Unknown Input Observers for SISO nonlinear systems

Jaime Moreno
Instituto de Ingeniería, UNAM.
Ciudad Universitaria, Apdo. Postal 70-472,
Coyoacán, 04510 México, D.F., Mexico.
Email: moreno@pumas.iingen.unam.mx

Abstract

The existence conditions for unknown input observers for LTI systems are well known and several methods for its design have been proposed in the literature. However, for nonlinear systems only sufficient conditions are known for certain classes of systems. In this paper sufficient conditions under which the construction of an state unknown input observer for nonlinear systems are derived. Furthermore, this conditions are also shown to be necessary, under some additional conditions. A method to design full order and reduced order unknown input observers is proposed, and its convergence is analyzed. Although in this paper the study is restricted to SISO systems, most of the results can be carried on to MIMO systems. These results are important in the design of Fault Detection and Isolation Filters, robust nonlinear observers, and decentralized control.

Keywords: Unknown Input Observers, Nonlinear robust observers, Nonlinear Observers.

1 Introduction

The classical theory of observers is concerned with the reconstruction of the state from the input and output of the system. For LTI systems it is a well-known fact that the necessary and sufficient condition for the existence of such a classical observer is detectability. When the input is not completely available for measurement the existence conditions for an unknown input observer (UIO) are more restrictive than detectability: necessary and sufficient conditions are now well-known [2, 5, 6]. Unknown input observers find a wide applicability in the design of robust observers, decentralized control, and for fault detection and isolation problems.

For nonlinear systems the problem of UIO is not yet solved. Usually the construction of Nonlinear UIO is treated as a part of the problem of fault detection and isolation (FDI) [1, 3, 9, 12, 13, 14]. In most of these references sufficient conditions are studied for a FDI system to exist. For a general class of nonlinear systems [13] propose a method to construct an UIO, but conditions for its convergence are not given. Furthermore, the necessity of such conditions is also not discussed.

The objective of this paper is to find conditions under which the construction of a state unknown input observer for nonlinear systems is possible. Furthermore, the necessity of the derived conditions is also studied. Although this study is restricted to SISO systems, most of the results can be carried on to MIMO systems. A method to design full order and reduced order unknown input observers is proposed, and its convergence is analyzed. A preassigned structure for the observer is not assumed. It is shown that under two basic conditions, that are necessary and sufficient in the LTI case, the UIO is (globally) convergent.

The paper is organized as follows. The next section poses the problem and the results in the LTI case are reviewed. In section 3 the problem is solved for systems in normal form, a reduced and a full order observer are proposed, and their global convergence is analyzed. Section 4 deals with the same problem when the system is given in a general form. Section 5 discusses the necessity of the basic conditions imposed on the system, and Section 6 illustrates the method and the results with a physically motivated example. Finally some conclusions are given.

2 Problem Formulation

Consider the nonlinear system Σ

$$\begin{aligned} \dot{x} &= f(x,u) + g(x)\,w\ , \quad x(0) = x_0 \\ y &= h(x)\ , \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^p$ is the known input vector, $w \in \mathbb{R}$ is the unknown input, and $y \in \mathbb{R}$ is the output of the system. f, g, and h are sufficiently smooth functions defined for $(x,u) \in D_x \times \mathbb{R}^p$ where D_x is a domain of $\mathbb{R}^n$. Denote as $x(t, x_0, u(t), w(t))$ the solution of (1) passing through x_0 at $t = 0$ and belonging to the functions $u(t)$ and $w(t)$, and as $y(t, x_0, u(t), w(t)) = h(x(t, x_0, u(t), w(t)))$ its corresponding output. If no confusion arises this functions will be denoted simply by $x(t)$ and $y(t)$.

Our objective is to design an UIO (Unknown Input Observer) for (1), i.e., an observer that does not require the information on the unknown input signal $w(t)$ to estimate asymptotically the state $x(t)$.

