Output Trajectory Tracking Using Dynamic Neural Networks

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Abstract

In this paper concerns the development of a robust asymptotic neuro observer (NN) for a class of unknown nonlinear systems with noise disturbances in the output. Son an output trajectory tracking from the estimated states is studied. The suggested asymptotic observer has three basic terms: the first one is introduced to approximate the unknown nonlinear dynamics, the second one is related with the innovation and the last one is a time delayed term introduced especially to assure the approximation of the unmeasured states derivatives. The Lyapunov-Krasovskii technique is used to proof the robust asymptotic stability "on average" of the obtained estimation error. A special "dead-zone" multiplier is introduced into the learning procedure to guarantee the boundness of the weight matrices of the dynamic NN.

Key words: dynamic neural network, robust neuro observer, time-delay, Dynamic neuro control, Krasovskii functional.

1 Introduction

The state observation and the control problems are one of the essential points in the modern control theory. Its solution for linear systems is well-known as the Kalman filter (in the presence of stochastic Gaussian noise) and the Luenberger observer (for the noises of the deterministic nature) [13]. Many papers have been published devoted to the theoretical analysis and practical implementations of nonlinear observers [7], [14], [21], [16] and [3]. Based on linearization techniques, extended Luenberger observers for nonlinear system were studied in [24]. Nonlinear observers analysis and synthesis, using Lie-algebra approach and Lyapunov methods, can be found in [7] and [21]. Sliding-mode observers for linear systems were studied in [20]. These techniques require *the exact knowledge of the nonlinear dynamics*, that implies that external disturbances are not allowed. For plants with unknown parameters the nonlinear adaptive observer was proposed in [14]. When we have *no complete modelling information*, a model-free nonlinear observer is required. If the nonlinear system is given in the normal linearized form, the high-gain observers may estimate the derivative of the output [16], [22], [23], [1] and [5] . They do their job well even if the plant is being considered as "a black box", but such observers loose their capability in presence of output unmeasured disturbances.

Neural networks can be considered as an alternative approach to high-gain method since they offer potential benefits for nonlinear modeling and control of a wide class of unknown system [15]. Based on function approximation theory via multilayer networks [6], control engineers can identify nonlinear systems using different techniques, such as off-line [15], on-line [12], static networks [12] and dynamic networks identifications [11] [19]. Neurocontrol, based on the neuro identifiers, turns out to be very useful for designing model-free controllers. Nonlinear adaptive control [2] [17], and feedback linearization [27] can be implemented by means of multilayer feedforward networks. Dynamic neural networks were also applied to design a Luenberger-like observer [10]. The stability of this observer with online updating of neural network weights was analyzed, but several restrictive assumptions were required: the nonlinear plant should contain a known linear part and verify a strictly positive real (SPR) condition to proof observation error stability. In this paper, following the ideas given in [18], the unknown nonlinear systems with both external disturbances and unmodelled dynamics is considered.

2 Nonlinear System and Neuro Observer

Consider the nonlinear systems given by

$$\dot{x}_t = f(x_t, u_t, t) + \xi_{1,t},\ y_t = Cx_t + \xi_{2,t} \qquad (1)$$

where $x_t \in \mathbb{R}^n$ is the state vector of the system, $u_t \in \mathbb{R}^q$ is a given control action, $y_t \in \mathbb{R}^m$ is the output vector measurable at each time t, $C \in \mathbb{R}^{m\times n}$ is a *known* output matrix; $f(\cdot) : \mathbb{R}^{n+q+1} \to \mathbb{R}^n$ is *unknown* vector valued nonlinear function describing the system dynamics and satisfying the following assumption

A1: *For a realisable bounded feedback control $\|u_t(x_t)\| \le \overline{u}$, the nominal (unperturbed) closed-loop nonlinear system is* ***quadratically stable****, that is, there exists a Lyapunov (may be, unknown) function $\overline{V}_t \ge 0$ and $\lambda_1\ \lambda_2 > 0$ satisfying*

$$\frac{\partial \overline{V}_t}{\partial x} f(x_t, u_t) \le -\lambda_1 \|x_t\|^2, \quad \left\|\frac{\partial \overline{V}_t}{\partial x}\right\| \le \lambda_2 \|x_t\|$$

Remark 1 *If a closed-loop system is exponentially stable and $f(x_t, u_t)$ is Lipshitzian, a converse Lyapunov theorem [8] implies* ***A1****. But this assumption is weaker and easy to be satisfied.*

The vectors $\xi_{1,t}$ and $\xi_{2,t}$ represent external unmeasured bounded disturbances.

