty} + \mathrm{b}_1\mathrm{a}_3 \|\dot{\mathbf{E}}\|_{T\infty} + \mathrm{b}_1\mathrm{a}_4 \|\dot{\mathbf{E}}\|_{T\infty}^2 + \mathrm{b}_1\mathrm{a}_5 \|\mathbf{E}\|_{T\infty} \|\dot{\mathbf{E}}\|_{T\infty} \tag{35}$$

$$\|\dot{\mathbf{E}}\|_{T\infty} \le \mathrm{b}_{02} + \mathrm{b}_2\mathrm{a}_0 + \mathrm{b}_2\mathrm{a}_1 + \mathrm{b}_2\mathrm{a}_2 \|\mathbf{E}\|_{T\infty} + \mathrm{b}_2\mathrm{a}_3 \|\dot{\mathbf{E}}\|_{T\infty} + \mathrm{b}_2\mathrm{a}_4 \|\dot{\mathbf{E}}\|_{T\infty}^2 + \mathrm{b}_2\mathrm{a}_5 \|\mathbf{E}\|_{T\infty} \|\dot{\mathbf{E}}\|_{T\infty}$$

By solving for $\|\boldsymbol{E}\|_{T\infty}$ from eq. (35), it is now clear that the system error trajectory stays within a region limited by the hyperbolas branches arising from these expressions. In eq. (36), we present the resulting region;

$$\|\mathbf{E}\|_{T\infty} \le \frac{\mathrm{b}_1\mathrm{a}_0 + \mathrm{b}_1\mathrm{a}_1 + \mathrm{b}_{01}}{1 - \mathrm{b}_1\mathrm{a}_2 - \mathrm{b}_1\mathrm{a}_5 \|\dot{\mathbf{E}}\|_{T\infty}} + \frac{\mathrm{b}_1\mathrm{a}_3 \|\dot{\mathbf{E}}\|_{T\infty}}{1 - \mathrm{b}_1\mathrm{a}_2 - \mathrm{b}_1\mathrm{a}_5 \|\dot{\mathbf{E}}\|_{T\infty}} + \frac{\mathrm{b}_1\mathrm{a}_4 \|\dot{\mathbf{E}}\|_{T\infty}^2}{1 - \mathrm{b}_1\mathrm{a}_2 - \mathrm{b}_1\mathrm{a}_5 \|\dot{\mathbf{E}}\|_{T\infty}}$$

$$\|\mathbf{E}\|_{T\infty} \ge \frac{-\mathrm{b}_2\mathrm{a}_0 - \mathrm{b}_1\mathrm{a}_1 - \mathrm{b}_{02}}{\mathrm{b}_2\mathrm{a}_3 + \mathrm{b}_3\mathrm{a}_5 \|\dot{\mathbf{E}}\|_{T\infty}} + \frac{1 - \mathrm{b}_2\mathrm{a}_3 \|\dot{\mathbf{E}}\|_{T\infty}}{\mathrm{b}_2\mathrm{a}_3 + \mathrm{b}_3\mathrm{a}_5 \|\dot{\mathbf{E}}\|_{T\infty}} - \frac{\mathrm{b}_2\mathrm{a}_4 \|\dot{\mathbf{E}}\|_{T\infty}^2}{\mathrm{b}_2\mathrm{a}_3 + \mathrm{b}_3\mathrm{a}_5 \|\dot{\mathbf{E}}\|_{T\infty}} \tag{36}$$

where

$$\mathrm{a}_0 = \left\| \boldsymbol{M}^{-1}\hat{\boldsymbol{M}}\left(\boldsymbol{x}(0)\boldsymbol{L} + \dot{\boldsymbol{x}}(0)\boldsymbol{L} + \boldsymbol{x}(0)\dot{\boldsymbol{L}}\right)\right\|_\infty$$

$$\mathrm{a}_1 = \left\| \boldsymbol{M}^{-1}\left(\tilde{\boldsymbol{M}}\ddot{\boldsymbol{x}}_{ref} + \dot{\boldsymbol{x}}_{ref}^T \tilde{\boldsymbol{C}}_i \dot{\boldsymbol{x}}_{ref} + \tilde{\boldsymbol{g}} + \tilde{\mathrm{j}}_i \dot{\boldsymbol{x}}_{ref}\right)\right\|_\infty + \left\|\boldsymbol{M}^{-1}\boldsymbol{K}\right\|_\infty + \frac{1}{g}\left\|\boldsymbol{M}^{-1}\dot{\boldsymbol{x}}_{ref}^T \hat{\boldsymbol{C}}_i\right\|_\infty \|\boldsymbol{L}\|_1 \|\boldsymbol{x}_{ref}\|_\infty$$

