Direct Adaptive Disturbance Accommodation

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Given any finite set of persistent disturbances that can be represented by a finite sequence of bounded, piecewise continuous functions, it is shown that stable disturbance rejection is possible for command generator tracker (CGT) model reference adaptive control if certain conditions are satisfied. If the system in question is disturbance-free, the addition of disturbance accommodating adaptive gains into the control law will not impede the asymptotic convergence of the tracking error. The special case of adaptive rejection only is derived and almost periodic disturbances are explored.

1. INTRODUCTION

With the relatively large amount of attention devoted to stochastic disturbances in systems, references are comparatively scarce on deterministic noise compensation. Disturbance accommodating control (DAC) has shown to be an effective state space method for deterministic noise suppression on linear time-invariant (LTI) systems using full state feedback.[6, 3] The internal model principle provides a much deeper result by allowing the construction of a controller within the framework of H^∞ optimal control that minimizes the effect of a disturbance transfer function on plant output.[5] These design techniques have the common element of representing disturbances as solutions of homogeneous LTI systems, where we term them persistent in the sense that they are considered bounded but not exponentially stable. It is easily seen that these are the non-repeated poles on the jω-axis, and both techniques offer a method of appending the controller with a disturbance rejecting filter.

If explicit knowledge of the plant is lost, it is known that disturbance rejection can be accomplished if the plant order is known.[13] However, direct model reference adaptive control does offer a different approach to disturbance suppression. For example, in the time domain the aforementioned jω disturbances are sinusoids of arbitrary amplitude and phase or steps of arbitrary amplitude, therefore each can be represented by a finite trigonometric polynomial with real coefficients. Since there is a bounded basis for these functions on the time interval $[0, \infty)$, it will be shown that there exists a stable disturbance accommodating adaptive control law for the command generator tracker (CGT) approach.[1, 2, 4]

The following derivations remove the assumption that persistent disturbances need to be generated by ordinary differential equations. It is noted that a proper subset of these functions are solutions to finite dimensional ODE's and such constructions are explored. A simple illustrative example is shown for almost periodic disturbances of known frequency.

2. DISTURBANCE-FREE CGT CONTROL

The following N^th order linear time-invariant system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx;\ x^0 \equiv x(0) \in \mathrm{R}^{\mathrm{N}} \end{aligned} \qquad (1)$$

is assumed to have norm $\|\cdot\|$ and $A : \mathrm{R}^{\mathrm{N}} \to \mathrm{R}^{\mathrm{N}}$, $B : \mathrm{R}^{\mathrm{M}} \to \mathrm{R}^{\mathrm{N}}$, and $C : \mathrm{R}^{\mathrm{N}} \to \mathrm{R}^{\mathrm{M}}$ are real valued matrix operators. We define (4) to be exponentially stable if

$$\mathrm{Re}(l_i) < 0 \ \text{ for each eigenvalue } \ l_i \in \mathrm{spec}(A). \qquad (2)$$

The above system is said to be output feedback stabilizable if there exists $G^* : \mathrm{R}^{\mathrm{M}} \to \mathrm{R}^{\mathrm{M}}$ such that the operator $(A+BG^*C) : \mathrm{R}^{\mathrm{N}} \to \mathrm{R}^{\mathrm{N}}$ is (exponentially) stable. Via the Kalman-Yacubovich lemma,[12] we state that the triple (A, B, C) is said to be strictly positive real (SPR) if A is stable, (A, B) controllable, and there exist symmetric positive definite matrix operators $P, Q \in \mathrm{R}^{\mathrm{N}\times\mathrm{N}}$ such that

$$\begin{aligned} A^T P + PA &= -Q \\ PB &= C^T . \end{aligned} \qquad (3)$$

For the non-adaptive CGT control design, we are given a plant model

$$\begin{aligned} \dot{x}_p &= A_p\, x_p + B_p\, u_p \\ y_p &= C_p\, x_p\ ;\ x_p^0 \equiv x_p(0) \in \mathrm{R}^{\mathrm{N}_p} \end{aligned} \qquad (4)$$

having $u_p, y_p \in \mathrm{R}^{\mathrm{M}}$ with the equations in (4) output feedback stabilizable by gain G_p^*. The above system is required to track the reference trajectory output of the following stable system:

