

Longitudinal Control Algorithm for Automated Vehicle Merging

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Abstract: This paper considers the theoretical part for longitudinal control of merging maneuver for Automated Highway System. The longitudinal control problem for merging is proposed for different road layouts. Then a unified mathematical model is established and a new concept *virtual platooning* is proposed, which effectively avoids a two point boundary value problem. Based on it, a rather general adaptive algorithm is provided and proved. This algorithm has been implemented and tested with automated cars.

1 Introduction

Nowadays, heavy traffic problem seems to become more and more prominent in highway driving. Automated highway systems (AHS) will be a promising alternative to improve traffic throughput and safety [7, 2, 9]. Efficient and reliable merging maneuver in traffic management and control for AHS will definitely increase traffic flow efficiency and reduce traffic congestion and accident.

This paper presents a newly developed algorithm with theoretical analysis for general vehicle merging in AHS from a control engineering point of view which is different from that adopted in [6]. The crux of the algorithm is to generate a reference trajectory for the merging vehicle based on the position and speed of the vehicles of a platoon in the main lane.

For safety, a deterministic approach is adopted in the merging algorithm. From this viewpoint, the problem of one vehicle merging with a platoon of vehicles in the main lane can always be abstracted as the entrance of the merging vehicle between two pre-fixed vehicles in the platoon in the main lane. The merging point is fixed as the intersection of the main lane and the merging lane. Several points need to be made clear.

(1) The choice of the two relevant vehicles is determined by a roadside manager (coordination layer) [9]. After making the decision, the roadside manager passes the rest of the merging task to the relevant merging vehicles (regulation layer).

(2) A separate algorithm is used for a splitting maneuver of the two relevant vehicles and those following them in the platoon such that there is a proper gap for the merging vehicle to enter when the merging vehicle reach the merging point [1, 3, 4, 8].

(3) This algorithm is actually a trajectory planning for

the merging vehicle, which determines the desired speed for the merging vehicle such that once it arrives at the merging point, a platoon of $n + 1$ vehicles is formed.

There are two major difficulties in trajectory planning for merging vehicle. One is the differences in road layouts. The other is the main lane vehicle speed variation. To cope with different road layouts, a new concept, i.e. *virtual platooning*, is introduced, which essentially shifts the time instant for platoon forming forward before real merging starts. This gives the merging vehicle more flexibility to adjust its speed and acceleration to $v_p(t)$ and $a_p(t)$, which are speed and acceleration of the leader vehicle in the main lane, as well as the relative distance. This idea effectively avoids a two point boundary value problem mathematically. This is one highlight of the algorithm.

To overcome the difficulty caused by $v_p(t)$ changing, a *closed-loop adaptive merging algorithm* is proposed. Essentially, the reference speed of the merging vehicle changes adaptively according to the speed of the leader vehicle (the first of the pre-fixed two vehicles). Meanwhile it takes into account the distance requirement for merging. This is the second highlight of the algorithm.

The third highlight is that the generated reference speed trajectory $v_{md}(t)$ has the same smoothness property as that of $v_p(t)$ — the leader vehicle speed in the main lane, which is important for real time implementation and safety.

This algorithm has been successfully implemented and tested using automated cars on test tracks with fixed merging point, which is always the case for magent guided steering control.

2 Control Problem Formulation

The following are notations and terminologies used throughout this paper:

- P_i , $i = 1, 3$ — vehicle ID in main lane
- P_2 — vehicle ID in merging lane
- $v(t)$ — merging vehicle speed, measurable
- $v_{md}(t)$ — desired speed of merging vehicle, to be determined
- $v_p(t)$ — platoon speed on main lane, measurable
- $x(t), x_{md}(t)$ — relative distance and desired relative distances between relevant vehicles
- t_{10} — time instant for P_1 to pass starting point on main lane