$$\mathrm{a}_2 = \frac{1}{g}\left\|\boldsymbol{M}^{-1}\dot{\boldsymbol{x}}_{ref}^T\hat{\boldsymbol{C}}_i + \left(\boldsymbol{I} - \boldsymbol{M}^{-1}\hat{\boldsymbol{M}}\right)\boldsymbol{K}_p\right\|_\infty + \left\|\frac{1}{g}\boldsymbol{M}^{-1}\dot{\boldsymbol{x}}_{ref}^T\hat{\boldsymbol{C}}_i - \boldsymbol{M}^{-1}\hat{\boldsymbol{M}}\boldsymbol{K}_p\right\|_\infty \|\boldsymbol{L}\|_1$$

$$\mathrm{a}_3 = \left\|\left(\boldsymbol{I} - \boldsymbol{M}^{-1}\hat{\boldsymbol{M}}\right)\boldsymbol{K}_v + \boldsymbol{M}^{-1}\hat{\boldsymbol{M}}\boldsymbol{L}(0) - 2\boldsymbol{M}^{-1}\dot{\boldsymbol{x}}_{ref}^T\tilde{\boldsymbol{C}}_i - \tilde{\mathrm{j}}_i\right\|_\infty + \left\|\boldsymbol{M}^{-1}\hat{\boldsymbol{M}}\right\|_\infty \left\|\dot{\boldsymbol{L}} + \boldsymbol{K}_v\boldsymbol{L}\right\|_1 + \frac{1}{g}\left\|\boldsymbol{M}^{-1}\hat{\boldsymbol{C}}_i\right\|_\infty \|\boldsymbol{L}\|_1 \|\boldsymbol{x}_{ref}\|_\infty$$

$$\mathrm{a}_4 = \frac{1}{g}\left\|\boldsymbol{M}^{-1}\hat{\boldsymbol{C}}_i\right\|_\infty \left(1 + \|L\|_1\right) \text{ and } \mathrm{a}_5 = \left\|\boldsymbol{M}^{-1}\tilde{\boldsymbol{C}}_i\right\|_\infty$$

In order to ensure that these expressions exist, it should be verified that,

$$\mathrm{b}_1\mathrm{a}_2 < 1 \quad \text{and} \quad \mathrm{b}_2\mathrm{a}_3 < 1 \tag{37}$$

However, these conditions are not enough to guarantee that the error trajectories stay within a closed region. Therefore, the branches of the hyperbolas should cut themselves in the first quadrant of the plane $\|\boldsymbol{E}\|_{T\infty}, \|\dot{\boldsymbol{E}}\|_{T\infty}$ (only this quadrant is of interest). Eq. (38) presents this condition that arises equating the regions presented in eq. (36) and verifying that the roots are real.

$$\mathrm{b}_1\mathrm{a}_2 + \mathrm{b}_2\mathrm{a}_3 + 2\sqrt{(\mathrm{b}_2\mathrm{a}_1 + \mathrm{b}_2\mathrm{a}_0 + \mathrm{b}_{02})(\mathrm{b}_1\mathrm{a}_5 + \mathrm{b}_2\mathrm{a}_4)} < 1 \tag{38}$$

In addition, the interaction force can be computed as: $F_e = K_m(x - x_m)$. It was shown that x is bounded, thus F_e is also bounded.

The conditions given in eqs. (37) and (38) can be satisfied in a practical problem as it is shown in the next section.

4. A numerical example

The following example was performed for the robotic teleoperation system described in Section 2. A step input was used as reference signal $\boldsymbol{x}_{ref}$. For computing the regions given in eq. (35) a Matlab program was performed with the following results:

$$\|\mathbf{L}\|_1 \le 0.22 \quad \text{and} \quad \|\dot{\mathbf{L}}\|_1 \le 0.68$$

$$a_0 = 0.4059,\, a_1 = 20.1435,\, a_2 = 40.0003,\, a_3 = 3.9451,$$

$$a_4 = 0.5503,\, a_5 = 0.1678,\, b_1 = 0.0025,\, b_2 = 0.0368$$

$$b_1 a_2 = 0.1002 \quad \text{and} \quad b_2 a_3 = 0.1451$$

$$b_1 a_2 + b_2 a_3 + 2\sqrt{b_2 (a_1 + a_0)(b_1 a_5 + b_2 a_4)} = 0.4951$$

Figure (5) shows the closed region and the error trajectory obtained. Note that the manipulator starts with an initial error of 0.05 m, whereas $\dot{E}(0) = 0$. In this example, the term given in eq.(26) was neglected because $\|L\|_\infty << 1$. Therefore the obtained bound is approximate.