$$\begin{aligned} \dot{x}_m &= A_m\, x_m + B_m\, u_m \\ y_m &= C_m\, x_m\ ;\ x_m^0 \equiv x_m(0) \in \mathrm{R}^{\mathrm{N}_m} \end{aligned} \quad \mathrm{N}_m \le \mathrm{N},\ y_m \in \mathrm{R}^{\mathrm{M}} \qquad (5)$$

with a marginally stable command generator model $(\mathrm{Re}(l(A_q)) \le 0)$:

$$\begin{aligned} \dot{q}_m &= A_q\, q_m \\ u_m &= C_q\, q_m\ ;\ q_m^0 \equiv q_m(0) \in \mathrm{R}^{\mathrm{N}_q} . \end{aligned} \qquad (6)$$

To this end, an ideal trajectory is introduced such that the ideal output matches that of the reference model:

$$\begin{aligned} \dot{x}_* &= A_p\, x_* + B_p\, u_* \\ y_* &= C_p\, x_* \\ y_* &= y_m \end{aligned} \qquad (7)$$

If there exists a transformation, (8), which satisfies the matching conditions given by (9a-d), we say that (5), (6), and (7) are *consistent* systems.

$$\begin{bmatrix} x_* \\ u_* \end{bmatrix} = \begin{bmatrix} S_{11}^* & S_{12}^* \\ S_{21}^* & S_{22}^* \end{bmatrix} \begin{bmatrix} x_m \\ u_m \end{bmatrix} \tag{8}$$

$$A_p S_{11}^* + B_p S_{21}^* = S_{11}^* A_m \tag{9a}$$

$$\left(A_p S_{12}^* + B_p S_{22}^*\right) C_q = S_{11}^* B_m C_q + S_{21}^* C_q A_q \tag{9b}$$

$$C_p S_{11}^* = C_m \tag{9c}$$

$$C_p S_{12}^* = 0 \tag{9d}$$

We define $e_y \equiv y_p - y_m$ to be the output error and introduce a control input for the system in (4) as:

$$u_p = S_{21}^* x_m + S_{22}^* u_m + G_p^* e_y \tag{10}$$

the closed loop can be shown to produce asymptotic output tracking, $e_y \to 0$ as $t \to \infty$.

In adaptive CGT control, the detailed knowledge of the plant assumed above is lost. However, making certain assumptions about the nature of (4) allows us to make the following conclusion that can be found in various references.[1, 2, 4]

Theorem 1*: If (4) is output feedback stabilizable by gain* G_p^*, $\left(A_p + B_p G_p^* C_p, B_p, C_p\right)$ *is SPR, and* (5), (6), *&* (7) *are consistent systems, then the adaptive gain laws,*

$$\begin{aligned} \dot{S}_{21} &= -e_y x_m^T \Delta_1 \\ \dot{S}_{22} &= -e_y u_m^T \Delta_2 \quad \Delta_i \ \textit{positive} \\ \dot{G}_p &= -e_y e_y^T \Delta_3 \quad \textit{definite} \end{aligned}$$

along with the control law, $u_p = S_{21} x_m + S_{22} u_m + G_p e_y$, *produce asymptotic output tracking* ($\lim_{t \to \infty} e_y = 0$) *with uniformly bounded adaptive gains.*

3. DISTURBANCE FUNCTION REPRESENTATIONS

Assume a real valued function, *h(t),* belongs to the space PC[0, ∞) of piecewise continuous functions with

$$\|h(t)\|_\infty = \operatorname*{ess\,sup}_{t \ge 0} |h(t)| < \infty \, ,$$

With this in mind, consider the space of R Cartesian products, $PC^R[0,\infty)$ having the property:

$$\|f\|_\infty \equiv \operatorname*{ess\,sup}_{t \ge 0} \left\{ |f_i| \mid 1 \le i \le R \right\} < \infty \tag{11}$$

Definition 1: *A uniformly bounded vector disturbance function $u_d(t)$ with norm has a finite representation* $s_l \equiv \sum_{k=1}^{l} a_k e_k f_k \in PC^R[0,\infty)$, *where $f_k(t)$ scalar functions with* $\|f_k\|_\infty = 1$, *a_k constant scalars, and* $e_k \in R^R[0,\infty)$ *constant unit vectors, if*

$$\|u_d - s_l\| = 0 \ . \tag{12}$$

This implies that $u_d(t) = s_l(t)$ *a.e.* $\forall t \ge 0$ *and therefore* u_d *Î* span{ $e_k f_k$ } ∀ $t \ge 0$.