A2: $\|\xi_{i,t}\|^2_{\Lambda_{\xi_i}} = \Upsilon_i < \infty,\ 0 < \Lambda_{\xi_i} = \Lambda^T_{\xi_i},\ i = 1, 2$

The normalizing matrices Λ_{ξ_i}, introduced to insure the possibility to work with components of different physical nature, are assumed to be a priori given.

If the nonlinear system (without unmodelled dynamics and external diturbances) model is known, then following to standard techniques [13],the folllowing structure for the corresponding nonlinear observer can be suggested:

$$\frac{d}{dt}\hat{x}_t = f(\hat{x}_t, u_t, t) + L_{1,t}[y_t - C\hat{x}_t] \tag{2}$$

The first term in the right-hand side of (2) repeats the known dynamics of the nonlinear system and the second one is intended to correct the estimated trajectory based on current residual values (the innovation term).

If $L_{1,t} = L_{1,t}(\hat{x}_t)$, this observer is named a "differential algebra" type observer (see [7], [14], and [3]). In the case of $L_{1,t} = L_1$ =constant, it is named as "high-gain" type observer, which was studied in [16], [22].

For applying this observer to a class of mechanical systems when only position measurements are available (velocities are unmeasured), as a rule, the corresponding velocity estimates turn out to be not so good because of the following:

1) The original dynamic mechanical system, in general, is given as

$$\ddot{z}_t = F(z_t, \dot{z}_t, u_t, t),\ y = z_t$$

or, in equivalent standard Cauchy form,

$$\dot{x}_{1,t} = x_{2,t}, \dot{x}_{2,t} = \tilde{F}(x_t, u_t, t), y_t = x_{1,t}$$

So, the corresponding nonlinear observer (2) has the form

$$\begin{pmatrix} \dot{\hat{x}}_{1,t} \\ \dot{\hat{x}}_{2,t} \end{pmatrix} = \begin{pmatrix} \hat{x}_{2,t} \\ \tilde{F}(\hat{x}_t, u_t, t) \end{pmatrix} + \begin{pmatrix} L_{1,1,t} \\ L_{1,2,t} \end{pmatrix} [y_t - \hat{x}_{1,t}] \tag{3}$$

It means that observable state components are estimated very well such that the residual term $[y_t - \hat{x}_{1,t}]$ turns out to be small and has no effect in (3). One of the possible solutions of this problem is to add the new delayed time term

$$L_{2,t}\left[h^{-1}(y_t - y_{t-h}) - Ch^{-1}(\hat{x}_t - \hat{x}_{t-h})\right]$$

which can be considered as a "derivative estimation error" and can be used for the tuning the velocity estimation. This new modified observer can be described as

$$\begin{array}{l} d\hat{x}_t/dt = f(\hat{x}_t, u_t, t) + L_{1,t}[y_t - C\hat{x}_t] \\ +L_{2,t}h^{-1}[(y_t - y_{t-h}) - C(\hat{x}_t - \hat{x}_{t-h})] \end{array}$$

2) If we have no complete information on the nonlinear function $f(\hat{x}_t, u_t, t)$, it seems to be natural to construct its estimate as $\hat{f}(\hat{x}_t, u_t, t \mid W_t)$,which depends on parameters W_t. These parameters can be adjusted on-line to obtain the best nonlinear approximation of unknown dynamic operator. This leads to the following observer scheme:

$$\begin{array}{l} \frac{d}{dt}\hat{x}_t = \hat{f}(\hat{x}_t, u_t, t \mid W_t) + L_{1,t}[y_t - C\hat{x}_t] \\ +L_{2,t}h^{-1}[(y_t - y_{t-h}) - C(\hat{x}_t - \hat{x}_{t-h})] \end{array}$$

with a special *updating* (learning) law

$$\dot{W}_t = \Phi(W_t, \hat{x}_t, u_t, t, y_t)$$

Such "robust adaptive observer" seems to be a more advanced device providing a good estimation under the absence of an exact dynamic model and incomplete state measurement.

In the next section a special observer structure, based on *Dynamic Neural Network* (see [10], [18]), will be introduced.