Definition 1 *Consider a finite dimensional nonlinear system*

$$\begin{aligned} \dot{z} &= \varphi(z, u, y) \ , \quad z(0) = z_0 \\ \hat{x} &= \chi(z, u, y) \ , \end{aligned} \tag{2}$$

where $z \in D_z \subset \mathbb{R}^m$ is the state vector, and φ, and χ are sufficiently smooth functions defined for $(z, u, y) \in D_z \times \mathbb{R}^p \times \mathbb{R}$ where D_z is a domain of $\mathbb{R}^m$. Denote as $z(t, z_0, u, y)$ the solution of (2) corresponding to the functions u, and y and passing through z_0 at $t = 0$. System (2) is called an Unknown Input Observer *(UIO) of system (1) if: (i) system (1-2) have unique and global solutions for each initial condition $x(0) = x_0 \in D_x$, $z(0) = z_0 \in D_z$, and for every admissible[1] $u(\cdot)$ and $w(\cdot)$; and (ii)*

$$\lim_{t\to\infty} \|\hat{x}(t, z_0, u, y) - x(t, x_0, u, w)\| = 0 \ . \tag{3}$$

Three classes of observers can be distinguished according to the dimension of z: reduced order, full order and expanded order observers.

To put the results in perspective the SISO LTI case will be first recalled. Consider the system

$$\begin{aligned} \dot{x} &= Ax + Bw \\ y &= Cx \end{aligned} \ , \tag{4}$$

where $x \in \mathbb{R}^n$, $w \in \mathbb{R}$, and $y \in \mathbb{R}$ are the state vector, the unknown input and the output of the system, respectively. In this case, without loss of generality, only the unknown input will be considered. Known inputs can be easily included in the observer.

A necessary and sufficient condition for the existence of an UIO observer for (4) is well-known

Lemma 2 *[5, Th. 1.12] The system (4) has an* unknown input observer (UIO) *if and only if*

$$rank \begin{bmatrix} sI - A & -B \\ C & 0 \end{bmatrix} = n + 1, \ \forall \ s \in \mathbb{C}_0^+ \ , \tag{5}$$

and

$$CB \neq 0. \tag{6}$$

$\mathbb{C}_0^+$ is the closed right half plane of the complex plane. Equation (5) can be interpreted as a minimum phase condition, while condition (6) is equivalent to the fact that the relative grade of the system is 1.

[1] They are called *admissible* if the trajectory of (1-2) stays for the future time in the region of definition of the system.

A full order UIO for (4) is (see for example [2]):

$$\begin{aligned} \dot{z} &= (A_I - KC)\, z + \left(A_I B\, (CB)^{-1}\right) y, \quad z(0) = z_0 \\ \hat{x} &= z + B\,(CB)^{-1}\, y \ , \end{aligned} \tag{7}$$

where $z \in \mathbb{R}^n$, and $A_I = \left[I - B\,(CB)^{-1}\, C\right] A$. The convergence of (7) is assured if the system matrix of the UIO, $A_o \triangleq A_I - KC$, is Hurwitz. We have

Lemma 3 *Suppose that system (4) satisfy the conditions of Lemma 2. Then A_o in (7) has $n-1$ eigenvalues at the (stable) positions of the zeros of the plant (4), that cannot be moved by the output injection vector K, and one further eigenvalue that can be arbitrarily placed by K.*

Proof: Since the relative degree of (4) is 1 its inverse, with system matrix A_I, has $n-1$ eigenvalues at the position of the zeros of (4), and one extra eigenvalue at $s = 0$, as can be easily seen by noting that $CA_I = 0$. Because of (5) A_I has all but one stable eigenvalues. The pair (C, A_I) is not observable, since $rank\,(\mathcal{O}_I) = rank \left[C^T, A_I^T C^T, \cdots, A_I^{(n-1)T} C^T\right]^T = rank\,(C) = 1$, where $\mathcal{O}_I$ is the observability matrix of the pair (C, A_I). Furthermore, the observability subspace of (C, A_I) has dimension 1, and the only observable eigenvalue of (C, A_I) is the one at $s = 0$. This can be concluded by noting that from $CA_I = 0$ one can conclude that $rank\left[A_I^T, C^T\right] = rank\,(A_I) + 1$, and since A_I has $n-1$ stable eigenvalues its rank is $n-1$. From this observability analysis it follows that A_o has $n-1$ eigenvalues at the (stable) positions of the zeros of the plant (4), that cannot be moved by the output injection vector K, and one further eigenvalue that can be arbitrarily placed by the observer vector K. ■

Note that all but one of the eigenvalues of the UIO (7) are fixed by the zeros of the plant.

