Hybrid Tracking Control for Spark–Ignition Engines.[1]

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Abstract

The design of a torque tracking controller for a spark ignition engine is presented. A hybrid model that describes the interacting behavior of the intake manifold, the engine, the power–train and the catalytic converter is illustrated. The proposed control is obtained by (i) decoupling the control problem into two subproblems, and (ii) relaxing each of the two subproblems to yield problems that can be solved with classical control techniques. The control law so obtained is mapped back into the hybrid domain. The quality of the proposed hybrid control feedback is demonstrated analytically and by simulations on the full-fledged hybrid model.

1 Introduction

The design of a torque tracking controller for a spark ignition direct injection[2] engine, equipped with an electronic–throttle valve, is considered. The desired torque signal is assumed to be produced on–line by the driver acting on the accelerator pedal. Fuel injection is subject to constraints on the amount of non–stoichiometric mix that can be supplied to the engine without saturating the catalytic converter[3].

Most of the published works on engine control synthesis are based on average–value models (e.g. [6]) and are devoted to the control of a particular phenomenon and/or subsystem during a particular operating condition. Cycle-accurate models are instead mainly used both to analyze the behavior of the engine, and to design mechanical parts as well as sensors and actuators (e.g. [8]). But they are rarely, if at all, used in the design of the control laws.

In our approach, the adoption of a hybrid formalism allows us to represent the cyclic behavior of the engine, thus capturing the effect of each fuel injection on the generated torque, and the interaction between the discrete torque generation and the continuous power–train and air dynamics. By means of the hybrid modeling of the engine, higher performances of the closed–loop system can be achieved. To obtain a satisfactory solution in the hybrid domain, we exploit the knowledge of the physical processes to control. We have been able to do so in a number of cases [2], including cut–off control [4], fast transient control [3, 5] and idle control [1]. In particular, in [3] we considered air-fuel ratio control during torque transients for non–GDI engines. The main problem was due to fact that, in a non–GDI engine, the fuel adheres in part to the walls of the manifold forming a liquid film, whose evaporation dynamics needs to be controlled to ensure air-fuel ratio constraint satisfaction. In this work, the synthesis of a torque tracking controller is formalized in the hybrid domain. The hybrid feedback laws are obtained from the solution to two simplified optimal control problems: torque generation control (in the discrete–time domain) and intake manifold control (in the continuous–time domain). In principle, the optimality properties of the two loops are lost when the control laws so obtained interact with each other in the full fledged hybrid model. However, the interesting point about our approach is that we are able to assess the properties of the hybrid feedbacks, such as stability of the tracking and velocity of convergence, so that a satisfactory behavior of the hybrid–closed loop system is guaranteed.

2 Hybrid model of the power–train

In this section, a hybrid model of a power–train equipped with a 4–cylinder in–line engine is presented. This model is derived from the N–cylinder engine model reported in [2], which is augmented here with the catalytic converter description. The power–train hybrid model is described in the tagged-signal model (TSM) formalism proposed in [7]. This formalism allows us to describe formally systems represented as in-

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[2]While in traditional engines the injectors deliver fuel in the intake manifold, in modern GDI (Gasoline Direct Injection) engines fuel is directly injected in the cylinders. This allows to reduce fuel consumption when a low engine torque is required.

[3]A device used in the exhaust system to reduce tail-pipe emissions in order to meet standards imposed by governments.

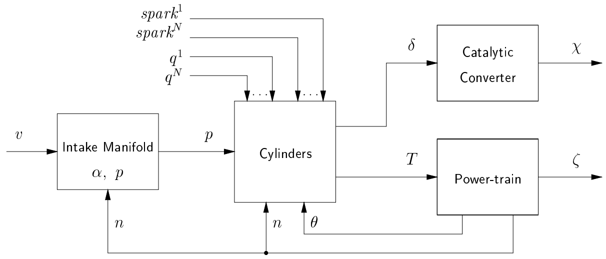

Figure 1: The engine and power–train model.

teracting processes of heterogeneous models of computation. In particular, we use a combination of Finite State Machines (FSMs), Discrete Event Systems (DESs) and Continuous–Time Systems (CTSs) to form a hybrid system that is the basis for our design. The overall system is composed of four main interacting blocks, namely the *intake manifold*, the *cylinders*, the *catalytic converter* and the *power–train* (Fig. 1). In the next section, we describe the catalytic converter model. The details of the other blocks can be found in [2].