3 A Dynamic Neuro Observer

The robust neuro observer, considered, uses the structure of dynamic (Hopfield's type) neural networks as in [25], [11] and [18]. We propose this Luenberger-like "second order" observer, with the new additional time-delay term:

$$\begin{array}{c}\dot{\widehat{x}}_t= A\widehat{x}_t + W_{1,t}\sigma(V_{1,t}\widehat{x}_t) + W_{2,t}\phi(V_{2,t}\widehat{x}_t)u_t \\ +L_1\left[y_t-\widehat{y}_t\right] + L_2/h\left[(y_t-y_{t-h})-(\widehat{y}_t-\widehat{y}_{t-h})\right] \\ \widehat{y}_t = C\widehat{x}_t\end{array} \quad (4)$$

The vector $\widehat{x}_t \in \mathbb{R}^n$ is the state of the neural network, $u_t \in \mathbb{R}^q$ is the input. $A \in \mathbb{R}^{n\times n}$ is a Hurtwitz stable constant matrix. The matrices $W_{1,t} \in \mathbb{R}^{n\times m}$ and $W_{2,t} \in \mathbb{R}^{n\times q}$ are the weights of the output layers. $V_1 \in \mathbb{R}^{m\times n}$ and $V_2 \in \mathbb{R}^{q\times n}$ are the weights of the hidden layers. $\sigma(\cdot) \in \mathbb{R}^m$ is a sigmoidal vector function, $\phi(\cdot)$ is $\mathbb{R}^{q\times q}$ diagonal matrix, that is,

$$\phi(\cdot) = \operatorname{diag}\left[\phi_1(V_{2,t}\widehat{x}_t)_1 \cdots \phi_q(V_{2,t}\widehat{x}_t)_q\right]$$

$L_1 \in \mathbb{R}^{n\times m}$ and $L_2 \in \mathbb{R}^{n\times m}$ are first and second order gain matrices. The scalar parameter h is assumed to be positive.

Remark 2 *The most simple structure without any hidden layers (containing only input and output layers), corresponds to the case when $m = n$, $V_1 = V_2 = I$, $L_2 = 0$. This single-layer dynamic neural networks with Luenberger-like observer was considered in [10].*

Remark 3 *The structure of the observer (4) consists of three parts:*

- the neural networks identifier $A\widehat{x}_t+ W_{1,t}\sigma(V_{1,t}\widehat{x}_t)+ W_{2,t}\phi(V_{2,t}\widehat{x}_t)u_t$;

- the Luenberger tuning term $L_1\left[y_t-\widehat{y}_t\right]$;

- the additional time-delay term

$$L_2/h\left[(y_t-y_{t-h})-(\widehat{y}_t-\widehat{y}_{t-h})\right]$$

where(y_t-y_{t-h}) and $(\widehat{y}_t-\widehat{y}_{t-h})$ are introduced to estimate $\dot{y}_t$ and $\dot{\hat{y}}_t$,

correspondingly.

Define the estimation error as $\Delta_t := x_t - \widehat{x}_t$. Then, the output error is equal to $e_t = y_t - \widehat{y}_t = C\Delta_t + \xi_{2,t}$ and, hence,

$$\begin{array}{c}C^Te_t = C^T\left(C\Delta_t+\xi_{2,t}\right) \\ = \left(C^TC+\delta I\right)\Delta_t - \delta I\Delta_t + C^T\xi_{2,t} \\ \Delta_t = C_\delta^+ e_t + \delta N_\delta\Delta_t - C_\delta^+\xi_{2,t}\end{array} \quad (5)$$

where $C_\delta^+ = \left(C^TC+\delta I\right)^{-1}C^T, N_\delta = \left(C^TC+\delta I\right)^{-1}$and δ is a small positive regularazing scalar parameter. It is clear that all sigmoid functions $\sigma(\cdot)$ and $\phi(\cdot)$, commonly used in NN, satisfy the Lipschitz condition. So, it is natural to assume that

