A Three-Phase AC Power Source using Robust Model Reference Adaptive Control

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***Abstract* – This paper presents the development of an equilibrated three-phase power source, which is able to generate sinusoidal waveforms with adjustable amplitudes and frequencies, as well as several arbitrary waveforms. The setup is based on a computer-controlled three-phase PWM inverter system, which model is rewritten using ab0 transformation. A robust model reference adaptive controller (RMRAC) is used to assure system robustness and performance. Simulation and experimental results are used to show the closed-loop system performance, when different waveforms are generated under several operation conditions.**

I. INTRODUCTION

Several standard tests in electrical power devices require the use of a three-phase AC power source at different voltages and frequencies. Induction motors, power and distribution transformers, uninterruptible power supplies (UPS) and standby power supplies (SPS) are examples of devices that need these tests. Some of these devices need additional harmonic tests such as the injection of harmonics in power and distribution transformers.

In developing an AC power source we may experience some problems. Due to harmonic distortion, model reference tracking errors may appear. To compensate these errors, elevated switching frequencies or high performance controllers are needed. Low [1], [2], proposes a single-phase AC power source using respectively sliding mode control and generalized predictive control (GPC), with inverter switching frequency at 25 kHz. However, in high power static converters, switching losses increase with the elevation of the switching frequency. Efforts have been done for that high power converters obtain good performance even operating at low switching frequencies [3]-[6]. Such systems demand high performance controllers to compensate eventual or periodic disturbances in the waveforms, such as the ones that appear under non-linear loads. In addition, modeling errors and unmodeled dynamics are quite common, due to simplifications of the model and characteristics of the plant. All these factors influence directly the converter performance, and, in more critical cases, they even compromise the controllers used in the system, as described in [7]. Gründling et al [7], introduce a robust model reference adaptive controller (RMRAC) applied for single-phase uninterrupted power supplies, which even operating at low switching frequencies presents results with good transient response. In [8], Carati *et al* apply RMRAC for a three-phase UPS structure, assuring robustness and good performance.

In this work it is implemented a three-phase AC power source that is able to generate equilibrated sinusoidal voltages with adjustable amplitudes and frequencies. Moreover, harmonics can be added to the fundamental sine wave, so arbitrary waveforms can be generated. The three-phase system model is obtained using $\alpha\beta0$ transformation. A RMRAC controller is designed to guarantee robustness and good performance. Simulation and experimental results are presented to verify the dynamic performance of the resulting closed-loop system.

This paper is organized as follows: the Section II contains the description of the plant while in the Section III presents the closed-loop system structure. The proposed RMRAC control law is described in the Section IV. The Section V presents the parameter adaptation algorithm. Simulation and experimental results are given in the section VI. The section VII concludes the paper.

II. PLANT DESCRIPTION

The proposed system shown in Fig. 1 is formed by a three-phase inverter, a delta (Δ) connected LC filter and three-phase load. System state space model is given by:

$$\begin{bmatrix} \dot{i}_R \\ i_S \end{bmatrix} = \frac{1}{3L_f}\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} V_{RS} - V_{UW} \\ V_{ST} - V_{VW} \end{bmatrix} - \frac{R_f}{L_f}\begin{bmatrix} i_R \\ i_S \end{bmatrix}, \qquad (2.1)$$

$$i_T = -(i_R + i_S)$$

and

$$\begin{bmatrix} \dot{V}_{RS} \\ V_{ST} \end{bmatrix} = \frac{1}{3C_f}\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -i_R - i_{RL} \\ -i_S - i_{SL} \end{bmatrix}, \qquad (2.2)$$

$$V_{TR} = -(V_{RS} + V_{ST}).$$

In (2.2), the load currents i_{RL} and i_{SL} depend of the configuration and balancing condition of the load. The resistive load is connected in wye (Y) configuration as shown in the Fig. 2a. The load currents i_{RL} and i_{SL}, in the Fig. 2a, are given by:

$$i_{RL} = \frac{2V_{RS}}{3R} + \frac{V_{ST}}{3R}, \; i_{SL} = -\frac{V_{RS}}{3R} + \frac{V_{ST}}{3R}. \qquad (2.3)$$