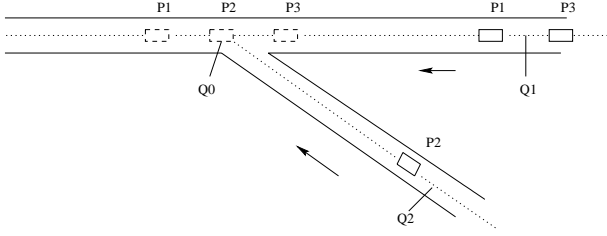


Figure 1: Road layout A

t_{20} – time instant for P_2 to pass starting point on merging lane

$t_{merg} = \max(t_{10}, t_{20})$ – time instant for merging maneuver to start

T_{merg} – time instant when practical merging completed and real platoon formed

T_{virt} – time instant when a virtual platoon formed but merging not starting

Q_1 – a point in main lane marked by magnets using special coding

Q_2 – a point in merging lane marked by magnets using special coding

Q_0 – crossing (merging) point of main lane and merging lane

$|Q_1Q_0|$ – the distance between Q_1 and Q_0 .

Q_{start_1}, Q_{start_2} – vehicle position on main lane and merging lane at $t = t_{merg}$

Q_{virt_1}, Q_{virt_2} – virtual position for P_1 and P_2 when virtual platoon is formed

l_0 – desired distance between the pre-fixed two vehicles for the merging vehicle to enter

l_{des_follow} – desired distance between consecutive vehicles in the platoon after merging

l_i – length of vehicle P_i , $i = 1, 2, 3$.

It is noted that merging maneuver starts means that the merging algorithm starts to activate, while merging starts means the real merging.

2.1 Geometric Layouts of the Road

Two different geometric road layouts which represent all possible practical cases lead to slightly different problem formulation.

In some drive way, there is no parallel lane for the merging vehicle to adjust its distance and speed. For convenience, it is called *layout A* as in Fig. 1. In this case, Q_1, Q_2, Q_0 are marked by specially coded magnets. At the time instant P_1 or P_2 passing them, magnetometer will acknowledge the vehicle, which determines the time instant and the vehicle position with respect to the merging point Q_0 . Once both P_1 and P_2 have passed Q_1 and Q_2 respectively, merging maneuver starts. Here the difficulty is that the merging point Q_0 is fixed. The merging vehicle has to arrive at this point at a right time instant and at proper speed and acceleration for a merging to be fulfilled safely and successfully.

In freeway, there is usually a short lane to help the merging vehicle to speed up. This can be abstracted as a parallel section between the main lane and the merging lane, which is called *layout B* as in Fig. 2.

Similarly, Q_1 and Q_{10} are marked by specially coded magnets on main lane and Q_2, Q_{10} and Q_{20} can be marked on merging lane. This road layout has more flexibility because merging can actually be carried out at any point between Q_{10} and Q_{20} in a much longer time period compared to the previous road layout.

Although the merging problems for these two different road layouts seem different, from a control design viewpoint, an algorithm suitable for road layout A will automatically apply to layout B.

Here $v_{md}(t)$ should be determined by the speed $v_p(t)$ of the first vehicle of the two in the main lane as well as relative positions of the relevant vehicles at the time instant when merging starts. This following mathematical model gives the conditions (equations and initial conditions) that v_{md} should satisfy. The key step here is to determine the merging time instant T_{merg} from which $v_{md}(t)$ is determined.

For convenience, one can assume that

$$|Q_{start_1}Q_0| - |Q_{start_2}Q_0| + l_1 + l_{des_follow} > 0$$

which can always be satisfied by proper arrangement of specially coded magnets at Q_{start_1} and Q_{start_2} . The control problem is formulated as follows.

Longitudinal Control Problem for Merging: Design a reference trajectory for merging vehicle P_2 such that

(1)

$$\begin{aligned} v_{md}(t_{merg}) &= v(t_{merg}) \\ v_{md}(T_{merg}) &= v_p(T_{merg}) \\ a_{md}(T_{merg}) &= a_p(T_{merg}) \end{aligned}$$

(2) three vehicle P_1, P_2, P_3 form a platoon of speed $v_p(t)$ for $t \geq T_{merg}$

(3) the relative distance between two consecutive vehicles is l_{des_follow} as prescribed

The following conditions are assumed:

(a) the platoon of vehicles P_1 and P_3 with fixed distance is formed and maintained by a separate controller;

(b) v_p is measured and passed to P_2 and P_3 by communication in real-time;

(c) the distance $|Q_1Q_0|, |Q_2Q_0|$ ($|Q_2Q_{01}|$ and $|Q_2Q_{02}|$ for road layout B) are known in advance;

(d) the time instant t_{01} when P_1 passing Q_1 can be measured and all are available for P_2 by communication. The time instant t_{02} when P_2 passing Q_2 is measured by P_2 itself. Thus virtual merging maneuver starting time instant t_{merg} and position Q_{start_1}, Q_{start_2} can be calculated on P_2 .

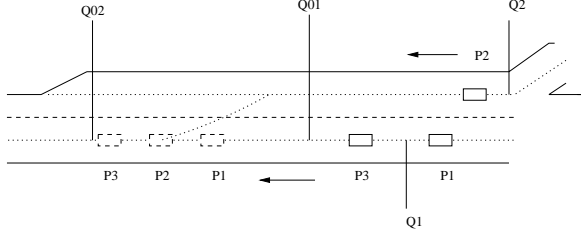


Figure 2: Road layout B

3 Mathematical Modelling for Merging

3.1 Road Layout A

As in Fig. 1, there is no parallel section between the main lane and the merging lane. T_{merg} is determined by the following rule

$$\int_{t_{merg}}^{T_{merg}} v_p(t)dt = |Q_{start_1}Q_0| + l_1 + l_{des_follow} = L_1$$

v_{md} is then determined by

$$\int_{t_{merg}}^{T_{merg}} v_{md}(t)dt = |Q_{start_2}Q_0|$$

The modelling problem is first discussed for a special case when $v_p(t)$ is constant and then for the general case.

3.1.1 Main Lane Vehicle Speed Fixed

In this case, the platoon speed is constant. i.e. $v_p(t) = const$ and T_{merg} is thus known before merging maneuver starts.

$$T_{merg} - t_{merg} = L_1/v_p$$

The trajectory planing problem becomes: to find a reference trajectory $v_{md}(t)$ such that

$$\begin{aligned} v_{md}(t_{merg}) &= v(t_{merg}) \\ \int_{t_{merg}}^{t_{merg}+L_1/v_p} v_{md}(t)dt &= |Q_{start_2}Q_0| \quad (3.1) \\ v_{md}(t_{merg} + L_1/v_p) &= v_p \\ a_{md}(t_{merg} + L_1/v_p) &= 0 \end{aligned}$$

The first equation are initial condition. The second is the merging requirement for distance compatibility. The last two are the platoon forming requirement. It is clear that if one of the following conditions

$$\begin{aligned} v_p(t_0) &= v_m(t_0) \\ L_1 &= v_p(t_0)(T_{merg} - t_{merg}) \end{aligned}$$

is violated, there is no trivial solution $v_{md}(t) = const$.

3.1.2 Main Lane Vehicle Speed Changing

If $v_p(t)$ is time varying, T_{merg} is unknown. The problem is to find a reference trajectory $v_{md}(t)$ such that

$$\begin{aligned} v_{md}(t_{merg}) &= v(t_{merg}) \\ \int_{t_{main}}^{T_{merg}} v_p(t)dt &= |Q_{start_1}Q_0| + l_1 + l_{des_follow} = L_1 \\ \int_{t_{merg}}^{T_{merg}} v_{md}(t)dt &= |Q_{start_2}Q_0| \quad (3.2) \\ v_{md}(T_{merg}) &= v_p(T_{merg}) \\ a_{md}(T_{merg}) &= a_p(T_{merg}) \end{aligned}$$

Again, the first equation is an initial condition. The second and the third conditions represent distance compatibility. From the second equation, one can determine T_{merg} implicitly. The third equation determines $v_{md}(t)$ once T_{merg} is known. The last two equations are the platoon forming requirement. Clearly, two implicitly coupled two boundary value functional equation need to be solved to get a solution. It is also expected that the solution here is not unique.