2.1 The catalytic converter model.

In order to achieve a proper combustion of the air-fuel mixture, the amount of fuel q^i injected into a cylinder has to meet specified constraints. These constraints are usually expressed in terms of the air-fuel ratio $A/F = \frac{m^i}{q^i}$. When $A/F = (A/F)_{stoic} = 14.64$, the mix is said to be at stoichiometry. Rich mixes $A/F < 14.64$ produce excess of CO and HC, while lean mixes $A/F > 14.64$ have excess of NO_X. Hence, for a proper combustion, the equivalence ratio

$$\gamma(k) = \frac{(A/F)_{stoic}}{A/F} = (A/F)_{stoic}\frac{q^i(k)}{m^i(k)}$$

has to satisfy[4]

$$\gamma(k) = [\gamma_{min}, \gamma_{max}] \cup \{0\} \; . \qquad (1)$$

The working principle of a three–way catalytic converter is based on the oxygen storage and release mechanism. The conversion efficiency of the converter drastically decreases when such mechanism reaches a saturation point, due to either excess of NO_X or HC and CO in the engine–out gas. Assuming that at start up the catalytic converter is balanced, when the air–fuel mixture loaded by the cylinders is at stoichiometry, i.e. $\gamma = 1$, the oxygen storage and release mechanism remains balanced. Engine outputs of lean mixtures ($\gamma < 1$) and rich mixtures ($\gamma > 1$) unbalance the converter towards excess and lack of oxygen, respectively. At each cycle, the i–th cylinder model (a DES) provides as an output the value of the mixture bias mass $\delta^i = m^i - (A/F)_{stoic}q^i = (1-\gamma^i)m^i$. A regulation of the storage mechanism of the catalytic converter is obtained by controlling the evolution of the mixture bias sequence $\{\delta^i(k_i)\}$ for $i = 1,\ldots,N$. The storage mechanism is modeled at the beginning of the exhaust pipe by a DES, whose state variable l represents the sum of mixture bias masses in the gas delivered by the engine up to the current time:

$$l(k^0+1) \;\; = \;\; l(k^0) + \sum_{i=1}^{N} \delta^i(k^i) \; . \qquad (2)$$

The sequence $\{t_{k^0}\}$ of times at which (2) is updated is defined as $\cup_{i=1}^{N}\{t_{k^i}\}$, where $\{t_{k^i}\}$ denotes the sequence of times at which the i–th cylinder DES makes a step. Dynamics (2) captures the storing nature of the catalytic converter and is subject to the saturation constraint[5]

$$l \in [l_{min}, l_{max}] \; . \qquad (3)$$

2.2 Hybrid engine and power–train model.

In this section, the overall hybrid model of a power–train equipped with a 4–cylinder in–line engine is presented. Assuming optimal spark ignition (to optimize fuel consumption), the cylinder FSM given in [2] degenerates to a single–state FSM. At each dead center, the FSM takes a self–loop transition and generates the sequence $\{t_k\}$ of transition times. Fuel injection actuation is modeled by a one–step delay in the cylinder DES. Let $\gamma(k)$ denote the value of the equivalence ratio of the cylinder that enters the intake stroke at time t_k. To handle more easily constraint (1), we consider γ as the injection input instead of q. The produced torque is $T(k) = G\gamma(k-2)m(k-1) = Gc_p\gamma(k-2)p(t_{k-1})$. Dynamics (2) evolves on the transition times sequence $\{t_k\}$ and receives as input the variable $\delta(k)$ denoting the mixture bias of the cylinder that enters the exhaust stroke at time t_k.

Summarizing, the hybrid model of the 4–cylinder in–line engine and the power–train is composed of:

1. a CTS, modeling throttle and intake dynamics[6]

$$\dot\alpha(t) = a_\alpha\alpha(t) + b_\alpha v(t), \;\; \dot p(t) = a_p p(t) + b_p\alpha(t) \qquad (4)$$

with $\alpha \in [0, 90]$ and $v \in [-V, +V]$;

2. a CTS, modeling the power–train dynamics

$$\dot\zeta(t) = A\zeta(t) + bT(t) - b_0, \;\; \dot\theta(t) = (0,\, 6,\, 0)\zeta(t) \qquad (5)$$

where $\zeta = (\alpha_e, n, \omega_p)^T$ includes the drive-line torsion angle, the crankshaft revolution speed, and the wheel revolution speed and θ is the crankshaft position; b_0 models resistant actions;