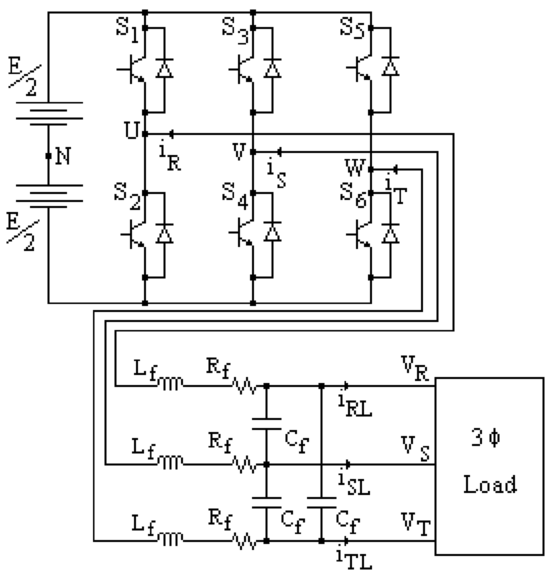


Fig. 1. Three-phase PWM inverter system.

When the load is connected in delta (Δ) configuration, as shown in the Fig. 2b, the load currents i_{RL} and i_{SL} become:

$$i_{RL} = \frac{2\,V_{RS}}{R} + \frac{V_{ST}}{R}, \quad i_{SL} = -\frac{V_{RS}}{R} + \frac{V_{ST}}{R}. \qquad (2.4)$$

The inverter switches $S_{1\text{-}6}$ are turned on and off once every sampling interval T, so that the line to line inverter voltages, V_{UV}, V_{VW} and V_{WU}, are pulses with amplitude E, 0 and –E. These inverter output PWM voltages generate the output system voltages, V_{RS}, V_{ST} and V_{TR}.

III. Closed-Loop System Structure

The three-phase reference model outputs, y_{mr}, y_{ms} and y_{mt}, are converted in αβ0 coordinates, $y_{m\alpha}$ and $y_{m\beta}$, respectively, by a T transformation, as follows:

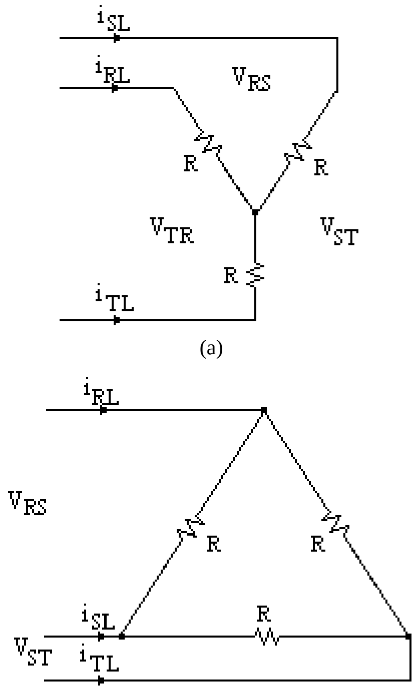
$$T = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \qquad (3.1)$$

$$T^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \qquad (3.2)$$

In this point is interesting to note that $y_{m\alpha}$ and $y_{m\beta}$ are independent orthogonal variables.

Such as shown in the Fig. 4, the control laws u_α and u_β and the tracking errors e_α and e_β are computed separately for each axis α and β, respectively, using a single RMRAC controller for each axis.

Using the T transformation, the load measured voltages variables, V_r, V_s and V_t are converted in orthogonal output voltage variables, V_α and V_β. The system output errors, e_α and e_β, are obtained through a comparison of these variables with that ones from the reference model, $y_{m\alpha}$ and $y_{m\beta}$, respectively. From these errors, the RMRAC algorithm computes separately the orthogonal control variables, u_α and u_β. Then, the three-phase control variables, u_u, u_v and u_w, obtained by the T^{-1} transformation, are converted in PWM signals, ΔTu, ΔTv and ΔTw, where $\Delta T_i(k) = T/2 + T\,u_i(k)/E$, $|u_i(k)| \le E/2$ and T is the sampling period. Then, the PWM inverter generates the LC filter input voltages, Vu, Vv and Vw, respectively.