The existence of such a sequence as described in this definition is stringent, for it is required that the partial sum needs to be almost equal along an infinite time interval as a direct result of the essential supremum norm appearing in (11). Each vector $f_i(t) \equiv e_i f_i(t)$ can be thought of as a vector in R^R that has time varying amplitude which cannot escape the unit ball. If $a_i \neq 0 \in R$ for all $1 \le i \le l$ and $\{e_i\}$ are a linearly independent set of vectors in R^R, then we have a spatial linear independence at any time t: $s_l(t) = 0$ iff $f_i(t) = 0$ for all $1 \le i \le l$. We can also approach this in a more general sense.

Definition 2: *A set of vectors* $\left\{f_k(t)\right\}_{k=1}^{l} \subset PC^R[0, \infty)$ *is said to be linearly independent if* $\sum_{k=1}^{l} a_k f_k(t) = 0$ *for all t* ≥ 0 *implies* $a_k = 0$ *for all* $1 \le k \le l$.

4. ADAPTIVE TRACKING WITH PERSISTENT DISTURBANCE REJECTION

Consider the following system with a persistent disturbance

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p u_p + \Gamma_p u_d \\ y_p &= C_p x_p \ ; \ x_p^0 \equiv x_p(0) \in R^{N_p} \end{aligned} \tag{13}$$

where $u_d \in PC^R$ and $\Gamma_p : PC^R \to R^{N_p}$ is a real valued matrix operator. For the tracking problem, making (13) follow the disturbance-free ideal system in (7) remains our control objective. In most cases, the presence of a persistent disturbance u_d will induce unacceptable levels of output tracking error in the closed loop. If this disturbance has a representation as defined in definition 1, it is shown that there is indeed an adaptive control strategy that produces asymptotic tracking. First we state a version of a lemma of Barbalat.[8]

Lemma: *If h(t) is a real-valued differentiable function on* [0, ∞) *with limit k* ∈ R *as t®¥ and* $\dot{h}(t)$ *uniformly continuous, then* $\lim_{t \to \infty} \dot{h}(t) = 0$.

We begin with the main result:

Theorem 2: *Assume that* span(G_p) ⊆ span(B_p) *and that* u_d *Î* PC^R *with finite representation* $u_d = \sum_{i=1}^{l} a_k e_k f_k$ *. Define* $f_d \equiv [f_1 \quad f_2 \quad \cdots \quad f_l]^T$ *. If (4) is output feedback stabilizable by gain* G_p^*, $\left(A_p + B_p G_p^* C_p, B_p, C_p\right)$ *is SPR, and* (5), (6), *&* (7) *are consistent systems, then the adaptive gain laws,*

$$\begin{aligned} \dot{S}_{21} &= -e_y x_m^T \Delta_1 \\ \dot{S}_{22} &= -e_y u_m^T \Delta_2 \quad \Delta_i \ \textit{positive} \\ \dot{G}_p &= -e_y e_y^T \Delta_3 \quad \textit{definite} \\ \dot{H}_p &= -e_y f_d^T \Delta_4 \end{aligned}$$

with the control law, $u_p = S_{21}x_m + S_{22}u_m + G_p e_y + H_p f_d$ *, produce asymptotic output tracking* ($\lim_{t\to\infty} e_y = 0$) *with bounded adaptive gains.*

Proof: Since span(Γ_p) $\subseteq$ span(B_p), there exists a transformation T such that $\Gamma_p = B_p$ T.
Take $E = -\begin{bmatrix} a_1 e_1 & a_2 e_2 & \cdots & a_l e_l \end{bmatrix}$ and define $H_p^* \equiv T \cdot E$. The state equation in (13) becomes

$$\dot{x}_p = A_p x_p + B_p(u_p - H_p^* f_d)$$

Let the trajectory error be defined as $e_* \equiv x_p - x_*$ and $e_y = C_p e_*$. Differentiating this error and evaluating the variables in (7) and (13), we have:

$$\begin{gathered}\dot{e}_* = A_p\, e_* + B_p G_p e_y \\ + B\left[\left(S_{21} - S_{21}^*\right)x_m + \left(S_{22} - S_{22}^*\right)u_m + \left(H_p - H_p^*\right)f_d\right]\end{gathered} \tag{14}$$