A3:

$$\begin{array}{c}\widetilde{\sigma}_t^T\Lambda_1\widetilde{\sigma}_t \le \Delta_t^T\Lambda_\sigma\Delta_t, \quad \left(\widetilde{\phi}_tu_t\right)^T\Lambda_2\left(\widetilde{\phi}_tu_t\right) \le \overline{u}^2\Delta_t^T\Lambda_\phi\Delta_t \\ \widetilde{\sigma}_t' = D_\sigma\widetilde{V}_{1,t}\widehat{x}_t + \nu_\sigma, \quad \widetilde{\phi}_t'u_t := D_\phi\widetilde{V}_{2,t}\widehat{x}_t + \nu_\phi\end{array}$$

where, based on Lemma 1 (see Appendix), the introduced variables satisfy

$$\begin{array}{c}\widetilde{\sigma}_t := \sigma(V_1^*x_t) - \sigma(V_1^*\widehat{x}_t), \widetilde{\phi}_t := \phi(V_2^*x_t) - \phi(V_2^*\widehat{x}_t) \\ \widetilde{\sigma}_t' := \sigma(V_1^*\widehat{x}_t) - \sigma(V_{1,t}\widehat{x}_t) \\ \widetilde{\phi}_t'u_t := \phi(V_2^*\widehat{x}_t)u_t - \phi(V_{2,t}\widehat{x}_t)u_t \\ D_\sigma = \frac{\partial\sigma^T(Z)}{\partial Z}\,|_{Z=V_{1,t}\widehat{x}_t}\in\mathbb{R}^{m\times m} \\ \|\nu_\sigma\|_{\Lambda_1}^2 \le l_1\left\|\widetilde{V}_{1,t}\widehat{x}_t\right\|_{\Lambda_1}^2, \quad l_1>0 \\ D_\phi = \frac{\partial(\phi_iu_i)^T(Z)}{\partial Z}\,|_{Z=V_{2,t}\widehat{x}_t}\in\mathbb{R}^{m\times m} \\ \|\nu_\phi\|_{\Lambda_2}^2 \le l_2\left\|\widetilde{V}_{2,t}\widehat{x}_t\right\|_{\Lambda_2}^2, \quad l_2>0 \\ \widetilde{V}_{1,t} := V_1^* - V_{1,t}, \quad \widetilde{V}_{2,t} := V_2^* - V_{2,t} \\ \widetilde{W}_{1,t} := W_1^* - W_{1,t}, \quad \widetilde{W}_{2,t} := W_2^* - W_{2,t}\end{array}$$

where Λ_1, Λ_2, Λ_σ and Λ_ϕ are positive define matrices.

For the general case, when the neural network

$$\dot{\widehat{x}}_t= A\widehat{x}_t + W_{1,t}\sigma(V_{1,t}\widehat{x}_t) + W_{2,t}\phi(V_{2,t}\widehat{x}_t)u_t$$

can not exactly match the given nonlinear system (1), this system can be represented as

$$\dot{x}_t= Ax_t + W_1^*\sigma(V_1^*x_t) + W_2^*\phi(V_2^*x_t)u_t + \widetilde{f}_t$$

where $\widetilde{f}_t$ is the unmodelled dynamic term and W_1^*, W_2^*, V_1^* and V_2^* are any known matrices which are selected below as the parameters of the designed differential learning law. To guarantee the global existence for the solution of (1), the following condition should be satisfied $\|f(x_t,u_t,t)\|^2 \le C_1 + C_2\|x_t\|^2$ where C_1 and C_2 are positive constants [4]. In view of this and taking into account that the sigmoid functions σ and ϕ are uniformly bounded, the following assumption, concerning the unmodeled dynamics $\widetilde{f}_t$, seems to be natural:

A4: *There exist positive constants $\overline{\eta}$ and $\overline{\eta}_1$such that*

$$\left\|\widetilde{f}_t\right\|_{\Lambda_f}^2 \le \overline{\eta} + \overline{\eta}_1\|x_t\|_{\Lambda_f}^2, \quad \Lambda_f = \Lambda_f^T > 0$$

The next fact plays a key role in this study. It is well known [26] that if the matrix A is stable, the pair $(A, R^{1/2})$ is controllable, the pair $(Q^{1/2}, A)$ is observable, and the special *local frequency condition* or its matrix equivalent

$$A^TR^{-1}A - Q \ge \frac{1}{4}\left[A^TR^{-1} - R^{-1}A\right]R\left[A^TR^{-1} - R^{-1}A\right]^T$$

is fulfilled, then the matrix Riccati equation

$$A^T P + PA + PRP + Q = 0 \tag{6}$$

has a positive solution. In view of this, we accept the following additional assumption.