(a)

(b)

Fig. 2. Load configuration: (a) wye -Y and (b) delta - Δ.

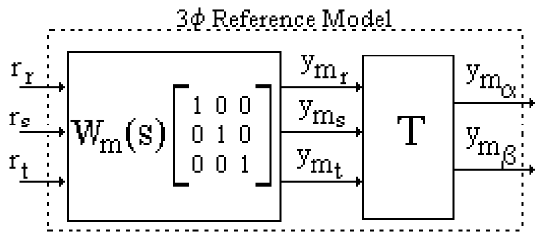


Fig. 3. Reference model of a three-phase UPS system.

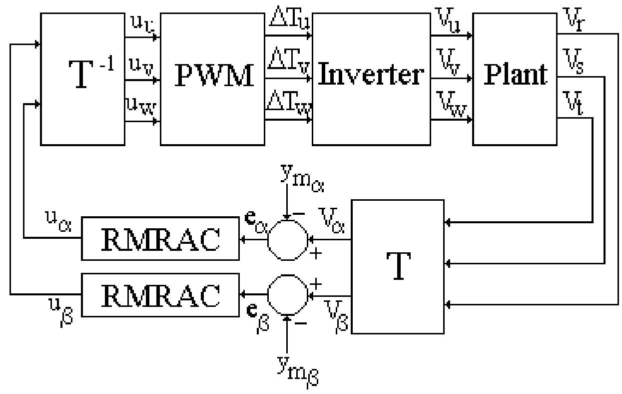


Fig. 4. Block diagram of a three-phase UPS system with RMRAC controller.

IV. RMRAC Controller Structure

Consider a single-input single-output plant (SISO) as presented in Fig. 5:

$$y = G(z)u$$
$$y = \left[G_0(z)\left[1+\mu\,\Delta_m(z)\right]+\mu\Delta_a(z)\right]\, u \qquad (4.1)$$

with

$$G_0(z) = k_p\,\frac{Z_0(z)}{R_0(z)} \qquad (4.2)$$

where G(z) is the transfer function of the plant, $G_0(z)$ is the strictly proper transfer function of the modeled part of the plant, $\mu\cdot\Delta_m(z)$ and $\mu\cdot\Delta_a(z)$ are additive and multiplicative perturbations, respectively. $Z_0(z)$ and $R_0(z)$ are monic polynomials with degree *m* and *n*, respectively.

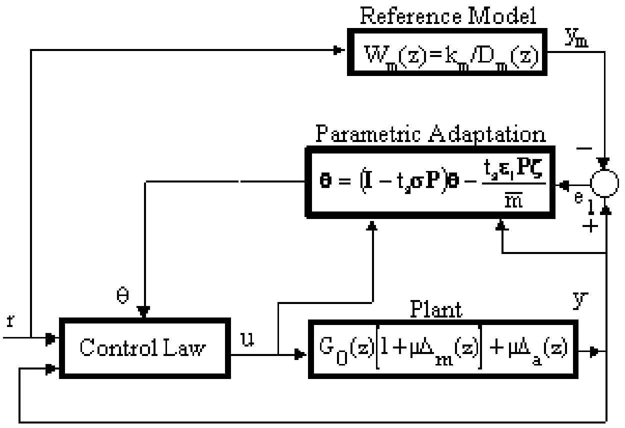


Fig. 5. RMRAC – UPS

Concerning the modeled part of the plant $G_0(z)$ the following assumptions are made:

S1. $Z_0(z)$ is a monic Hurwitz polynomial of degree *m* ($\leq n$-1).
S2. $R_0(z)$ is a monic polynomial of degree *n*.
S3. The sign of k_p and the values of *m* and *n* are known.