3. a single–state FSM, which generates dead–center events when the piston position $\phi(t) = \theta(t) \mathrm{mod}\, 180$ is 0;

4. a DES, active at each FSM transition and modeling the torque generation and the catalytic converter

$$T(k+1) = Gc_p\gamma^C(k)p(t_k), \;\; \gamma^C(k+1) = \gamma(k) \qquad (6)$$

$$\delta^E(k+1) = c_p\;(1-\gamma^C(k))\;p(t_k) \qquad (7)$$

$$l(k+1) = l(k) + \delta(k), \;\; \delta(k+1) = \delta^E(k) \qquad (8)$$

[4] where $\gamma = 0$ when fuel is not injected.

[5] In a model refinement, a nonlinear input map $\Psi(m^i, q^i)$, such that $\Psi(m^i, (A/F)_{stoic}m^i) = 0$, can be used in place of δ^i.

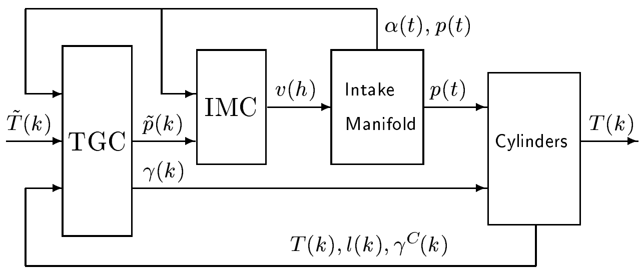

Figure 2: Control scheme: Torque Generation Control (TGC) and Intake Manifold Control (IMC).

with γ bounded as in (1) and l subject to (3). Torque T in (5) is obtained from (6) by a zero-order holder.

3 Hybrid control design

The complexity of the design of a torque–tracking controller for the power–train hybrid model described in Sec. 2.2 is attacked by decomposing the problem into the synthesis of two nested control loops (see Fig. 2):

- a *torque generation control* — in the outer loop;
- an *intake manifold control* — in the inner loop.

The aim of torque generation control is to find an appropriate policy of distribution of the control actions, between fuel injection and air loading, which achieves both torque tracking and proper catalytic converter management. The intake manifold inner control loop has to ensure tracking of a desired manifold pressure signal ($\tilde{p}$) received from the outer control loop. The two subproblems are solved by "relaxing" them respectively to the discrete–time domain (see Sec. 3.1) and continuous–time domain (see Sec. 3.2), where optimal solutions can be devised by classical control techniques. In Sec. (3.3), a hybrid feedback control for the hybrid model described in Sec. 2 is derived from the two previously designed control loops. Finally, in Sec. 4, the behavior of the closed–loop hybrid system is analyzed in depth and sufficient conditions that guarantee convergence and torque tracking as well as proper catalytic converter management are presented.

3.1 Torque generation control (TGC)

In this section, a feedback control that achieves tracking of a given reference torque signal $\hat{T}(k)$ is devised. We assume that the reference torque profile is the ramp $\hat{T}(k+1) = \hat{T}(k) + m\tau$, where m and τ are constant parameters and the latter stands for the time between two dead centers. The torque generation DES model is obtained from (6–8). The TGC produces the value $\tilde{p}(k)$ of desired manifold pressure. To decouple the two loops, we assume that the IMC will be able to produce the desired value $\tilde{p}(k)$ at the next dead center. Hence, dynamics (4) are abstracted away and replaced by $p(k+1) = \tilde{p}(k)$. Further, by considering the mixture bias at the intake phase instead of at the exhaust phase, the delay equations (7–8) are removed. The TGC optimal control problem is:

Problem 3.1.1 *Given an initial state* $(\hat{T}(0), p(0), \gamma^C(0), T(0), l(0)) = (\hat{T}_0, p_0, {\gamma^C}_0, T_0, l_0)$

$$\min_{\gamma(k),\tilde{p}(k)} \sum_{i=0}^{\infty} \frac{W}{2}(T(i) - \hat{T}(i))^2 \tag{9}$$

subject to

$$\begin{array}{l} \hat{T}(k+1) = \hat{T}(k) + m\tau, \quad p(k+1) = \tilde{p}(k) \\ T(k+1) = Gc_p p(k)\gamma^C(k), \quad \gamma^C(k+1) = \gamma(k) \\ l(k+1) = l(k) + c_p\ (1-\gamma^C(k))\ p(k) \end{array} \tag{10}$$

$$\begin{array}{l} \gamma(k) \in [\gamma_{min}, \gamma_{max}], \quad \tilde{p}(k) \in [\tilde{p}_{min}, \tilde{p}_{max}], \\ l(k) \in [l_{min}, l_{max}], \quad \lim_{k\to\infty} l(k) = 0 \end{array} \tag{11}$$

