Decentralized Hybrid Traffic Network Routing Using Optimal Sliding Modes

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Abstract

In this paper, three hybrid network routing strategies using Optimal Sliding Modes method are presented. First a two-road hybrid routing strategy using optimal sliding modes is proposed. In this method, a finite state machine is defined and the Optimal Sliding Mode method is used to find the optimal routing under different states. Then the hybrid method is extended to large networks by three approaches: centralized, decentralized and partial-decentralized. A comparison of these three approaches is also given.

1 Model

The dynamics of a node with L^i incoming links and L^o outgoing links can be written as

$$\dot{q}(t) = \sum_{k=1}^{L^i} f_{kn_k}(t) - \sum_{l=k}^{L^o} u_k(t), \tag{1}$$

$$\dot{\rho}_{11}(t) = \frac{1}{\delta_{11}}[u_1(t) - f_{11}(t)], \tag{2}$$

$$\dot{\rho}_{1i}(t) = \frac{1}{\delta_{1i}}[f_{1(i-1)}(t) - f_{1i}(t)], \tag{3}$$

$$f_{1i}(t) = v_{f1}\rho_{1i}(t)\left[1 - \left(\frac{\rho_{1i}(t)}{\rho_{max1}}\right)^{l_1}\right]^{m_1}, \tag{4}$$

$$\vdots \tag{5}$$

$$\dot{\rho}_{L^o1}(t) = \frac{1}{\delta_{L^o1}}[u_{L^o}(t) - f_{L^o1}(t)], \tag{6}$$

$$\dot{\rho}_{L^oi}(t) = \frac{1}{\delta_{L^oi}}[f_{L^o(i-1)}(t) - f_{L^oi}(t)], \tag{7}$$

$$f_{L^oi}(t) = v_{fL^o}\rho_{L^oi}(t)\left[1 - \left(\frac{\rho_{L^oi}(t)}{\rho_{maxL^o}}\right)^{l_{L^o}}\right]^{m_{L^o}} \tag{8}$$

where $q(t)$ denotes the queue length formed in the node, ρ_{ij} and f_{ij} are the traffic density and traffic flow at the jth segment of the ith road. v_{fi} is the free speed of the ith road. δ_{ij} is the length of the jth segment at the ith road. f_{kn_k} is the traffic flow at the n_kth segment of the kth coming link of the node, and u_k is the traffic into the kth leaving link. l_i and m_i are constants related to the road shape.

Based on the theory of kinematic waves and extensive measurement-based verification studies, we know when traffic density of a road is less than a certain value named critical density ρ_{cr}, the traffic flow f of the road is approximately proportional to the traffic density ρ and the slope is the free speed of the road. When a traffic system is in this situation, it is called "stable". And when the traffic density of over the critical density, the traffic flow becomes, rapidly and without any obvious reason, more and more congested. This is called "unstable". Thus, when the road is "stable," Eqn. (4) and (8) can be approximately written as

$$f_i(t) = v_f \rho_i(t) \tag{9}$$

The dynamics of the node becomes linear.

2 Hybrid Routing

Consider a two-road network, in which a queue is formed by the coming traffic (Figure 1). Since the

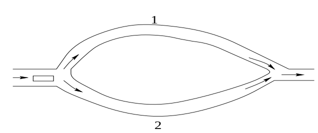

Figure 1: Node with a queue and two leaving links

length of the queue and the traffic density of every segment of the road represent the traffic performance of the network, this optimization problem is to minimize all these variables, i.e.,

$$J = \int_0^\infty \left[x^T Q x\right] dt \tag{10}$$

with constraints

$$0 \leq u_1(t) \leq c_1 \quad 0 \leq u_2(t) \leq c_2 \tag{11}$$

where $x^T = [q(t), \rho_{11}(t), \cdots \rho_{1n_1}(t), \rho_{21}(t), \cdots, \rho_{2n_2}(t)]$ and Q is a positive definite matrix.