Concerning the unmodeled plant part is assumed that:

S4. $\Delta_a(z)$ is a strictly proper stable transfer function.
S5. $\Delta_m(z)$ is a stable transfer function.
S6. A lower bound $p_0 > 0$ for which the poles of $\Delta_a(z\text{-}p)$ and $\Delta_m(z\text{-}p)$ are stable are known.

The control objective is: Given the reference model

$$y_m = W_m(z)\cdot r = \left(K_m / D_m(z)\right)r \qquad (4.3)$$

where $D_m(z)$ is a Hurwitz polynomial of degree $n^* = n - m$ and r(t) is uniformly bounded, design a adaptive controller so that for some $\mu^* > 0$ and any $\mu \in [0, \mu^*)$ the resulting closed-loop plant is stable and the output plant *y* tracks the reference model output y_m as closely as possible for all perturbations $\Delta_a(z)$ and $\Delta_m(z)$ satisfying **S4 - S6.**

As in [7] the input *u* and the output *y*, in the discrete time domain, are used to generate *n*-1 dimensional auxiliary vectors, so that

$$\omega_1(k) = (zI - F)^{-1} q\, u(k)$$
$$\omega_2(k) = (zI - F)^{-1} q\, y(k) \qquad (4.4)$$

where F is a matrix stable and (F,q) is a controllable pair.

The plant input is taken as

$$u(k) = \mathbf{q}^T(k)\mathbf{w}(k) + c_0 r(k) \qquad (4.5)$$

where $\boldsymbol{\theta}^T(k) = \left[\boldsymbol{\theta}_1^T(k), \boldsymbol{\theta}_2^T(k), \theta_3(k)\right]$ is a (2*n*-1) dimensional vector of control parameters, $c_0(t)$ is a scalar gain, $\boldsymbol{\omega}^T(k) = \left[\boldsymbol{\omega}_1^T(k), \boldsymbol{\omega}_2^T(k), y(k)\right]$. The reference model tracking error is $e_1(k) = y_m(k) - y(k)$.

V. Parameter Adaptation Algorithm

There are a number of well-known parameter estimation techniques which have been successfully applied to identification problems [10], [1]. In these scheme is considered a recursive least-squares (RLS) modified algorithm:

$$\mathbf{q}(k+1) = \left(\mathbf{I} - t_s\sigma\mathbf{P}(k)\right)\mathbf{q}(k) - \frac{t_s\varepsilon_1(k)\mathbf{P}(k)\mathbf{z}(k)}{\overline{m}(k)} \qquad (5.1)$$

$$\mathbf{P}(k+1) = (1 + t_s\lambda\overline{\mu}^2)\mathbf{P}(k) - \cdots$$
$$\cdots - t_s\left(\frac{\mathbf{P}(k)\mathbf{z}(k)\mathbf{z}^T(k)\mathbf{P}(k)}{\overline{m}(k)} + \overline{\mu}^2\frac{\mathbf{P}^2(k)}{R^2}\right) \qquad (5.2)$$

where $\mathbf{P} = \mathbf{P}^T$ is so that

$$0 < \mathbf{P}(0) \leq \lambda R^2\mathbf{I} \quad , \quad \mu^2 \leq k_\mu\overline{\mu}^2 ,$$
$$\overline{m}(k) = 1 + \alpha_1\left[m(k)\right]^2 \quad , \quad \mathbf{z}(k) = W_m(z)\mathbf{I}\mathbf{w}(k) , \qquad (5.3)$$

$$m(k+1) = (1 - t_s\delta_0)m(k) + t_s\delta_1\left(|u(k)| + |y(k)| + 1\right),$$
$$m(0) > \frac{\delta_1}{\delta_0} , \quad \delta_1 \geq 1 , \qquad (5.4)$$

where α_1, δ_0, δ_1, λ, $\overline{\mu}$ and R^2 are positive constants and δ_0 satisfies $\delta_0 + \delta_2 \leq \min\left[p_0, q_0\right]$, $q_0 \in \Re^+$ is such that the poles of $W_m(z\text{-}q_0)$ and the eigenvalues of $\mathbf{F} + q_0\mathbf{I}$ are stable and δ_2 is a positive constant. $p_0 > 0$ is defined in **S6** and σ in (5.1) is given by