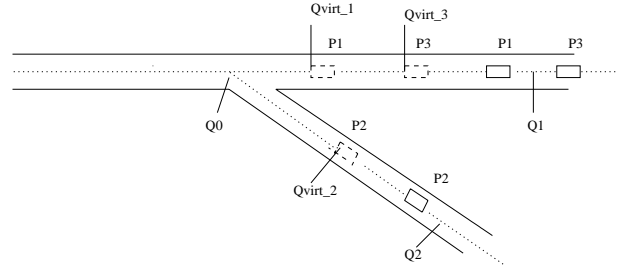


Figure 3: A unified road layout

3.2 Road Layout B

As in Fig. 2, one way to formulate the problem is this. Use the same model developed above for road layout B except that the merging point Q_0 can be any point between Q_{01} and Q_{02} . Thus (3.2) can still be used for this road layout. It is noted that, the flexibility in choosing Q_0 greatly reduces the difficulty in solving the problem. This point will become clear in next section.

3.3 Unified Modeling for Two Road Layouts

For road layout A, why should one form a platoon only after merging? There is no reason one has to do so. The idea is that, before P_1 and P_2 arrive at the merging point, a *virtual platoon* can be formed.

A virtual platoon means that at some time instant T_{virt}

$$t_{merg} \leq T_{virt} \leq T_{merg}$$

P_i arrives at Q_{virt_i} as shown in Fig. 3. At this point, the following conditions are satisfied:

$$\begin{aligned} v_{md}(T_{virt}) &= v_p(T_{virt}) \\ a_{md}(T_{virt}) &= a_p(T_{virt}) \\ |Q_{virt_1}Q_0| + l_1 + l_{des_follow} &= |Q_{virt_2}Q_0| \end{aligned}$$

Then for $t \in [T_{virt}, T_{merg}]$, it is sufficient that

$$\begin{aligned} v_{md}(t) &= v_p(t) \\ a_{md}(t) &= a_p(t) \end{aligned}$$

for a real platoon to be formed at the time instant when P_2 arrives at Q_0 . It is noted that, as for road layout A, the concept of virtual platoon before merging brings much more flexibility in finding a solution in theoretical analysis. It also greatly increases safety by avoiding the last second jump. In this way, the longitudinal control problem of merging for these two seemingly quite different road layouts can be unified. The condition corresponding to (3.2) can be stated as follows:

There exist time instant T_{virt} and point Q_{virt_1} between Q_1 and Q_0 (Q_{01} for road layout B) and Q_{virt_2} between Q_2 and Q_0 (Q_{02} for road layout B) such that

$$\begin{aligned} v_{md}(t_{merg}) &= v(t_{merg}) \\ \int_{t_{merg}}^{T_{virt}} v_p(t) dt &= |Q_{start_1}Q_{virt}| \\ \int_{t_{merg}}^{T_{virt}} v_{md}(t) dt &= |Q_{start_2}Q_{virt}| \end{aligned} \quad (3.3)$$

$$\begin{aligned} |Q_{virt_1}Q_0| + (l_1 + l_{des_follow}) &= |Q_{virt_2}Q_0| \\ v_{md}(T_{virt}) &= v_p(T_{virt}) \\ a_{md}(T_{virt}) &= a_p(T_{virt}) \end{aligned}$$

Remark 1 The longitudinal control problem for vehicle safe merging is eventually a distance and speed control problem. It is noted that these conditions are necessary and sufficient for the merging problem. In fact, any trajectory planning for automated vehicle merging should satisfy conditions (3.3) for safety. Mathematically, if any of these condition is not satisfied, the problem is not trivial.

Next section will present some adaptive solutions of the problem, which are particularly suitable for real-time implementation.

4 Adaptive Solutions

This is a speed based closed-loop adaptive method. It uses a variable structure approach to design a reference speed $v_{md}(t)$ for the merging vehicle, which has the same smoothness property as that of $v_p(t)$. The smoothness of $v_{md}(t)$ is important in practical implementation because non-smooth of $v_{md}(t)$ will cause jumps in the calculation of $\frac{d}{dt}(v_{md}(t))$ in control design, which might cause unease

of passengers and real-time instability when fed into the longitudinal controller.