Let

$$x|_{[x_1,x_2]} \equiv \begin{cases} x_1 & \text{if } x < x_1 \\ x & \text{if } x_1 \le x \le x_2 \\ x_2 & \text{if } x > x_2 \end{cases} .$$

By applying standard optimal control techniques in the discrete–time domain, a solution to the optimal control Problem 3.1.1 is determined as follows:

$$\tilde{p}(k) = \left.\frac{\hat{T}(k+2)}{Gc_p\bar{\gamma}(k)}\right|_{[\tilde{p}_{min},\tilde{p}_{max}]} \qquad \gamma(k) = \left.\frac{\hat{T}(k+2)}{Gc_p\tilde{p}(k)}\right|_{[\widehat{\gamma}_{min},\widehat{\gamma}_{max}]} \tag{12}$$

where, for a given value of the parameter $Q \in (0,1]$,

$$\begin{array}{l} \bar{\gamma}(k) = \left.\frac{\hat{T}(k+2)}{\hat{T}(k+2) - QG(l(k)+c_p p(k)(1-\gamma^C(k)))}\right|_{[\gamma_{min},\gamma_{max}]} \\ \widehat{\gamma}_{max} = \min\{1 + \frac{l(k)+c_p p(k)(1-\gamma^C(k)) - l_{min}}{c_p\tilde{p}(k)}, \gamma_{max}\} \\ \widehat{\gamma}_{min} = \max\{1 + \frac{l(k)+c_p p(k)(1-\gamma^C(k)) - l_{max}}{c_p\tilde{p}(k)}, \gamma_{min}\} \end{array}$$

Lemma 3.1.1 *If the reference signal $\hat{T}(k)$ is such that there exists a $\bar{k} > 0$ for which*

$$\hat{T}(k) \in\ (Gc_p\tilde{p}_{min}, Gc_p\tilde{p}_{max}) \quad \forall\, k > \bar{k}, \tag{13}$$

then[7] *under feedback (12), the sequence of $l(k)$ can be bounded as follows*

$$0 \le (1-Q)|l(k)| \le |l(k+1)| < |l(k)| \quad \forall k > \bar{k}+1 \tag{14}$$

By the above lemma, the velocity of convergence is lower bounded by $1-Q$.

3.2 Intake manifold control (IMC)

When the engine torque T has approached the reference torque ramp $\hat{T}$ and the catalytic converter has reached the equilibrium point $l = 0$, the feedback (12) produces $\tilde{p}(k+1) = \tilde{p}(k) + \frac{m\tau}{Gc_p}$ and $\gamma(k) = 1$. Hence, assuming

[6] In (4), we assume that an inner control loop linearizes the intake manifold dynamics and compensates its dependency on the crankshaft speed n, so that a_p and b_p can be considered constant.

[7] This is a very mild hypothesis since it requires that the plant should be able to track $\hat{T}$ with $\gamma \in (\gamma_{min}, \gamma_{max})$.

small variations in the crankshaft speed, the desired pressure sequence can be approximated by a ramp signal. Then, the intake manifold control is obtained by solving a minimum–time control problem with respect to the reference pressure

$$\hat{p}(t) = m_p t + n_p \ . \quad (15)$$

The corresponding reference for the throttle angle is:

$$\hat{\alpha}(t) \quad = \quad -\frac{a_p}{b_p}\hat{p}(t) + \frac{m_p}{b_p} = m_\alpha t + n_\alpha \quad (16)$$

where $m_\alpha = -\frac{a_p m_p}{b_p}$ and $n_\alpha = \frac{m_p - a_p n_p}{b_p}$. The formalization of the optimal control problem for the reference trajectory (15–16) is obtained by augmenting the state space with a state variable β denoting the elapsed time:

Problem 3.2.1 *Given an initial state* $(\alpha, p, \beta) = (\alpha_0, p_0, 0)$,

$$\min_{v(t)} \beta(t_f) \quad (17)$$

subject to (4) and

$$\dot{\beta}(t) = 1, \ \ (\alpha(t_f), p(t_f), \beta(t_f)) \in \mathcal{S} \quad (18)$$

with target set $\mathcal{S} \equiv \left\{ \begin{array}{l} p - m_p\beta - n_p = 0 \\ \alpha - m_\alpha\beta - n_\alpha = 0 \end{array} \right.$.