$$\sigma = \begin{cases} 0 & \text{if} \quad \|\mathbf{q}\| < M_0 \\ \sigma_0\left(\dfrac{\|\mathbf{q}\|}{M_0} - 1\right) & \text{if} \quad M_0 \leq \|\mathbf{q}\| \leq 2M_0 \\ \sigma_0 & \text{if} \quad \|\mathbf{q}\| \geq 2M_0 \end{cases} \qquad (5.5)$$

where $M_0 > \|\mathbf{q}^*\|$ and $\sigma_0 > 2\bar{\mu}^2/R^2 \in \Re^+$ are design parameters. The modified error in (5.1) is defined in [7] and is given by

$$\varepsilon_1(k) = e_1(k) + \theta^T(k)\mathbf{z}(k) - W_m(z)\,\theta^T(k)\mathbf{w}(k)$$

or

$$\varepsilon_1(k) = \mathbf{f}^T(k)\mathbf{z}(k) + \mu\eta(k) \qquad (5.6)$$

The recursive least-squares (RLS) used as the parameter adaptation algorithm has fast convergence if compared with others algorithms.

VI. RESULTS

In order to verify the analysis and to demonstrate the performance of the proposed three-phase AC power source using RMRAC controller (Fig. 5), both simulation and experimental results have been carried out. Table I presents the parameters of the reference model, three-phase PWM inverter, LC filter and load, used to in the simulation and experimental results.

TABLE I
REFERENCE MODEL, 3ϕ PWM INVERTER , LC FILTER AND LOAD PARAMETERS

Reference Model Parameters	LC filter inductance	L_m = 10 mH
	LC filter capacitance	C_m = 60 μF
	Load	R_m = 12 Ω
System Parameters	LC filter inductance	L_f = 5.4 mH
	LC filter capacitance	C_f = 75μF
	Inductors resistance	R_f = 0.1 Ω
	Load	R = 17 Ω
	Output rectifier capacitor	C_{rec} = 330μF
	Sampling interval	t_s = 1/1800 s

A. Simulation Results

Simulation results were obtained to demonstrate the effectiveness of the control scheme to track reference model outputs in the three-phase AC power source, even with sinusoidal or more complex waves.

Reference model, with the parameters presented in Table I, has the following transfer function:

$$W_m(z) = \frac{0.1940\,z + 0.1495}{z^2 - 1.1187\,z + 0.4623}.$$

The three-phase AC power source was simulated with a linear delta (Δ) connected load, such as in Fig. 2b. Simulated results are presented in Figs. 6 – 10.

Fig. 6 show the variation of the identified parameters in the direct RMRAC control applied to the system. The recursive least-squares algorithm guarantees fast parameters convergence. Three-phase output voltages are presented in Fig. 7. An amplitude step is applied to show the regulation capability of the system. Fig. 8 shows α and β voltages, where solid lines represent the output voltages and dashed lines represent reference model voltages. As it can be seen, tracking error is very small.

To show tracking capability to arbitrary waveforms, Fig. 9 presents a complex equilibrated three-phase set of reference model voltages, generated by the three shifted references with the following waveform: 250 (0.6 sin (ωt) + 0.2 sin(4ωt) + 0.2 sin (5ωt)), where ω = 2π12 rad/s. Respective output voltages are presented in Fig. 10. Almost no difference can be seen between the waveforms in Fig. 9 and Fig. 10. DC components were added to the first and the third waveforms intending to make clear the results.

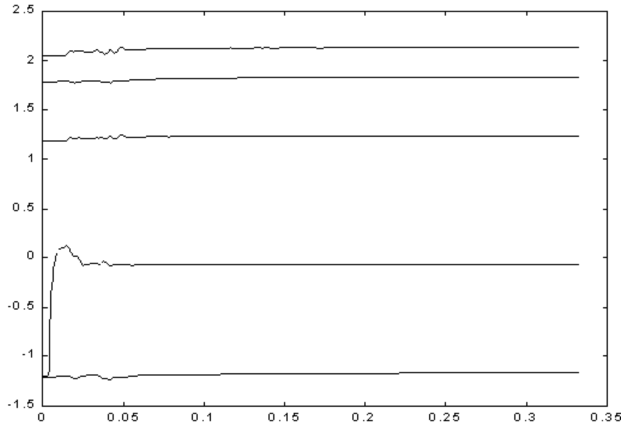


Fig. 6. Parameter adaptation (**q**).