A5: *There exist a stable matrix A and a positive parameter δ such that the matrix Riccati equation (6) with*

$$\begin{array}{c} R = 2\overline{W}_1 + 2\overline{W}_2 + \Lambda_f^{-1} + \Lambda_{\xi_1}^{-1} + \delta R_1 \\ Q = \Lambda_\sigma + \overline{u}^2 \Lambda_\phi, + P_1 + Q_1 - 2C^T \Lambda_\xi C \\ \overline{W}_1 := W_1^{*T} \Lambda_1^{-1} W_1^*, \quad \overline{W}_2 := W_2^{*T} \Lambda_2^{-1} W_2^* \end{array} \tag{7}$$

has a positive solution P, where Q_1 satisfies

$$\begin{array}{c} \lambda_{\max}(Q_1) \geq \lambda_{\min}(Q_1) \geq \|I - \delta N_\delta\|^2 \\ R_1 = 2N_\delta K_1^T \Lambda_\sigma^{-1} K_1 N_\delta^T + 2N_\delta K_1^T \Lambda_\phi^{-1} K_1 N_\delta^T + \\ N_\delta K_3^T \Lambda_\sigma^{-1} K_3 N_\delta^T + N_\delta K_4^T \Lambda_\phi^{-1} K_4^T N_\delta^T \end{array}$$

This conditions is easily verified if we select A as a stable diagonal matrix. Let denote $\mathcal{H}$ as the class of unknown nonlinear systems satisfying **A1-A5.**

4 Learning Algorithm and Neuro

4.1 Observer Analysis

The main contribution of this study is the new dynamic learning law which can be expressed by the following system of matrix differential equations:

$$\begin{array}{c} \dot{W}_{1,t} = -K_1 \left[s_t P C_\delta^+ e_t \left(\sigma^T + \widehat{x}_t^T \widetilde{V}_{1,t}^T D_\sigma \right) \right. \\ + s_t \delta \left(\tilde{W}_{1,t} \Lambda_\sigma^{-1} \sigma \sigma^T + \widehat{x}_t \widehat{x}_t^T \widetilde{W}_{1,t}^T D_\sigma \Lambda_\sigma^{-1} D_\sigma \right) \\ \left. + 2 \left\| P C_\delta^+ e_t \sigma^T \right\| \frac{\widetilde{W}_{1,t}}{tr\{\widetilde{W}_{1,t}^T \widetilde{W}_{1,t}\}} \right] \\ \dot{W}_{2,t} = -K_2 \left[s_t P C_\delta^+ e_t \left((\phi u_t)^T + \widehat{x}_t^T \widetilde{V}_{1,t}^T D_\phi \right) \right. \\ + s_t \delta \left(\widetilde{W}_{2,t} \Lambda_\phi^{-1} (\phi u_t)(\phi u_t)^T + \widehat{x}_t \widehat{x}_t^T \widetilde{W}_{2,t}^T D_\phi \Lambda_\phi^{-1} D_\phi \right) \\ \left. + 2 \left\| P C_\delta^+ e_t u^\intercal \phi^\intercal \right\| \frac{\widetilde{W}_{2,t}}{tr\{\widetilde{W}_{2,t}^T \widetilde{W}_{2,t}\}} \right] \\ \dot{V}_{1,t} = -K_3 \left[s_t P C_\delta^+ e_t \widehat{x}_t^T W_{1,t} D_\sigma \right. \\ + \frac{l_1}{2} s_t \Lambda_1 \widehat{x}_t \widehat{x}_t^T \widetilde{V}_{1,t} + s_t \delta \widehat{x}_t \widehat{x}_t^T \widetilde{V}_{1,t}^T D_\sigma \Lambda_\sigma^{-1} D_\sigma \\ \left. + 2 \left\| P C_\delta^+ e_t \widehat{x}_t^T W_{1,t} D_\sigma \right\| \frac{\widetilde{V}_{1,t}}{tr\{\widetilde{V}_{1,t}^T \widetilde{V}_{1,t}\}} \right] \\ \dot{V}_{2,t} = -K_4 \left[s_t P C_\delta^+ e_t \widehat{x}_t^T W_{2,t} D_\phi \right. \\ + \frac{l_2}{2} s_t \Lambda_2 \widehat{x}_t \widehat{x}_t^T \widetilde{V}_{2,t} + s_t \delta \widehat{x}_t \widehat{x}_t^T \widetilde{V}_{2,t}^T D_\phi \Lambda_\phi^{-1} D_\phi \\ \left. + 2 \left\| P C_\delta^+ e_t \widehat{x}_t^T W_{2,t} D_\phi \right\| \frac{\widetilde{V}_{2,t}}{tr\{\widetilde{V}_{2,t}^T \widetilde{V}_{2,t}\}} \right] \end{array} \tag{8}$$