The minimum–time control is obtained by applying the Pontryagin Maximum Principle [9]:

$$v_O(\alpha,p) = \begin{cases} +V & \text{if } [(\sigma(\alpha,p) < 0) \vee ((\sigma(\alpha,p) = 0) \\ & \quad \wedge (\alpha < \hat{\alpha}))] \wedge (\alpha < 90) \\ -90\frac{a_\alpha}{b_\alpha} & \text{if } [(\sigma(\alpha,p) < 0) \vee ((\sigma(\alpha,p) = 0) \\ & \quad \wedge (\alpha < \hat{\alpha}))] \wedge (\alpha = 90) \\ -V & \text{if } [(\sigma(\alpha,p) > 0) \vee ((\sigma(\alpha,p) = 0) \\ & \quad \wedge (\alpha > \hat{\alpha}))] \wedge (\alpha > 0) \\ 0 & \text{if } [(\sigma(\alpha,p) > 0) \vee ((\sigma(\alpha,p) = 0) \\ & \quad \wedge (\alpha > \hat{\alpha}))] \wedge (\alpha = 0) \end{cases} \quad (19)$$

where

$$\sigma(\alpha,p) = p - \frac{b_p}{a_\alpha - a_p}\left[\frac{\hat{\alpha}-\alpha_v}{\alpha-\alpha_v} - \left(\frac{\hat{\alpha}-\alpha_v}{\alpha-\alpha_v}\right)^{\frac{a_p}{a_\alpha}}\right](\hat{\alpha}+\alpha_v)$$
$$\left(\frac{\hat{\alpha}-\alpha_v}{\alpha-\alpha_v}\right)^{-\frac{a_p}{a_\alpha}}\hat{p} + \left[1 - \left(\frac{\hat{\alpha}-\alpha_v}{\alpha-\alpha_v}\right)^{-\frac{a_p}{a_\alpha}}\right]p_v \quad (20)$$

3.3 Hybrid torque–tracking control

In this section a hybrid feedback control that solves the torque tracking problem for the hybrid model of the engine described in Sec. 2 is derived from the TGC and the IMC, presented respectively in Sec. 3.1 and Sec. 3.2.

3.3.1 Outer loop TGC: The following points need to be considered:

1. in Problem 3.1.1, the discrete–time model of the plant has been assumed to evolve at a fixed sampling period τ, i.e. fixed crankshaft speed;
2. in (12), the bounds $\tilde{p}_{min}$ and $\tilde{p}_{max}$ on the desired manifold pressure $\tilde{p}$ are supposed to be fixed and known, while in the overall model they depend on the intake manifold dynamics;
3. the feedback law needs to be generalized to the case where a generic (non–ramp) reference torque signal is applied.

To address points 1, 2 and 3, the TGC (12) is modified as described below. At each time t_k of the sequence of dead centers times $\{t_k\}$, the TGC (12) gives the current value of the equivalence ratio $\gamma(k)$ for fuel injection and the current value of desired intake manifold pressure $\tilde{p}(k)$ to be used by the inner–loop IMC. In order to include reference torque signals $\hat{T}$ different from ramp–signals, the linear 2–step ahead estimation

$$\hat{T}(k+2) = \hat{T}(k) + 2m_{\hat{T}}(k)\ \tilde{\tau}(k) \quad (21)$$

is used in (12). In (21), the current estimate of the dead center period $\tilde{\tau}(k)$ is $30/n(t_k)$ and

$$m_{\hat{T}}(k) = \frac{\hat{T}(k) - \hat{T}(k-1)}{\tau(k-1)} \quad (22)$$

with $\tau(k-1)$ the measured time between the current and the previous dead center. Moreover, bounds $\tilde{p}_{min}$ and $\tilde{p}_{max}$ required in (12) are computed at each dead center as follows