3 Unknown Input Observers for systems in normal form

For simplicity we will consider in this section the design of UIO for nonlinear systems (1) in normal form:

$$\dot{y} = \alpha(y, \xi, u) + \gamma(y, \xi)\, w \ , \quad y(0) = y_0 \tag{8}$$

$$\dot{\xi} = \eta(y, \xi, u) \ , \quad \xi(0) = \xi_0 \tag{9}$$

where $\xi \in \mathbb{R}^{n-1}$, α, γ, and η are sufficiently smooth functions defined for $(y, \xi, u) \in D_y \times D_\xi \times \mathbb{R}^p$ where D_ξ is a domain of $\mathbb{R}^{n-1}$, and D_y is a domain of $\mathbb{R}$. It will be assumed that $\gamma(y, \xi)$ is not identically zero for all $(y, \xi) \in D_y \times D_\xi$, since then one would have the trivial case when w does not have any influence on the system.

Denote as $y(t, y_0, \xi_0, u(t), w(t))$ and $\xi(t, y_0, \xi_0, u(t))$ the solutions of (8-9) passing through (y_0, ξ_0) at $t = 0$ and belonging to the functions $u(t)$ and $w(t)$. If no confusion arises this functions will be denoted simply by $y(t)$ and $\xi(t)$. The subsystem (9) driven by y and u will be called the **tracking dynamics** of the plant.

Condition 4 *For system (8-9) it will be assumed that* $\gamma(y, \xi) \neq 0$ *for all* $(y, \xi) \in D_y \times D_\xi$.

Note that, under condition 4, system (8-9) has the following invertibility property:

Proposition 5 *Suppose that Condition 4 is satisfied. For any (admissible) pair of functions* $(y(t), u(t))$ *and an (admissible) initial state* ξ_0*, there is a corresponding trajectory of (9),* $\xi(t) = \xi(t, y(t), \xi_0, u(t))$*, and a function* $w(t)$ *such that* $y(t)$ *and* $\xi(t)$ *are solutions of (8-9) corresponding to the initial states* $y_0 = y(0)$, $\xi(0) = \xi_0$ *and to* $u(t)$.

Proof: The necessary $w(t)$ is given by

$$w(t) = \frac{\dot{y}(t) - \alpha(y(t), \xi(t), u(t))}{\gamma(y(t), \xi(t))} ,$$

that is well defined since $\gamma(y, \xi) \neq 0$ for all $(y, \xi) \in D_y \times D_\xi$. ∎

In other words, any (admissible) pair $(y(t), u(t))$ can be synthesized by selecting $w(t)$ for any (admissible) initial condition ξ_0. This shows that the pair $(y(t), u(t))$ does not carry any information on the initial state ξ_0, when $w(t)$ is unknown.

For the (tracking) dynamics (9) the following (strong) stability property will be assumed, in order to be able to determine the true state $\xi(t)$ from the measurable external variables:

Condition 6 *Fix any (admissible) pair of functions* $(y(t), u(t))$ *and an (admissible) initial state* ξ_0 *for system (8-9). Then the corresponding trajectory of (9),* $\xi^*(t) = \xi(t, y(t), \xi_0, u(t))$*, is uniformly and asymptotically stable. This means that for any (admissible)* $\tilde{\xi} \in D_\xi$ *and the same pair* $(y(t), u(t))$ *the corresponding trajectory converges uniformly and asymptotically to* $\xi^*(t)$*, i.e.* $\left\| \xi(t) - \xi\left(t, y(t), \tilde{\xi}, u(t)\right) \right\| \to 0$. *An equivalent form of expressing this condition is: for every (admissible) pair* $(y(t), u(t))$ *and* $\xi^*(t)$ *given as above the equilibrium point* $\varepsilon = 0$ *of*

$$\dot{\varepsilon} = [\eta(y, \varepsilon + \xi^*, u) - \eta(y, \xi^*, u)] , \quad \varepsilon(0) = \varepsilon_0 \quad (10)$$

is uniformly and asymptotically stable for every (admissible) ε_0 *such that* $\varepsilon_0 + \xi_0 \in D_\xi$.