Based on the characteristics of traffic flow and density, we know that a road has its "stable" and "unstable" states. To handle the routing of the two-road system under different states, a hybrid control system is proposed, which has a structure shown in Figure 2. The

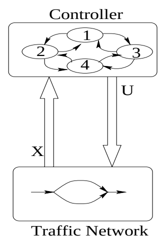

Figure 2: Hybrid control system

finite state machine has four states:

State 1: The two roads are stable, i.e.,

$$\begin{array}{lll} \rho_{1i} & < & \rho_{cr1} \quad i = 1, \cdots, n_1 \\ \rho_{2i} & < & \rho_{cr2} \quad i = 1, \cdots, n_2 \end{array} \tag{12}$$

State 2: Road 1 is stable, but road 2 is unstable, i.e.,

$$\begin{array}{lll} \rho_{1i} & < & \rho_{cr1} \quad i = 1, \cdots, n_1 \\ \rho_{2i} & > & \rho_{cr2} \quad i = 1, \ or \cdots \ or \ n_2 \end{array} \tag{13}$$

State 3: It is similar to State 2. The difference is that the states of two roads switch, i.e.,

$$\begin{array}{lll} \rho_{1i} & > & \rho_{cr1} \quad i = 1, \ or \cdots, \ or \ n_1 \\ \rho_{2i} & < & \rho_{cr2} \quad i = 1, \cdots, n_2 \end{array} \tag{14}$$

State 4: Both of the roads are unstable. Whatever our control is, the system will become unstable. State 4 is uncontrollable.

Because we know that if a road is "unstable" and traffic continues to be directed into the road, the road will go into stop-and-go state, traffic routed into an "unstable" road should be zero. And when a road is in the "stable" state, the optimal sliding modes approach [1] is applied to minimize the criterion Eqn. (10) since the dynamics is approximately linear. The reason why we use the optimal sliding modes is based on the characteristics of this problem. First, in our criterion, only the quadric term of system state is included. Without the quadric term of control vector in the criterion, many linear optimization methods cannot be used. Second, the control of the system, i.e., traffic directed into the two roads, is bounded. These two characteristics determine that the optimal sliding modes approach is used.

Three approaches exist to extend the above hybrid routing strategy to large networks: centralized, decentralized and partial-decentralized.

Centralized hybrid network routing is directly extended from the above hybrid two-road routing strategy. Based on the dynamics of the whole network, including the dynamics of queue formed in every node and the dynamics of the every link, the amount of traffic routed into every link can be decided by using the optimal sliding modes method. Using Centralized hybrid network routing strategy, the traffic system can normally achieve optimal performance. However, because of the huge amount of calculation and requiring all the traffic information, this approach is not applicable for large networks.

To overcome the problems of the centralized hybrid network routing, we propose a decentralized routing strategy, in which, every node uses the optimal sliding modes method to decide its routing only by its own traffic information. This decentralized hybrid network routing strategy is much simpler and easier to be applied than the centralized one. However, since every node only considers its own traffic condition in this strategy, the performance of the system may not be optimal. In some cases, the whole traffic system is even unstable.

In an effort to combine the advantages of centralized and decentralized hybrid network routing methods, we propose the partially decentralized hybrid network routing strategy. Its basic idea is dividing a large network into several small sub-networks and using the hybrid routing strategy for these small networks. One possible definition of subnets is given as: every node, its downstream nodes and their corresponding links form a subnet. Assuming all the traffic information of the subnet of the ith node is available for the node, i.e., all downstream notes of the ith node send their traffic information (the queue lengths and the traffic densities of the segments of their outgoing links) back to the ith node , we can use the hybrid method discussed above to construct the finite state machine for the subnet and apply the optimal sliding modes method to solve it. Note that, in the ith subnet, not only the routing of the ith node but also the routing of its downstream nodes are found. However, only the routing of the ith node is useful. This is because if we decide the routing of the downstream nodes by the subnet of the ith node, the traffic information of the downstream nodes of these nodes is not considered. Then the partial-decentralized routing strategy will have the same disadvantage as the decentralized routing.

3 Conclusion

Centralized, decentralized and partial-decentralized hybrid network routing strategies using the optimal sliding modes method are proposed in this paper. The theoretical analysis and comparison of the three routing strategies show that the partial-decentralized hybrid routing is the best one.

References

[1] V. Utkin, *Sliding Modes in Control and Optimization*, Springer Verlag, 1992.