$$\begin{aligned} \tilde{p}_{min}(k) &= \left[A_p(\tilde{\tau}(k))\begin{pmatrix}\alpha(k)\\ p(k)\end{pmatrix} - B_p(\tilde{\tau}(k))V\right]\Bigg|_{(2)} \\ \tilde{p}_{max}(k) &= \left[A_p(\tilde{\tau}(k))\begin{pmatrix}\alpha(k)\\ p(k)\end{pmatrix} + B_p(\tilde{\tau}(k))V\right]\Bigg|_{(2)} \end{aligned} \quad (23)$$

where $x|_{(2)}$ denotes the second component of vector x,

$$A_p(t) = \begin{bmatrix} e^{a_\alpha t} & 0 \\ \frac{b_p e^{(a_\alpha - a_p)t}}{a_\alpha - a_p} & e^{a_p t}\end{bmatrix}, \ B_p(t) = (I - A_p(t))\begin{bmatrix} -\frac{b_\alpha}{a_\alpha} \\ \frac{b_p}{a_p}\frac{b_\alpha}{a_\alpha}\end{bmatrix}$$

3.3.2 Inner loop IMC: Since the engine control unit is implemented by a digital system, in this section a discrete version of the intake manifold control developed in Sec. 3.2 is derived. Let τ_0 denote the sampling period of the discrete–time IMC and let $\{t_h\}$ denote the sequence of sampling times. The intake manifold discrete–time model is

$$\begin{bmatrix}\alpha(h+1)\\ p(h+1)\end{bmatrix} = A_p(\tau_0)\begin{bmatrix}\alpha(h)\\ p(h)\end{bmatrix} + B_p(\tau_0)v(h) \quad (24)$$

The asynchrony between the TGC and the IMC is handled by interpolating the values of the sequence $\tilde{p}(k)$, generated at dead–center times $\{t_k\}$, on the fixed time base $\{t_h\}$ of the intake manifold control. Given a time t_k, let $\bar{h}(k) = \text{argmin}_h(t_h > t_k)$. For all h with $t_h \in [t_{\bar{h}(k)}, t_{\bar{h}(k+1)})$, the reference trajectory $(\hat{\alpha}, \hat{p})$ is defined as in (16) and (15), where t is replaced by $(h - \bar{h}(k))\tau_0$ and

$$m_p = m_{\tilde{p}}(k) = \frac{\tilde{p}(k) - \tilde{p}(k-1)}{\tau(k-1)}, \ n_p = n_{\tilde{p}}(k) = \tilde{p}(k) \quad (25)$$

To avoid chattering around the point $(\hat{\alpha},\hat{p})$, the bang-bang control (19) is used to steer the state to the set

$$\mathcal{R}(\hat{\alpha},\hat{p})=\{(\alpha,p):(\alpha-\hat{\alpha})^2+(p-\hat{p})^2\le\rho_1^2\wedge|\sigma(\alpha,p)|\le\rho_2\} \tag{26}$$

centered on $(\hat{\alpha},\hat{p})$, where ρ_1,ρ_2 are control parameters. Inside $\mathcal{R}(\hat{\alpha},\hat{p})$ the linear feedback $v_{\mathcal{R}}(\alpha,p) = (B_p(\tau_0)\ A_p(\tau_0)B_p(\tau_0))^{-1}\left[\begin{pmatrix}\hat{\alpha}(h)\\ \hat{p}(h)\end{pmatrix}-A_p(\tau_0)^2\begin{pmatrix}\alpha(h)\\ p(h)\end{pmatrix}\right]\Big|_{(2)}$ is applied[8]. The discrete–time IMC is:

$$v(h)=\begin{cases} v_O(\alpha,p) & \text{if } (\alpha,p)\notin\mathcal{R}(\hat{\alpha},\hat{p}) \\ v_{\mathcal{R}}(\alpha,p) & \text{if } (\alpha,p)\in\mathcal{R}(\hat{\alpha},\hat{p}) \end{cases} \tag{27}$$

The overall hybrid tracking feedback control is:

Torque Generation Control: dead–center times $\{t_k\}$
compute $\tilde{\hat{T}}(k+2)$ according to (21)
compute $\tilde{p}_{min},\tilde{p}_{max}$ according to (23) (28)
compute $\tilde{p}(k)$ and set $\gamma(k)$ according to (12)
compute $m_{\tilde{p}}(k), n_{\tilde{p}}(k)$ according to (25)

Intake Manifold Control: sampling $\{t_h\}$
compute $\sigma(\alpha,p)$ according to (20), (15), (16), (25) (29)
set $v(h)$ according to (27)

4 Tracking performances of the hybrid closed–loop system

According to the design methodology described in [4], the hybrid design is concluded with an analytic discussion on the performances achieved by the hybrid tracking feedbacks (28–29), when applied to the hybrid engine model presented in Sec. 2. The following cases are considered:
R.1 fixed engine speed and ramp reference torque;
R.2 fixed engine speed and generic reference torque;
R.3 hybrid plant model and generic reference torque.