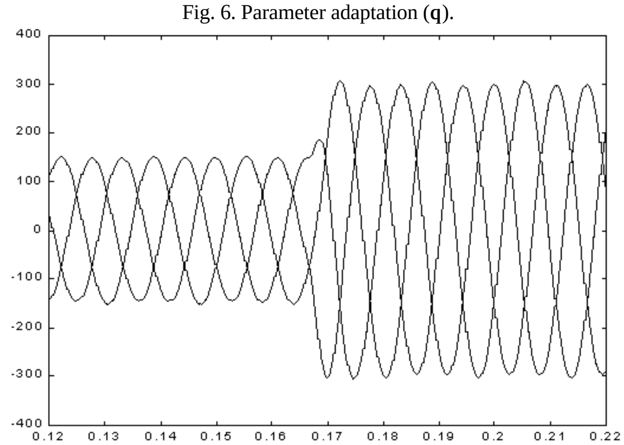


Fig. 7. Three-phase 60 Hz output voltages with a step reference.

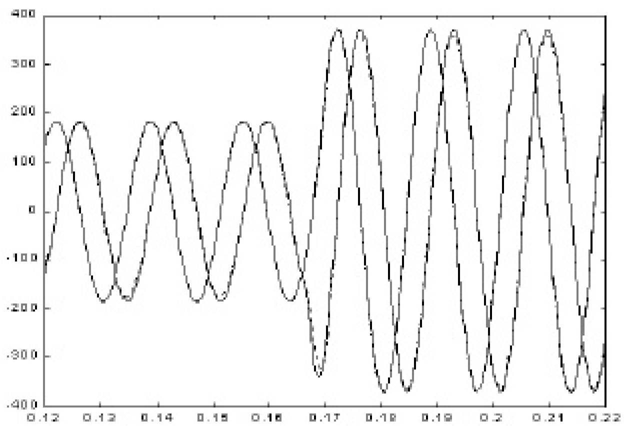


Fig. 8. α and β output (solid) and reference model (dashed) voltages.

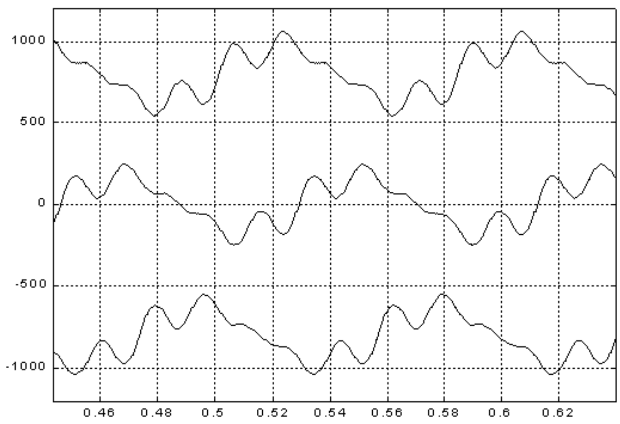


Fig. 9. Three-phase reference model output voltages following 250 (0.6 sin (ωt) + 0.2 sin(4ωt) + 0.2 sin (5ωt)).