Remark 7 *Condition 6 is global. One can also consider local versions of it.*

A full order UIO for system (8-9) is given by

$$\begin{aligned} \dot{\zeta} &= -K_y \zeta, \quad \zeta(0) = \zeta_0 \\ \dot{\hat{\xi}} &= \eta\left(y, \hat{\xi}, u\right), \quad \hat{\xi}(0) = \hat{\xi}_0 \\ \hat{y} &= y - \zeta , \end{aligned} \quad (11)$$

where $K_y > 0$ is a design parameter. A reduced order $(m = n - 1)$ UIO is given by

$$\dot{\hat{\xi}} = \eta\left(y, \hat{\xi}, u\right), \quad \hat{\xi}(0) = \hat{\xi}_0. \quad (12)$$

In this case there are no design parameters, since the UIO is just a copy of the tracking dynamics (9) of the plant. The convergence of these observers is stated in the following

Theorem 8 *Assume that for system (8-9) Condition 6 is satisfied. Then the reduced order Unknown Input Observer (12) is a global (with respect to* D_ξ*) UIO for the system (8-9). If* $K_y > 0$ *the same is true for the full order UIO (11).*

Proof: It is immediate that if the reduced order UIO converges so does the full order UIO. For this last one the dynamics of the observation error is given by 10, and because of condition 6 the affirmation follows. ∎

Remark 9 *For local versions of condition 6 one obtains local versions of the theorem, i.e. local UIOs.*

Remark 10 *If in the full order observer (11) in the equation for* $\hat{\xi}$ *the output* y *is replaced by its estimated value* $\hat{y}$*, as is usually done for observers, then the global result does not follow from Condition 6. The observation error equation would have an exponentially decaying perturbation term, that could destroy the stability of the observer. This problem does not appear in the LTI case.*

Remark 11 *These results are in concordance with those for the LTI case.*

4 Unknown Input Observers for systems in general form

When the system (1) is not in normal form, one can design an UIO by transforming it to that form. We will give some sufficient conditions for this to be possible. Let us recall from [7, 10] some well known concepts.

Definition 12 *(Relative degree with respect to w). The relative degree of system (1) with respect to w, ρ_w, is defined as the integer such that*

$$L_g L^i_{f_0} h(x) = 0 \ , \qquad \forall x \in U_0 \ , \ 0 \le i \le \rho_w - 2$$
$$L_g L^{\rho_w - 1}_{f_0} h(x) \neq 0 \ , \qquad \forall x \in U_0 \ ,$$

where U_0 is a neighborhood of the origin, and $f_0(x) = f(x, 0)$. If $U_0 = \mathbb{R}^n$, then ρ_w is a global relative degree.

In the present paper it will be assumed that in the region of interest of the state space the plant has relative degree 1. In this case the system can be brought to the normal form. Adapting the results from [10, Lemmas 4.1.2 and 4.3.1] it is easy to obtain