R.1: fixed crankshaft speed and ramp reference torque. Consider an approximated hybrid model of the engine where: the crankshaft speed is supposed to be constant, i.e. $n(t)=30/\tau$, and the reference torque is a ramp of type (10). We have

Proposition 4.0.1 *If the reference signal $\hat{T}$ satisfies the hypothesis of Lemma 3.1.1, where $\tilde{p}_{min}$ and $\tilde{p}_{max}$ are given by (23), and if $Q\in(0,1]$ and*

$$Q\le\min\left\{\left|\frac{\tilde{p}_{max}-\hat{T}(k+2)}{l(k+1)}\right|,\left|\frac{\tilde{p}_{min}-\hat{T}(k+2)}{l(k+1)}\right|\right\}$$

then
- $\exists K<\infty$ *such that* $T(k)=\hat{T}(k)$, *for all* $k\ge K$, *and*
- $\lim_{k\to\infty} l(k)=0$.

Futher, if the manifold pressure is affected by multiplicative and additive disturbances $\varepsilon_M(k)$ and $\varepsilon_A(k)$, i.e. $p(k)=\tilde{p}(k)\varepsilon_M(k)+\varepsilon_A(k)$, with $|\varepsilon_M(k)-1|\le d_M<1$ and $|\varepsilon_A(k)|\le d_A$, then $\lim_{k\to\infty} l(k)=L<\infty$.

[8] For unconstrained v, $v_{\mathcal{R}}(\alpha,p)$ is a dead–beat control that achieves convergence of dynamics (24) to $(\hat{\alpha},\hat{p})$ in two steps. Since in our case v is bounded, the parameters ρ_1 and ρ_2 that define the set $\mathcal{R}(\hat{\alpha},\hat{p})$ in (26) are chosen such that $\mathcal{R}(\hat{\alpha},\hat{p})$ be a controlled invariant for dynamics (24).

Note that, if Q is small then the upper bounds d_A and d_M are small.

R.2: fixed crankshaft speed and generic reference torque. When a generic reference torque signal $\hat{T}(t)$ is applied, feedback (28) computes the next value of the requested torque by locally approximating the reference signal according to (21). If the crankshaft speed is fixed at $30/\tau$ then, at a given dead center time t_k, the hybrid control (28–29) produces a torque $T(t_k)$ that exactly matches the value of the ramp signal starting at time t_{k-2} with value $\hat{T}(t_{k-2})$. If the second derivative with respect to time of the reference torque $\hat{T}(t)$ is bounded by M'', then the estimation error is upper bounded by

$$|T(t_k)-\hat{T}(t_k)|\le 4M''\tau^2\ . \tag{30}$$

In the IMC loop, the discontinuities due to a non–ramp torque signal are added to those due to the variations of $\tilde{p}(h)$ introduced by the equivalence ratio modulation. Hence, non–ramp reference signals do not qualitatively change the closed–loop behavior.

R.3: hybrid plant model and generic reference torque. Consider now the case where the hybrid feedback control (28–29) is applied to the hybrid engine model in Sec. 2. Let $\tau_d(k)$ denote the difference between the time $\tau(k)$ up to the next dead center from the current dead center, and its estimated value $\tilde{\tau}(k)$, i.e. $\tau_d(k)=\tau(k)-\tilde{\tau}(k)$. By (10) and (22), $T(k+2)=G[\tilde{p}(k+1)+m_{\tilde{p}}(k+1)\tau_d(k+1)]\gamma(k)$. Assuming that feedback (12) set $\gamma(k)=\frac{\hat{T}(k+2)}{Gc_p\tilde{p}(k+1)}$, i.e. the control has locked the reference signal $\hat{T}(k)$, we have

$$\begin{aligned} T(k+2) &= \hat{T}(k+2)+m_{\hat{T}}(k)\tau_d(k+2)+ \\ &\quad \left(m_{\hat{T}}(k)+G\frac{m_p(k+1)}{\tilde{p}(k+1)}\right)\tau_d(k+1)\ . \end{aligned}$$