Theorem Suppose that

(1) time response of the known longitudinal controller is fast enough and speed and distance tracking error is small enough, i.e.

$$\begin{aligned} a(t) &\approx a_{md}(t) \\ v(t) &\approx v_{md}(t) \\ x(t) &\approx x_{md}(t) \end{aligned}$$

for $t \in [t_{merg}, T_{merg}]$;

(2) $|Q_1Q_0|$ and $|Q_2Q_0|$ are long enough, or equivalently, the time interval $[t_{merg}, T_{merg}]$ is long enough to adjust the speed and distance of the merging vehicle;

(3) The positions of the specially coded magnets and vehicle length satisfy the following condition

$$dist_para = |Q_{start_1}Q_0| - |Q_{start_2}Q_0| + l_1 + l_{des_follow} > 0;$$

(4) There exists $\delta > 0$ such that

$$v_p(t) > v(t_{merg}) + \delta, \quad t \in [t_{merg} + \eta, T_{merg}]$$

where $0 < \eta \ll 1$ This means that there exists a constant $\epsilon > 0$ such that

$$\int_t^{t+\epsilon} (v_p(t) - v(t_{merg})) dt \geq \epsilon\delta$$

as long as $[t, t + \epsilon] \subset [t_{merg} + \eta, T_{merg}]$.

(5) The following reference speed for P_2 is fed into the longitudinal controller

$$\begin{aligned} v_{md}(t) &= \begin{cases} (1 - \alpha(t))v(t_{merg}) + \alpha(t)v_p(t), & t_{merg} \leq t \leq T_{virt} \\ v_p(t), & T_{virt} < t \leq T_{merg} \end{cases} \\ \alpha_0(t) &= \frac{\int_{t_{merg}}^t v_p(s) ds}{\int_{t_{merg}}^t v(s) ds + dist_para} \\ \alpha(t) &= \alpha_0^\beta(t), \quad \beta > 0 \end{aligned}$$

Then virtual platoon is guaranteed to be formed for some $\beta > 0$. i.e. there exist a constant $\beta > 0$, a time instant $T_{virt} \in [t_{merg}, T_{merg}]$ and points Q_{virt_1} and Q_{virt_2} such that

$$|Q_{virt_i}Q_0| \leq |Q_iQ_0|, \quad i = 1, 2$$

and all the conditions in (3.3) are satisfied. Besides, $v_{md}(t)$ has the same smoothness property as $v_p(t)$.

Remark 2 Before proving the theorem, physical meaning of the algorithm and the conditions in the theorem are briefly explained as follows.

(a) $\alpha(t)$ is time variant, which is closely related to the speed of merging vehicle and that of the platoon. The reference trajectory can be divided into two phases according to time:

Phase 1. $t_{merg} \leq t \leq T_{virt}$, this is the essential part of this trajectory planning. The purpose is to adjust both speed $v(t)$ and distance $|Q_{virt_1}Q_0|$ such that the

formation of the virtual platoon actually completed at the end of this phase.

Phase 2. $T_{virt} < t \leq T_{merg}$, a virtual platoon keeping control. As long as the merging vehicle follows the speed and acceleration of the platoon in main lane, real merging will be guaranteed at time instant T_{merg} .

(b) The assumptions in the theorem are basically the physical constraints which can be set as required in practice. Condition (1) depends on the controller adopted. Conditions (2) and (3) can be implemented by proper infrastructure of magnet marking in both merging lane and main lane. Condition (4) can be weakened as that, for most of the time, the overall platoon speed during merging is higher than the speed of the merging vehicle after the time instant t_{merg} , which is obviously true in freeway traffic in general or can be made true.

Proof of Theorem. Clearly $\alpha(t_{merg}) = 0$ and so $v_{md}(t_{merg}) = v(t_{merg})$. Thus the first condition in (3.3) is satisfied.