Lemma 13 *Assume that for system (1) the relative degree with respect to w is 1. Then there exist $n-1$ functions $\xi_i(x)$, $i = 2, \cdots, n$, such that: (i) The functions $\{h(x), \xi_2(x), \cdots, \xi_n(x)\}$ form a local diffeomorphism about the origin; (ii) $L_g \xi(x) = 0$. In local coordinates $(y, \xi) = (y(x), \xi(x)) = (h(x), \xi_2(x), \cdots, \xi_n(x)) = \Phi(x)$ system (1) is expressed in* ***normal form*** *(8-9), where $\alpha(y, \xi, u) = L_f h(\Phi^{-1}(y, \xi))$, $\gamma(y, \xi) = L_g h(\Phi^{-1}(y, \xi))$, $\eta(y, \xi, u) = L_f \xi(\Phi^{-1}(y, \xi))$. If, in addition, the global relative degree $\rho_w = 1$ is well defined and (iii) the vector fields $\tilde{f} = f_0 - \frac{L_{f_0} h}{L_g h} g(x)$, $\tilde{g} = \frac{1}{L_g h} g(x)$ are complete, then there exists a global diffeomorphism transforming (1) into the normal form (8-9).*

Remark 14 *Note that Condition 4 implies that for system (1) $\rho_w = 1$ in $U_0 = \Phi^{-1}(D_y \times D_\xi)$.*

The full (11) and reduced order (12) Unknown Input Observers derived for systems in normal form together with the static mapping

$$\hat{x} = \Phi^{-1}\left(y, \hat{\xi}\right) \ , \qquad (13)$$

are UIOs for the estimation of the state in original coordinates. Note that for the realization of these observers the knowledge of the transformation map $\Phi(x)$ is necessary. Since this mapping can be computed by solving a system of partial differential equations, the calculation of the proposed observers can be in general very difficult.

However, in some particular cases those PDE can be solved explicitly. The easiest one is that of LTI systems. Moreover, for SISO bilinear systems an explicit expression for the UIO can be given, under a (weak) additional restriction (see [11]). It is interesting that the obtained observer is in general not bilinear, as is often assumed in the literature.

5 Necessity of the proposed conditions

For systems in the normal form (8-9) it was shown that Condition 6 is sufficient to design a global reduced order UIO. In this case the relative degree condition 4 (see remark 14) is not necessary, as long as the normal form is given. In the extreme case when $\gamma(y, \xi) = 0$ for all $(y, \xi) \in D_y \times D_\xi$ in (8-9), condition 6 is still sufficient for the design of an UIO. However, in this case the design of an UIO is equivalent to the design of an observer without unknown inputs, and then condition 6 is very restrictive and, by no means, necessary. It turns out, that if condition 4 is satisfied, then condition 6 is necessary for the existence of an UIO:

Lemma 15 *Assume that for system (8-9) condition 4 is satisfied. Assume, furthermore, that condition 6 is not met in the sense that there exist time functions $y(t)$ and $u(t)$ and two vectors ξ^1_0 , $\xi^2_0 \in \mathbb{R}^{n-1}$ such that the corresponding solutions of (9), $\xi_1(t) = \xi\left(t, \xi^1_0, u(t), y(t)\right)$ and $\xi_2(t) = \xi\left(t, \xi^2_0, u(t), y(t)\right)$ are such that $\lim_{t\to\infty} \|\xi_1(t) - \xi_2(t)\| \neq 0$. Then there does not exist an UIO for the system.*

Proof: Because $\gamma(y, \xi) \neq 0$ there exist two inputs, $\frac{\dot{y} - \alpha(y, \xi_i) + \beta(y, \xi_i) u}{\gamma(y, \xi_i)} = w_i(t)$, $i = 1, 2$, such that applied to the plant (together with $u(t)$ and the initial state ξ^i_0) produces the state trajectories $(y(t), \xi_1(t)) \neq (y(t), \xi_2(t))$ in the plant. Since the only information of the UIO are the signals $y(t)$ and $u(t)$ the estimated state by the UIO for these two signals has to converge to two different signals, what is impossible. ∎

The conclusions of this lemma are also valid for systems in general form.