Hence, the generated torque is affected by two disturbances caused by the error $\tau_d(k)$: the first is due to manifold pressure estimation errors, the second is due to torque signal interpolation[9]. Further, $l(k)$ is affected by the manifold pressure measurement time error only:

$$l(k+2)=l(k+1)(1-Q)+m_{\tilde{p}}(k+1)(1-\gamma(k))\tau_d(k)$$

Being $1-\gamma(k)$ and $m_{\tilde{p}}(k+1)$ bounded, for any $Q\in(0,1)$, $\lim_{k\to\infty}|l(k)|<L$, with L proportional to $\frac{\max|\tau_d(k)|}{1-Q}$.

5 Simulations

The tracking performances achievable by the hybrid feedbacks (28–29) are illustrated in this section. As the simulation results depicted in Figure 3 demonstrate, a

[9] Recall that, the error on $\hat{T}(k+2)$ due to the tangent approximation of the reference torque is upper bounded as in (30).

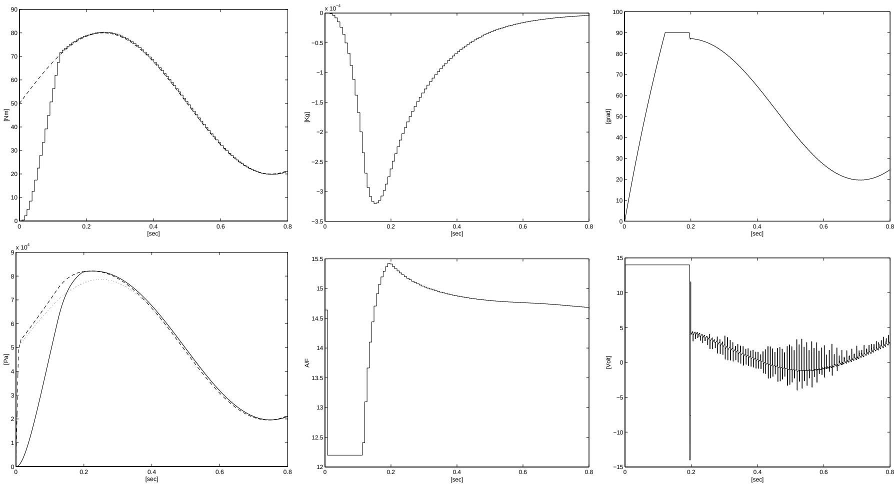

Figure 3: Evolution of the closed–loop hybrid engine model with a sinusoidal reference torque. On the top: produced torque (solid) and reference torque (dashed); catalytic converter state $l(k)$; throttle valve angle $\alpha(t)$. On the bottom: desired pressure $\tilde{p}$ (dashed), intake manifold pressure $p(t)$ (solid), pressure for stoichiometric mix (dotted); equivalence ratio $\gamma(k)$; throttle–motor voltage $v(t)$.

satisfactory tracking behavior is achieved by the proposed control. A sinusoidal reference torque is applied. The control action is composed of two subsequent parts. First, overloading the catalytic converter, the TGC attempts to reach the reference torque by using an equivalence ratio γ greater than the stoichiometric value 1. Then, keeping the torque locked on the reference signal, the control balances the catalytic converter by driving the manifold pressure to values that both guarantee torque tracking and, at the same time, produce lean mixture, until the catalytic converter state is steered to 0. At each dead center, a new reference signal is provided to the IMC. The errors on the estimation of the next dead center time produce a noise on the reference manifold pressure signal $\tilde{p}(h)$. To reduce the negative effects of such noise in the intake manifold inner loop, signal $\tilde{p}(h)$ is filtered through an one–order low–pass filter. The discontinuities of $\tilde{p}(h)$ produced at each dead center appear also in the input signal $v(h)$. Such discontinuities decrease when the catalytic converter state converges to 0 because, in this case, $\tilde{p}(h)$ depend only on the torque reference signal that is smooth.

Conclusions

The design of a torque–tracking controller for a spark ignition engine equipped with an electronic–throttle valve, has been addressed using a hybrid model for the engine. The control strategy distributes the control actions between fuel injection and air loading achieving both torque tracking and proper catalytic converter management. The robustness of the solution has been discussed and simulation results have been shown.

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