

A Structural Analysis Method Formulation for Fault-tolerant Control System Design

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Abstract

An analysis of structural model representation has been used to extract available inherent redundant information in the system. The paper presents a refined structural model representation based on bipartite directed graph definition and the necessary condition for structural observability are proposed.

1 Introduction

The structural approach ([3, 2]), constitutes a general framework for providing information when the system becomes complex. The main objective of applying the structural approach is to identify the subsystems with inherent redundant information.

Consider the system S as a set of components $\bigcup_{i=1}^m C_i$, each imposing one (several) relations f_i between a set of variables and parameters $z_j, j = 1, \dots, n$ i.e.

$$f_i(z_1, \dots, z_p) = 0, \quad 1 < p \leq n \quad (1)$$

f_i can represent any kind of constraint/relation (dynamic, static, linear, or non-linear). [2, 3] The system's structural model is represented by the set of constraints $\mathcal{F} = \{f_1, f_2, \dots, f_m\}$ and the set of variables $\mathcal{Z} = \mathcal{K} \cup \mathcal{X} = \{z_1, z_2, \dots, z_n\}$. \mathcal{X} is the set of unknown variables and \mathcal{K} is the set of known variables/parameters. The system's structural model is represented by a structure graph [1]:

Definition 1 *The structure graph of a system is a bipartite graph $G(\mathcal{F}, \mathcal{Z}, \mathcal{A})$ where elements in the set of arcs $\mathcal{A} \subset \mathcal{F} \times \mathcal{Z}$ are defined by : $(f_i, z_j) \in \mathcal{A}$ iff f_i applies to z_j , ($f_i \in \mathcal{F}$ and $z_j \in \mathcal{Z}$).*

The set of constraints \mathcal{F} is separated into $\mathcal{F}_{\mathcal{K}}$, those that apply *only* to known variables, and $\mathcal{F}_{\mathcal{X}} = \mathcal{F} \setminus \mathcal{F}_{\mathcal{K}}$ that is the set of those constraints that include at least one unknown variable. Let \mathcal{E} denote a set (such as $\mathcal{F}_{\mathcal{X}}$ or \mathcal{Z}) and $\mathbb{P}(\mathcal{E})$ denote the power set of \mathcal{E} . Then a

subsystem $(F, Q(F))$, $F \in \mathbb{P}(\mathcal{F}_{\mathcal{X}})$ will be defined as

$$\begin{aligned} \mathbb{P}(\mathcal{F}_{\mathcal{X}}) &\xrightarrow{Q} \mathbb{P}(\mathcal{Z}) \\ F \rightarrow Q(F) &= \{z_j \mid \exists f_i \in F \text{ such that } (f_i, z_j) \in \mathcal{A}\} \end{aligned}$$

Matching Consider a graph $G(F_X, X, A_X)$ representing a restricted part of the system's structured graph. Let $a = (F_X(a), X(a))$ be the arc that connects a constraint $F_X(a)$ with an unknown variable $X(a)$.

Definition 2 *The (sub)graph $G(F_X, X, A)$ is a matching on $G(F_X, X, A_X)$ iff:*

- 1) $A \subset A_X$,
- 2) $\forall a_1, a_2 \in A \mid a_1 \neq a_2 \uparrow \downarrow$
 $F_X(a_1) \neq F_X(a_2) \wedge X(a_1) \neq X(a_2)$.

A complete matching w.r.t. F_X :

$$\forall f \in F_X \exists a \in A_X : F_X(a) = f.$$

A complete matching w.r.t. X :

$$\forall x \in X \exists a \in A_X : X(a) = x.$$

By applying matching one can decompose the system into three parts according to the following theorem:

Theorem 1 [4] *Any bipartite graph of finite external dimension can be uniquely decomposed:*

- $G^+ = (F^+, X^+, A^+)$ such that $Q(F^+) = X^+$ and a complete matching exists on X^+ but not on F^+ .
- $G^= = (F^=, X^=, A^=)$ such that $Q(F^=) = X^= \cup X^+$ and a complete matching exists on $X^=$ as well as on $F^=$.
- $G^- = (F^-, X^-, A^-)$ such that $Q(F^-) = X^- \cup X^= \cup X^+$ and a complete matching exists on F^- but not on X^- .