Remark 16 *Condition 4 can be weakened somehow for the necessity of condition 6. Needed is the invertibility condition of proposition 5.*

For systems in general form (1) with well-defined relative degree $\rho_w = 1$ the system can be transformed (locally) to the normal form. Condition 4 is then satisfied, and Condition 6 is again sufficient for the design of the UIO. However, if the relative degree (with respect to w) is not well-defined it can still be possible to transform system (1) to the normal form and to design an UIO, although Condition 4 is not satisfied. For LTI systems, however, condition 4 is necessary for the existence of an UIO for *every* input. Therefore, Condition 4 cannot be dropped in general. It turns out that for systems with a well defined relative degree (with respect to w) Condition 4 is also necessary for the existence of an UIO:

Lemma 17 *Assume that system (1) satisfies the global* *conditions of Lemma 13. Suppose, moreover, that the system is* complete, *i.e. for every time function $u(t) \in \mathcal{U}$, a set of functions that includes the zero function, for every function $w(t)$ bounded in finite time intervals, and for every initial condition x_0 there exists a trajectory of the system $x(t, x_0, u(t), w(t))$ for every $t \geq 0$. If $\rho_w \geq 2$ then there is no a global UIO for the system in the sense of definition 1.*

Proof: Without lost of generality let us assume that $f(0,0) = 0$, and $h(0) = 0$. Let us also suppose that $\rho_w = 2$, but the arguments are the same for bigger values. Let us put the system in the normal form [10]

$$\begin{aligned} &\dot{v}_1 = v_2 \\ &\dot{v}_2 = \alpha(v, \xi, u) + \gamma(v, \xi) w \ , \quad v(0) = v_0 \\ &\dot{\xi} = \eta(v, \xi, u) \ , \quad \xi(0) = \xi_0 \\ &y = v_1 \end{aligned} \tag{14}$$

Now suppose that there exists a global UIO as given by definition 1. If we apply to the plant $u = 0$, $w = 0$, and the initial state is $x_0 = 0$, then $y = 0$ for all the time. Then the equilibrium point of the UIO (suppose $z = 0$) has to be a globally and asymptotically stable point for the dynamics

$$\dot{z} = \varphi(z, 0, 0) \ , \quad z(0) = z_0 \ .$$

By Lemma 5.4 in [8] the system $\dot{z} = \varphi(z, 0, y(t))$ is LISS and as $y(t) \to 0$ then $z(t) \to 0$. Now let us suppose that the plant in normal form (14) has as input $u = 0$, as initial condition for the tracking dynamics $\xi(0) = \xi_0$, and as unknown input

$$w(t) = \frac{\ddot{y}(t) - \alpha(y(t), \dot{y}(t), \xi(t), 0)}{\gamma(y(t), \dot{y}(t), \xi(t))} \ ,$$

where $y(t) = \epsilon(t+\delta)^{-1} \sin(t+\delta)^2$, $\epsilon, \delta > 0$, and $\xi(t)$ is the corresponding solution of the tracking dynamics. $w(t)$ is well defined since $\gamma \neq 0$. Since $y(t) \to 0$ and $\sup_{t \geq 0} \|y(t)\|$ can be made as small as desired by selecting ϵ small, for the UIO it is necessary that $z(t) \to 0$. Also the estimation $\hat{x}(t) \to 0$. But $x(t) \nrightarrow 0$ since $\dot{y}(t) \nrightarrow 0$. A contradiction. ■

Remark 18 *Condition 4 is the same as for the linear case, cfr. (6). In the linear case if (5) is satisfied, then Condition 6 is satisfied. So these conditions are generalizations of the ones in the linear case.*

6 Example

Consider a D.C. motor [7, Sec. 4.10] in which the rotor voltage is kept constant can be described by equations (1) with state variables $x_1 = I_s$, stator current, $x_2 = I_r$, rotor current, and $x_3 = \Omega$, angular velocity of the motor shaft, without known input u and as unknown input $w = V_s$, stator voltage,

$$\begin{aligned} f(x,u) &= \begin{bmatrix} -\frac{R_s}{L_s} x_1 \\ -\frac{R_r}{L_r} x_2 + \frac{V_r}{L_r} - \frac{KL_s}{L_r} x_1 x_3 \\ -\frac{F}{J} x_3 + \frac{KL_s}{J} x_1 x_2 \end{bmatrix} , \\ g(x) &= \begin{bmatrix} \frac{1}{L_s} & 0 & 0 \end{bmatrix} , \quad h(x) = x_1 \ . \end{aligned}$$