It is sufficient to prove that T_{virt} and Q_{virt} exist such that the rest of the conditions in (3.3) are satisfied. Equivalently, it is necessary and sufficient to prove that there exists a time instant $T_{virt} \in [t_{merg}, T_{merg}]$ such that

$$\alpha(T_{virt}) = 1$$

or equivalently

$$\alpha_0(T_{virt}) = 1$$

In fact, $\alpha_0(T_{virt}) = 1$ if and only if

$$\int_{t_{merg}}^{T_{virt}} v_p(s) ds = \int_{t_{merg}}^{T_{virt}} v(s) ds + dist_para$$

Or equivalently

$$\begin{aligned} & |Q_{start_1} Q_{vert_1}| - |Q_{start_1} Q_0| - (l_1 + l_{des_follow}) \\ &= |Q_{start_2} Q_{vert_2}| - |Q_{start_2} Q_0| \end{aligned}$$

i.e.

$$|Q_{virt_1} Q_0| + l_1 + l_{des_follow} = |Q_{virt_2} Q_0|$$

This is exactly the distance compatibility condition, the 4th in (3.3).

The 5th condition in (3.3) is clearly satisfied. To prove the 6th condition in (3.3), i.e. the continuous of acceleration, is satisfied, differentiae $v_{md}(t)$ to obtain

$$\dot{v}_{md}(t) = -\dot{\alpha}(t) v(t_{merg}) + \dot{\alpha}(t) v_p(t) + \alpha(t) a_p(t)$$

Because

$$\alpha(T_{virt}) = 1.$$

It is sufficient to prove that

$$[-\dot{\alpha}(t) v(t_{merg}) + \dot{\alpha}(t) v_p(t)]_{t=T_{virt}} = 0$$

or equivalently

$$\dot{\alpha}(t) \Big|_{t=T_{virt}} = 0$$

because

$$v_p(t) - v(t_{merg}) \geq \delta$$

by assumption. Now

$$\dot{\alpha}(t) = \alpha_0^{\beta-1}(t) \dot{\alpha}_0(t)$$

and

$$\begin{aligned} \dot{\alpha}_0(t) \Big|_{t=T_{virt}} &= \frac{v_p(t) \left(\int_{t_{merg}}^t v(s) ds + dist_para \right) - v(t) \int_{t_{merg}}^t v_p(s) ds}{\left(\int_{t_{merg}}^t v(s) ds + dist_para \right)^2} \Big|_{t=T_{virt}} \\ &\approx \frac{v_p(t) \left[\int_{t_{merg}}^t v(s) ds + dist_para - \int_{t_{merg}}^t v_p(s) ds \right]}{\left(\int_{t_{merg}}^t v(s) ds + dist_para \right)^2} \Big|_{t=T_{virt}} = 0 \end{aligned}$$

because

$$v(T_{virt}) \approx v_{md}(T_{virt}) = v_p(T_{virt})$$

$$\alpha_0(T_{virt}) = 1.$$

Now it is to prove the existence of T_{virt} such that $\alpha(T_{virt}) = 1$ by contradiction. Suppose that such a T_{virt} does not exist in $[t_{merg}, T_{merg}]$.

First, it is noted that the following facts are true:

(a) $\alpha_0(t_{merg}) = 0$.

(b) $\alpha_0(t)$ is continuous for $t \in [t_{merg}, T_{merg}]$ because $\int_{t_{merg}}^t v_p(s) ds$ and $\int_{t_{merg}}^t v(s) ds$ are continuous.

Condition (3) implies that $0 \leq \alpha_0(t)$. Thus if there exists $t_1 \in [t_{merg}, T_{merg}]$ such that $\alpha_0(t_1) > 1$, then there exists $T_{virt} \in [t_{merg}, T_{merg}]$ such that $\alpha_0(T_{virt}) = 1$ because $\alpha_0(t)$ is continuous. By assumption, it must be true that $\alpha_0(t) < 1$, $t \in [t_{merg}, T_{merg}]$. Then there exists a $\mu > 0$ such that $\alpha_0(t) \leq 1 - \mu$ because $\alpha_0(t)$ is continuous and $[t_{merg}, T_{merg}]$ is a closed interval. Thus for $\varepsilon > 0$ sufficiently small, there exists β such that

$$\alpha(t) = \alpha_0^\beta(t) \leq \varepsilon$$

Now

$$v_p(t) - v(t_{merg}) \geq \delta > 0, \quad t \in [t_{merg} + \eta, T_{virt}]$$