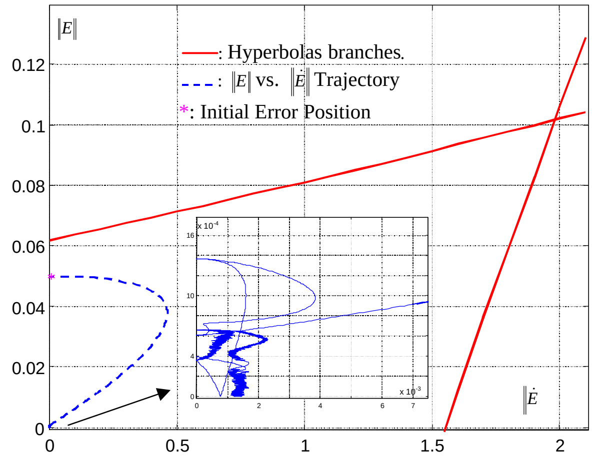

Figure 5: Region and error trajectory.

Conclusions

In this paper a stability analysis for a non-linear teleoperation system with force and position feedback and fixed time delay has been presented. The analysis takes into account both, the human operator and the environment. Although it was performed for a teleoperation system, it may be adapted to any system that can be represented according to the scheme of Fig. (4), where the operator **R** is non linear (robot-manipulator + robust impedance controller) and **L** is linear.

It is guaranteed that the signal of the error position stays within a closed region stated by eq. (35) and subject to the conditions given in eqs. (37) and (38). Likewise, the interaction force with the environment is also bounded. It is possible as well to assure that the internal signals of the system are bounded.

Acknowledgments

The authors thanks the help given by Prof. Carlos Calvo, Dept. of Mathematics, U.N.S.J and by Prof. Alberto Garcia, U.N.S.J. They also gratefully acknowledge the financial support from CONICET (National Council for Scientific Research and Technology) and ANPCyT, Argentina.

References

[1] Antsaklis P., Lemmon M., Stiver J., “Hybrid System Modeling and Event Identification”, Technical Report of the ISIS Group at the University of Notre Dame. ISIS – 93 – 002 (1993).

[2] Craig J. Adaptive Control of Mechanical Manipulators. Addison – Wesley. U.S.A. 1995.

[3] Desoer C.A. and Vidyasagar M. Feedback Systems: Input-Output Properties. Academic Press, Inc. Orlando, Florida. ISBN 0-12-212050-7. 1975

[4] Garcia C. y Carelli R. Control Hibrido de un sistema de Teleoperacion Robotico. Internal Report. Biblioteca. Instituto de Automatica. U.N.S.J. http://www.pino.unsj.edu.ar/public.

[5] Garcia C., Morales B., Carelli R., Postigo J., Cury J. Supervosiry Control for a Teleoperation System. Hybrid Control Approach. Published CD of MCPL 2000. Grenoble. France.

[6] Hogan N. Impedance Control: An approach to manipulation: Part i, part ii, part iii. ASME J. Dynamic Syst., Measurement, Control, vol 107, (1985).

[7] Lawrence D. Stability and Transparency in Bilateral Teleoperation. IEEE Transaction on Robotics and Automation, Vol. 9, No. 5. October 1993.

[8] Mut V., Postigo J. and R. Carelli. "Robust Motion-Force Control of Robots With Uncertain Environment". Proceedings of "II Latin American Seminar on Advanced Control (LASAC '95) and Fourth Seminar on System Identification, Parameter Estimation and Adaptive Control (SISEPCA '95)", pp. 245 -250. Santiago, Chile. Setiembre 27 - 29, 1995. http://www.pino.unsj.edu.ar/public

[9] Sukhan Lee. Modelling, Design and Evaluation of Advance Teleoperator Control Systems with Short Time Delay. IEEE Transaction on Robotics and Automation, Vol. 9, No. 5. October 1993.

[10] Sutton R. Modelling Human Operators in Control System Design. J. Wiley& Sons Inc. Ney York 1990.

[11] Xi N., Tarn T. Action Synchronization and Control of Internet Based Telerobotic Systems. Proceedings of the 1999 IEEE ICRA, Vol. 1, pp 219-224. Detroit, Michigan. May 1999.

[12] Zhu W., Salcudean S. Teleoperation with Adaptive Motion/Force Control. Proceedings of the 1999 IEEE ICRA, Vol. 1, pp 231-237. Detroit, Michigan. May 1999.