R_s and L_s are the resistance and, respectively, inductance of the stator winding. R_r and L_r are the resistance and, respectively, inductance of the rotor winding. V_r is the rotor voltage, J the inertia of the load, F the viscous friction constant, and K a conversion constant. The objective is the reconstruction of the whole state x by measuring one state variable x_1, the stator current, and considering $w = V_s$, the stator voltage, as an unknown perturbation signal. Since this system is already in the regular form (8-9) one can readily see that Condition 4 is satisfied. Now let us check if Condition 6 is fulfilled. The tracking dynamics (9) takes the form

$$\dot{\xi} = \eta(y, \xi, u) = \begin{bmatrix} -\frac{R_r}{L_r} & -\frac{KL_s}{L_r} y(t) \\ \frac{KL_s}{J} y(t) & -\frac{F}{J} \end{bmatrix} \xi + \begin{bmatrix} \frac{V_r}{L_r} \\ 0 \end{bmatrix} , \tag{15}$$

with $\xi^T = [x_2 \ , \ x_3]$. Condition 6 is satisfied if the equilibrium point $\varepsilon = 0$ of (see (10))

$$\dot{\varepsilon} = \begin{bmatrix} -\frac{R_r}{L_r} & -\frac{KL_s}{L_r} y(t) \\ \frac{KL_s}{J} y(t) & -\frac{F}{J} \end{bmatrix} \varepsilon \ , \quad \varepsilon(0) = \varepsilon_0 \ , \tag{16}$$

is uniformly and asymptotically stable for every (admissible) ε_0 and every $y(t)$. Note that this equation is, for fixed $y(t)$, a linear time-varying equation in ε. The quadratic candidate Lyapunov function $V(\varepsilon) = \frac{1}{2}\left(L_r \varepsilon_1^2 + J \varepsilon_2^2\right)$, that is positive definite since $J > 0$ and $L_r > 0$, and with time derivative along the trajectories of (16) $\dot{V}(\varepsilon) = -R_r \varepsilon_1^2 - F \varepsilon_2^2$, which is negative definite since $R_r, F > 0$. Therefore (16) is global and exponentially stable for every $y(t)$, and Condition 6 is satisfied globally. Furthermore both full-order (11) and reduced order (12) Unknown Input Observers converge globally. The reduced order observer has the bilinear structure given by (15).

Simulations corresponding to these two observers for an arbitrary unknown input function w are given in Figure 1. The error dynamics for both observers is the same, except obviously for the output error. The parameters used for the simulations were: $R_s = 25.2\ \Omega$, $L_s = 63.5\ H$, $R_r = 0.05\ \Omega$, $L_r = 0.003\ H$, $V_r = 100\ V$, $J = 15\ Kg \cdot m^2$, $F = 0.1$, and $K = 0.0166$. Furthermore $K_y = 10$, $x_0^T = [10 \ , \ -5 \ , \ 10]$ (for plant), $z_0^T = [20 \ , \ -30 \ , \ 10]$ (full order UIO), and $z_0^T = [-30 \ , \ 10]$ (reduced order UIO).