G^+ represents the part of the system with possible redundant information as $|F^+| > |X^+|$, where $|F|$ denotes the cardinality of F . The unknown variables in X^+ can be calculated in several ways by using the known variables. $G^=$ and G^- represent the parts with no redundant information [3, 1], for application see [5].

2 Bipartite directed graph model

To obtain usable results from matching some calculability properties need to be addressed:

Definition 3 *Calculability property:* Let $z_j, j = 1, \dots, p, \dots, n$ be variables that are related through a constraint f_i , e.g. $f_i(z_1, \dots, z_n) = 0$. The variable z_p is calculable if its value can be determined through the constraint f_i under the condition that the values of the other variables $z_j, j = 1, \dots, n, j \neq p$ are known.

This property, in general, has restrictive effects on representing components which contain terms such as sinusoidal and polynomials. One solution is to investigate the relations around the possible operating points of the system. Taking calculability properties into considerations, the systems structural model is now represented by a bipartite directed graph:

Definition 4 The structure graph of the system is a bipartite directed graph $((\mathcal{K} \cup \mathcal{X}), \mathcal{F}, \mathcal{A})$ where the elements in the set of $\mathcal{A} \subset (\mathcal{K} \cup \mathcal{X}) \times \mathcal{F}$ are defined by:

$$\begin{cases} a_{ij} = (f_i, x_j) = 1 & \text{iff } f_i \text{ applies to } x_j, \\ a_{ij}^* = (x_i, f_j) = 1 & \text{iff } x_i \text{ is calculable through } f_j \\ kf_i = (\mathcal{K}, f_j) = 1 & \text{iff } f_j \text{ applies on a known var.} \\ 0 & \text{Otherwise.} \end{cases}$$

The corresponding incidence matrix \mathbf{I}_{md} has the following compact form:

$$\begin{array}{c} K \\ \mathcal{F}_{\mathcal{X}} \\ \mathcal{X} \end{array} \begin{array}{c} K \quad \mathcal{F}_{\mathcal{X}} \quad \mathcal{X} \\ \left[\begin{array}{ccc} 0 & \mathbf{KF} & \mathbf{0} \\ \mathbf{KF}^T & \mathbf{0} & \mathbf{A} \\ \mathbf{0} & \mathbf{A}^* & \mathbf{0} \end{array} \right] \end{array} = \mathbf{I}_{\text{md}} \quad (2)$$

A dedicated matching algorithm can be used to decompose the system into different parts according to theorem 1.

3 Sensor fusion considerations

Determining the value of a variable in the system is (structurally) possible if there exists a path from the unknown variable to a set of known variables, i.e.:

Definition 5 (*Structural observability*): A $x_j \in \mathcal{X}, j \in \{1, \dots, |\mathcal{X}|\}$ is structurally observable if and only if it is reachable only from a set in $\mathbb{P}(\mathcal{K})$.

Structural observability is in fact a generalization of calculability definition. An unknown variable can be structurally observed (and hence constructed) if it fulfills the following condition:

Proposition 1 The necessary condition for a variable $x \in \mathcal{X}$ in the system S to be structurally observable is:

$$x \in Q(F^+ \cup F^-).$$

The purpose of applying sensor fusion in fault tolerant control systems is to enable an active system reconfiguration when a sensor fault occurs. For a structural model, sensor fusion is possible for an unknown variable if there is at least two paths from the variable to the known sets. Assume that a variable, x , is structurally observable via f_i . One path from the known set of variables to the unknown variable x is obviously through f_i . To investigate that there are other paths to x , one should simply remove f_i from $\mathcal{F}_{\mathcal{X}}$ and perform the matching on the remaining system. If x fulfills the necessary condition stated in proposition 1, then there is a possibility for performing sensor fusion.

4 Conclusion

A bipartite directed graph reformulation of structural model of systems has been presented. The paper addressed sensor fusion possibility from (graph's) structural observability point of view. The necessary condition for structural observability were established. Developing dedicated matching algorithm and detailed consideration of observability w.r.t. reconfigurability is in the scope of future work.

References

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