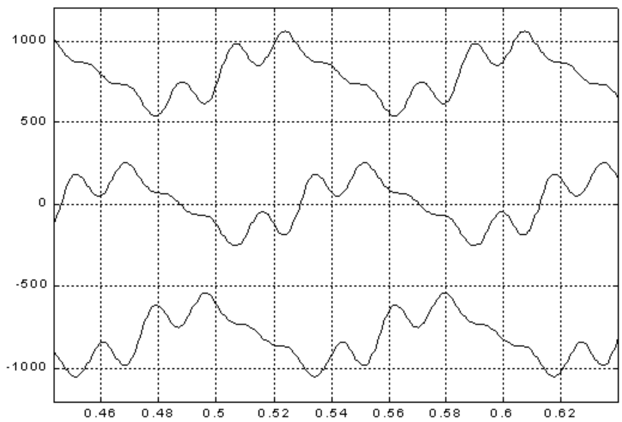


Fig. 10. Three-phase output voltages following 250 (0.6 sin (ωt) + 0.2 sin(4ωt) + 0.2 sin (5ωt)).

B. Experimental Results

In the implemented three-phase AC power source prototype it was used a system as in Fig. 1, with parameters of Table I. A linear wye (Y) connected load was used, as illustrated in Fig. 2a.

Fig. 11 shows the three-phase AC power source response for a three-phase set of reference model voltages, following 75 (0.6 sin (ωt) + 0.2 sin(4ωt) + 0.2 sin (5ωt)), where ω = 2π12 rad/s. Results are similar to that obtained in the simulation shown in Fig. 10.

To demonstrate system capability to generate arbitrary waveforms Fig. 12 presents the generation of a fundamental plus second and seventh harmonics waveform in each phase. Spectral analysis was made and is presented in Fig. 13, where it can be seen the presence of second and seventh harmonics.

A non-linear load is formed by a three-phase full-bridge rectifier with an output filter capacitor (C_{rec}) as shown in Fig. 14. Connecting the non-linear load together with the previous load (Fig. 2b), the three-phase AC power source response is presented in Fig. 15. For simplicity, just one phase voltage and current responses are presented.

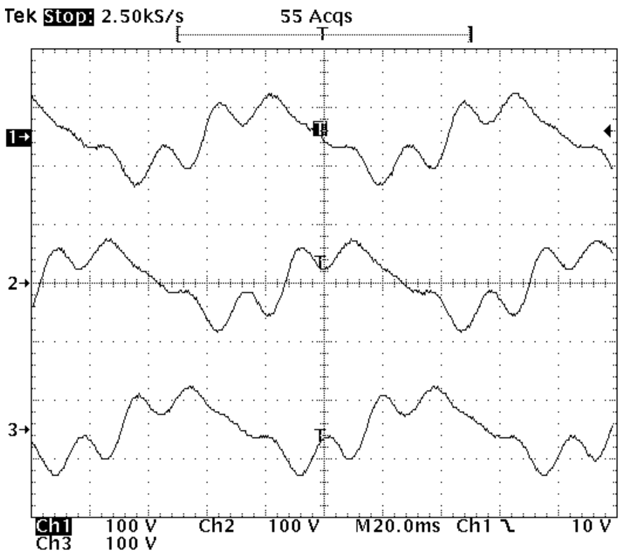


Fig. 11. Three-phase output voltages following 75 (0.6 sin (ωt) + 0.2 sin(4ωt) + 0.2 sin (5ωt)).

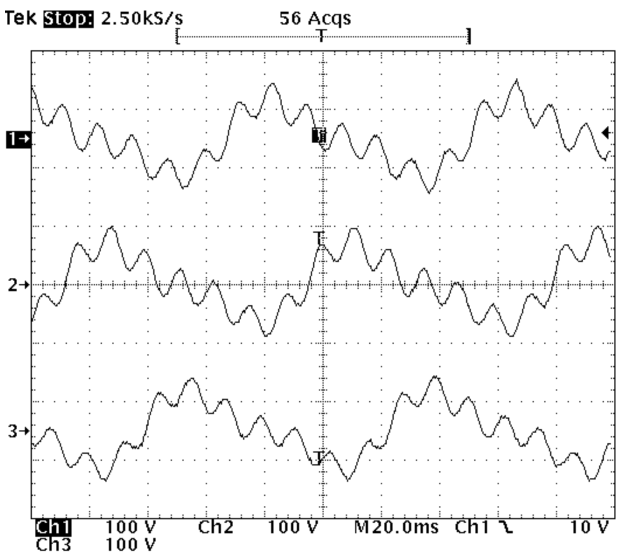


Fig. 12. Three-phase output voltages following 75 (0.6 sin (ωt) + 0.2 sin(2ωt) + 0.2 sin (7ωt)).