Because $\eta > 0$ is small, it can be ignored for convenience without loss of generality.

$$\begin{aligned} v_{md}(t) &= (1 - \alpha(t)) v(t_{merg}) + \alpha(t) v_p(t) \\ &= v(t_{merg}) + \alpha(t) (v_p(t) - v(t_{merg})) \\ &\leq v(t_{merg}) + \varepsilon v_p(t) \end{aligned}$$

Compare the numerator and the denominator of $\alpha_0(t)$ as follows.

$$\begin{aligned} \int_{t_{merg}}^t v(s) ds &\approx \int_{t_{merg}}^t v_{md}(s) ds \\ &\leq \int_{t_{merg}}^t (v(t_{merg}) + \varepsilon v_p(s)) ds \end{aligned}$$

$$0 \leq \int_{t_{merg}}^t v(s) ds + dist_para$$

$$\leq \int_{t_{merg}}^t (v(t_{merg}) + \varepsilon v_p(s)) ds$$

The difference between the denominator and the numerator of $\alpha_0(t)$ is

$$\begin{aligned} & \int_{t_{merg}}^t (v(t_{merg}) + \varepsilon v_p(s)) ds + dist_para - \int_{t_{merg}}^t v_p(s) ds \\ & \int_{t_{merg}}^t (v(t_{merg}) + (\varepsilon - 1)v_p(s)) ds + dist_para \\ \leq & \int_{t_{merg}}^t (v(t_{merg}) + (\varepsilon - 1)(v(t_{merg}) + \delta)) ds + dist_para \\ & = \int_{t_{merg}}^t (\varepsilon v(t_{merg}) + (\varepsilon - 1)\delta) ds + dist_para \\ & = (\varepsilon [v(t_{merg}) + \delta] - \delta)(t - t_{merg}) + dist_para \end{aligned}$$

Thus when ε is small enough and $t - t_{merg}$ is large enough, it is negative. This is a contradiction. i.e. if T_{merg} is large enough, $\alpha_0(T_{merg}) < 1$ is violated. This contradiction implies the existence of T_{merg} such that $\alpha_0(T_{merg}) = 1$.

Furthermore, if $v_p(t)$ is continuously differentiable n times, then $\alpha(t)$ is continuously differentiable $n + 1$ times. Thus $v_{md}(t)$ and $v_p(t)$ have the same smoothness property.

This completes the proof. \diamond

Remark 3 It is noted that the adaptive solution above is not unique. One may ask the question: is it possible to find an optimal reference trajectory for merging vehicle with respect to some performance index? The answer is not obvious. The difficulty here is that optimization will lead to a non-trivial two point boundary value problem which certainly brings a great difficulty in finding a solution or in the implementation even if a solution can be proved to exist. However, one can expect to choose β properly to reduce the demanding for vehicle acceleration. This will be addressed in future work.

5 Concluding Remarks

This paper considers a general longitudinal control problem for vehicle safe merging. First, a longitudinal control problem is raised based on performance requirements. Second, a mathematical model is proposed, which characterizes the essence of merging problem. Any safe and smooth automated merging algorithm should satisfy the model. Then a general adaptive closed-loop algorithms is given as the solution of the problem, which is suitable for different road layouts. To unify merging algorithm for different road layouts, a concept of virtual merging is proposed, which is essentially that a virtual platoon is to be formed before merging vehicle arrives at the real merging point. The idea of virtual platooning also effectively avoids a two point boundary value problem. The reference trajectory generated for the merging vehicle based on the speed of the leading vehicle in the main lane has the same smoothness property as that of the platoon speed. This algorithm has been successfully implemented and tested using automated cars with magnetometer based speed and distance control for speed between $21.0km/h \sim 56.4km/h$ [5].

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