where $K_i \in \mathbb{R}^{n \times n}$ $(i = 1 \cdots 4)$ are positive defined matrices, P is the solution of the matrix Riccati equation given by (6). The initial conditions are $W_{1,0} \neq W_1^*$, $W_{2,0} \neq W_2^*$, $V_{1,0} \neq V_1^*$, $V_{2,0} \neq V_2^*$. The new variable $s_\mu(z)$ given by

$$s_t := s_\mu(P^{1/2} C_\delta^+ e_t), \; s_\mu(z) = \left[1 - \frac{\mu}{\|z\|} \right]_+$$

is introduced. It represents a "cutting function" (or the "dead zone") which assures the boundness of the weights during the learning process.

Define also the parameters $\mu = \overline{\eta} / \lambda_{\min}(P^{-1/2} Q_0 P^{-1/2})$ and $\overline{d} = \overline{\eta} + \Upsilon_1 + 10\Upsilon_2$.

Remark 4 *One can see that the learning law (8) of the neuro observer (4) consists of three parts: the first term $PC_\delta^+ e_t \sigma^T$ exactly corresponds to the backpropagation scheme as in multilayer networks [12]; the second term$PC_\delta^+ e_t \widehat{x}_t^T \widetilde{V}_{1,t}^T D_\sigma$ is intended to assure the robust stable learning law. Even though the proposed learning law looks like the backpropagation algorithm, global asymptotic error stability is guaranteed because it is derived based on the Lyapunov approach (see next Theorem). So, the global convergence problem does not arise in this case.*

Theorem 1 *If the gain matrices are selected as*

$$L_1 = P^{-1} C^T \Lambda_{\xi_2}, \quad L_2 = hP^{-1} C^T \Lambda_\xi \tag{9}$$

and the weights are adjusted according to (8), then, by the assumptions ***A1-A5****, for a given class of nonlinear systems given by (1), the following properties hold:*

· *(a) the weight matrices remain bounded, that is,*

$$W_{1,t} \in L^\infty, \quad W_{2,t} \in L^\infty, \quad V_{1,t} \in L^\infty, \quad V_{2,t} \in L^\infty \tag{10}$$

· *(b) for any $T > 0$ the state estimation error fulfills the following*

$$\frac{1}{T} \int_0^T s_\mu(P^{1/2} \Delta_t) \|\Delta_t\|_{Q_1}^2 \, dt \leq \overline{d} \tag{11}$$

Remark 5 *The main purpose of the dead zone controller is to handle the modeling error $\overline{\eta}$ and disturbances Υ_1, Υ_2. The proved boundness property is global, i.e. initial conditions for the state estimation error can be selected arbitrarily.*

Remark 6 *For a system without any unmodelled dynamics, that is., neural network matches the given plant exactly ($\overline{\eta} = 0$) and without any external disturbances ($\Upsilon_1 = \Upsilon_2 = 0$), the propsed neuro-observer (4), with the matrix gain given by (9), guarantees " stability in average" of the state estimation error, i.e.*

$$\lim_{\mathcal{H}} \sup_{T \to \infty} \frac{1}{T} \int_0^T s_\mu(P^{1/2} \Delta_t) \Delta_t^T Q_1 \Delta_t dt = 0$$

that is equivalent to the fact $\lim_{t\to\infty} s_\mu(P^{1/2}\Delta_t) = 0$

The error upper bound (11) is valid for any positive value of the time delay h and the gain matrices L_1 and $L_2 h^{-1}$ (9) are independent on h.

Remark 7 *Similar to high-gain observers [22], the proved theorem stays only the fact that the estimation error is bounded asymptotically and does not say anything about a bound for a finite time that obligatory demands fulfilling a local uniform observability condition [3]. In our case, some observability properties are contained in **A5** (for example, if $C = 0$ this condition can not be fulfilled for any matrix A).*

5 TRACKING ERROR ANALYSIS

The control system behaviour expectected is to move the states to track a signal response generated by a nonlinear reference model given by $\dot{x}_m = f_m(x_m, t)$ defining the following seminorms:

$$|z|_Q^2 = \limsup_{\tau\to\infty} \frac{1}{\tau} \int_0^\tau z^T(t) Q z(t) dt$$

where $Q = Q^T > 0$, the state trajectory tracking can be formulated as:

$$J_{\min} = \min_{u(t)} J \quad J = |x_t - x_m|_{Q_c}^2 + |u_t|_{\widetilde{R}_c}^2$$