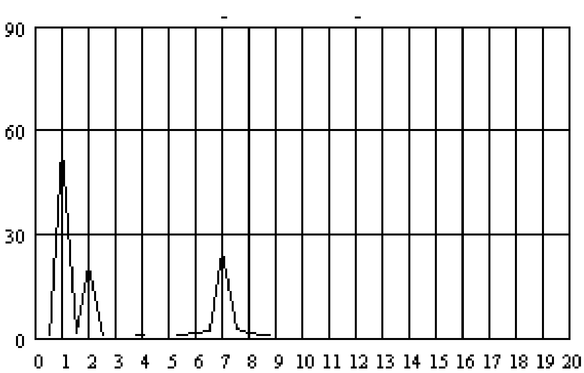


Fig. 13. Spectral analysis of one waveform of Fig. 12, showing 2^{nd} and 7^{th} harmonics.

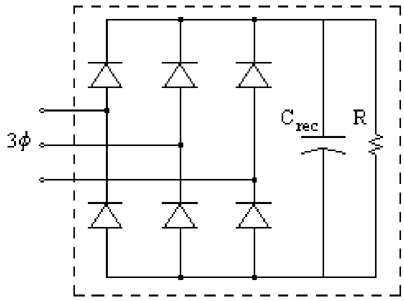


Fig. 14. Three-phase full-bridge rectifier with output filter capacitor (C_{rec}) and resistive load (R).

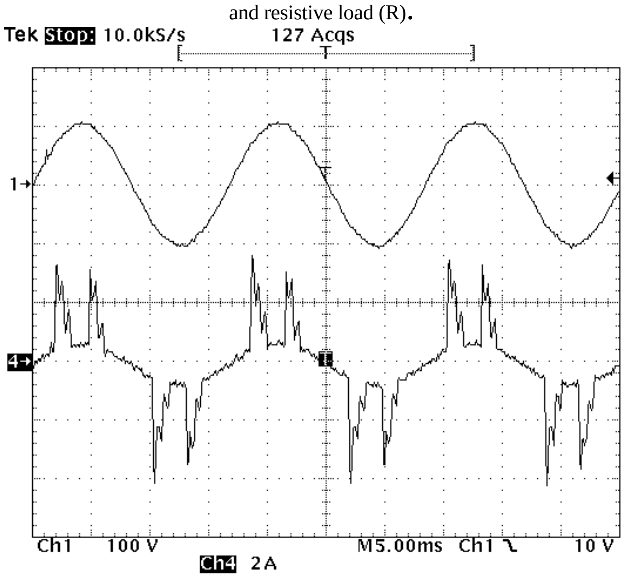


Fig. 15. One phase output voltage (up) and line current (down) for a non-linear load.

The presented experimental results are limited in frequency and amplitude due to the switching frequency capabilities and the amplitude voltage prototype limitations. Better results are obtained if a higher inverter switching frequency and a higher LC filter cutoff frequency are used. In this way, the proposed scheme can generate better quality waveforms as well as more harmonics can be added to the waveform.

At this time a new prototype with 6 kHz switching frequency is being developed, that can generate a high quality 60 Hz sinusoidal waveform, with up to 20 harmonics.

VIII. CONCLUSIONS

This paper proposes a high performance three-phase AC power source formed by a computer controlled PWM inverter with a LC filter. The scheme is controlled by a robust model reference adaptive controller (RMRAC), which guarantees robustness. The system is able to generate three equilibrated phases with pure sinusoidal or arbitrary waveforms. Simulation and experimental results demonstrate the effectiveness of the proposed scheme, generating several waveforms with good response for different loads. Moreover, this scheme can be designed for a reduced order plant, without the *a priori* knowledge of exact model of the load, LC filter and PWM inverter system. The proposed three-phase AC power source is particularly applicable to high power AC systems that require testing under variable amplitudes and frequencies, as well as the presence of harmonics.

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