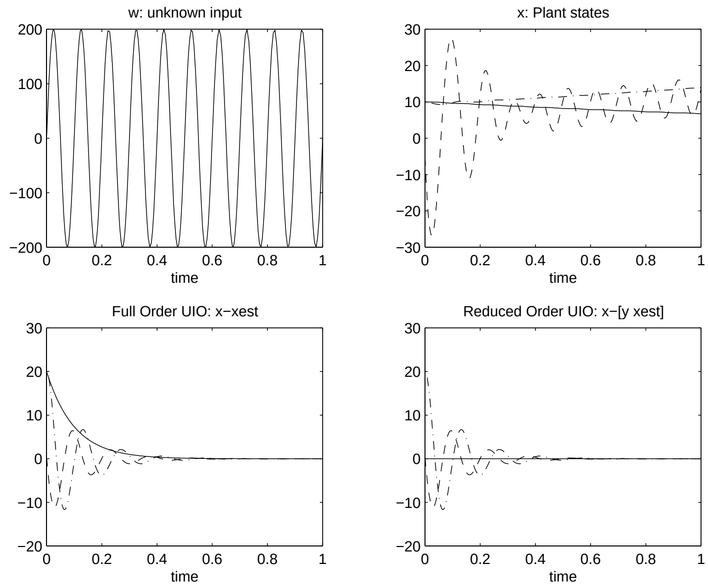

Figure 1: Simulation results: Error of the Full Order UIO (Down-Left) and of the Reduced Order UIO (Down-Right) of the C.D. Motor. Unknown input signal $w(t)$ (Up-Left), plant states of the plant (Up-Right) x_1: — , x_2: − − , x_3: − · − . Shown is $(\hat{x}_2 - x_2)/100$.

7 Conclusions

In this paper the problem of UIO to reconstruct the complete state for nonlinear systems has been addressed. Although the study has been made for SISO systems, most of the results can be carried on to MIMO systems. A method for constructing full and reduced order UIOs was proposed, and their convergence analyzed. The global convergence of both observers was established. These conditions are necessary and sufficient in the linear case and they were shown to be necessary in the nonlinear case, under some additional conditions. Since unknown input observers find a wide applicability in the design of robust observers, decentralized control, and for fault detection and isolation problems it is expected that the results of this paper can have an impact on these areas.

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References

[1] Ashton, S.A., and Shields, D.N. "Fault Detection Observer for a Class of Nonlinear Systems". In [12], pp. 353-373.

[2] Darouach, M.; Zasadzinski, M. and Xu, S.J. "Full-order Observers for linear systems with unknown inputs". *IEEE Trans. on Automatic Control*, Vol. 39, No. 3, (1994): 606-609.

[3] Frank, P.M., Schreier, G. and Alcorta García, E. "Nonlinear Observers for Fault Detection and Isolation". In [12], pp. 399-422.

[4] Gauthier, J.P., Hammouri, H. and Othman, S. "A simple Observer for Nonlinear Systems. Applications to Bioreactors". *IEEE Trans. on Automatic Control*, Vol. 37, No. 6, (1992): 875-880.

[5] Hautus, M. L. J. "Strong detectability and observers". *Linear Algebra and its Applications*, 50 (1983), 353-368.

[6] Hou, M. and Mueller, P.C. "Disturbance Decoupled Observer Design: A Unified Viewpoint". *IEEE Trans. on Automatic Control*, Vol. 39, No. 6, (1994): 1338-1341.

[7] Isidori, A. *Nonlinear Control Systems.* 3th ed. Berlin, Springer Verlag, 1995.

[8] Khalil, H. K. *Nonlinear Systems.* 2nd. ed. Upper Saddle River, N.J., Prentice Hall, 1996.

[9] Kinnaert, M. "Robust fault detection based on observers for bilinear systems". *Automatica* 35 (1999):1829-1842.

[10] Marino, R. and Tomei, P. *Nonlinear Control Design; Geometric, Adaptive & Robust.* London, Prentice Hall, 1995.

[11] Moreno, J. "Unknown Input Observers for SISO Bilinear Systems". *Proceedings of the (IFAC) IX Congreso Latinoamericano de Control Automático*, Cali, Colombia, Nov. 2000.

[12] Nijmeijer, H. and Fossen, T.I. (Eds.) *New Directions in Nonlinear Observer Design.* Lecture Notes in Control and Information Sciences 244. London, Springer-Verlag, 1999.

[13] Seliger, R. and Frank, P.M. "Fault-Diagnosis by Disturbance Decoupled Nonlinear Observers". *Proceedings of the 30th IEEE Conference on Decision and Control*, Brighton, England, Dec. 1991. pp. 2248-2253.

[14] Shields, D.N. and Yu, D.L. "A review of observer approaches for fault diagnosis based on bilinear systems". *IEE Colloquium on Qualitative and Quantitative Modelling Methods for Fault Diagnosis.* (1995): 6/1-6/19.