So, for any $\eta > 0$ it follows

$$J \leqslant (1+\eta) |\widehat{x}_t - x_m|_{Q_c}^2 + |u_t|_{\widetilde{R}_c}^2$$

The minimun of the term $|x_t - x_m|_{Q_c}^2$ can be solved selecting $\widetilde{R}_c = (1+\eta^{-1}) R_c$, so we can reformulate the control goal as follows: minimize the term $|\widehat{x}_t - x_m|_{Q_c}^2 + |u_t|_{\widetilde{R}_c}^2$ for this pourpose, we define the state trajectory error as: $\Delta_m = \widehat{x}_t - x_t$ and for this, we define the *energetic function* $\Psi_t(u)$ as:

$$\Psi_t(u) = \gamma^T(u) W_{2,t} P_L \Delta_m + u^T R_c u$$

where P_L is the solution of the following differential Riccati equation:

$$-\dot{P}_L = P_L A + A^T P_L + 2\Lambda_\sigma + Q$$
$$+ P_L \left(W_{1,t}^T Z^{-1} W_{1,t} + \Lambda_\xi^{-1} + W_{2,t}^T W_{2,t} \right) P_L$$

With the initial condition $P_L(0)$ equal to the positive solution of the algebraic riccati equation corresponding to (some equation) at time $t = 0$ with zero right hand side.

Proposition 1: We will select the control action $u(t)$ such a way to minimize the energetic function $\Psi_t(u)$ ay each time t i.e. $u_t^* = \arg\min_u \Psi_t(u)$

To calculate the control action $u(t)$, wich minimize $\Psi_t(u)$, we have to fulfill $(d\Psi_t(u)/du) = 0$. To perform this minimization, we assume that, at the given positive t (positive), $x_m(t)$, and $\widehat{x}(t)$ are known and do not depend on $u(t)$. We name the u_t^* as *locally optimal control* because it is calculated based on local information available at time t[28]. To solve this optimization problem. let us consider the following recursive gradient scheme[29].

$$u_k(t) = u_{k-1}(t) - \tau_k \frac{d\Psi_t(u_{k-1}(t))}{du}, \qquad u_0(t) = 0$$

where the gradient $d\Psi_t(u)/du$ is calculated as:

$$\frac{d\Psi_t(u)}{du} = 2\frac{d\gamma^T(u)}{du} W_{2,t} P_L(t) \Delta_m(t) + 2R_c u$$

and the sequence of the scalar parameter τ_k satisfies the condition: $\tau_k \geqslant 0$, $\sum_{k=0}^{\infty} \tau_k = \infty$, $\tau_k \to 0$. For example, we can select $\tau_k = (1/(1+k)^\tau)$, $\tau \in (0,1]$.

Lemma 2 *The $u^*(t)$ can be calculated as the limit of the sequence $u_k(t)$, i.e., $u_k(t) \to u^*(t)$, $k \to \infty$.*

Proof: it directly follows from the properties of gradient method [29].■

Corollary 1: If nonlinear input function to the DNN depends linearly on $u(t)$, we can select $d\gamma^T(u)/du = \Gamma$, and we can compensate the measurable signal $\xi^*(t)$ by the modified control law $u(t) = u_{comp}(t) + u^*(t)$ where $u_{comp}(t)$ satisfies the relation

$$W_{2,t}^T u_{comp}(t) + \xi^*(t) = 0,\ \xi^*(t) = A x_m - f_m(x_m, t)$$

and u^* is selected according to the *linear squares optimal control law* [30]

$$u^*(t) = -R_c^{-1} \Gamma^{-1} W_{2,t}^T P_c(t) \Delta_m(t) \qquad (12)$$

Theorem 3 *For the nonlinear system (1), the given neural network (4), the nonlinear reference model (**??**) and the control law (12), the following property holds:*

$$|\Delta_m|_Q^2 + |u^*|_Q^2 \leqslant 2 |x_m|_{\Lambda_\sigma}^2 + \limsup_{\tau\to\infty} \frac{1}{\tau} \int_0^\tau \Psi_t(u^*(t)) dt$$

This equation fixes a tolerance level for the trajectory tracking error. On the final structure of the DNN the weights are learned on line.

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