Let
$\Delta K = \left[\left(S_{21} - S_{21}^*\right) \quad \left(S_{22} - S_{22}^*\right) \quad \left(G_p - G_p^*\right) \quad \left(H_p - H_p^*\right)\right]$
and $z = \begin{bmatrix} x_m^T & u_m^T & e_y^T & f_d^T \end{bmatrix}^T$. Equation (14) and the adaptive gain laws in the statement of the theorem can be written in the following form:

$$\begin{gathered}\dot{e}_* = (A_p + B_p G_p^* C_p)e_* + B_p \Delta K\, z \\ \Delta\dot{K} = \dot{K} = -e_y z^T \Phi\end{gathered} \tag{15}$$

Define the Lyapunov function for the adaptive system in (15) as follows:

$$V \equiv e_*^T P e_* + \text{trace}\,\left[\Delta K \Phi^{-1} \Delta K^T\right] \geq 0 \tag{16}$$

where $\Phi = \text{diag}\left(\Delta_1, \Delta_2, \Delta_3, \Delta_4\right)$ is a block-diagonal positive definite matrix and P is positive definite such that $\left(A_p + B_p G_p^* C_p, B_p, C_p\right)$ satisfies (3) for some Q positive definite. Taking the time derivative of (16) and substituting (15) into the result yields

$$\begin{gathered}\dot{V} = e_*^T\left(\left(A_p + B_p G_p^* C_p\right)^T P + P\left(A_p + B_p G_p^* C_p\right)\right)e_* \\ + 2 \cdot z^T \Delta K^T B^T P e_* + 2 \cdot \text{trace}\left[\Delta\dot{K}\Phi^{-1}\Delta K^T\right]\end{gathered}$$

Invoking the equalities in the definition of SPR in (3) along with the fact that
$x^T y = \text{trace}[y x^T]$,

$$\begin{aligned}\dot{V} &= -e_*^T Q e_* + 2 \cdot \text{trace}\left[\left(e_y z^T + \Delta\dot{K}\Phi^{-1}\right)\Delta K^T\right] \\ &= -e_*^T Q e_* \leq -g\, \left\|e_*\right\|^2 \leq 0 \ \text{ for some } \gamma > 0.\end{aligned} \tag{17}$$

As a result of the inequalities stated in (16) and (17), Lyapunov theory guarantees the stability of the zero equilibrium point of (15) and we have e_* and *DK* uniformly bounded. Since x_m, u_m, f_d, and $e_y = C_p e_*$ are bounded, this implies that z is bounded. The second derivative of the Lyapunov function is

$$\begin{gathered}\ddot{V} = -2e_*^T Q \dot{e}_* \\ = -2e_*^T Q\left(\left(A_p + B_p G_p^* C_p\right)e_* + B_p \Delta K\, z\right)\end{gathered} \tag{18}$$

$$\begin{gathered}\leq 2\left\|e_*\right\|\left\|Q\right\|\left(\left\|A_p + B_p G_p^* C_p\right\|\left\|e_*\right\| + \left\|B_p\right\|\left\|\Delta K\right\|\left\|z\right\|\right), \\ \leq a\end{gathered} \tag{19}$$

for some $a > 0$

Equation (18) is bounded because each term in (19) is bounded in the appropriate norm. Invoking the mean value theorem, we have $\left|\dot{V}(t_1) - \dot{V}(t_2)\right| \leq a\left|t_1 - t_2\right| \ \forall t_1, t_2 \in \mathrm{R}$. Hence $\dot{V}(t)$ is uniformly continuous, so by the above lemma, $\lim_{t\to\infty}\dot{V}(t) = 0$. We have $\lim_{t\to\infty} e_* = 0$ because Q is positive definite in (17), therefore the tracking error has the property of asymptotic stability with $\lim_{t\to\infty} e_y = \lim C_p e_* = 0$. Furthermore, since *DK* is uniformly bounded, we have that the gains S_{21}, S_{22}, G_p, and H_p are uniformly bounded.

Hidden in this result is the fact that we already know what the representation for the disturbance is for all $t \geq 0$, an impossible task to accomplish in the general case! Even if one were to know the disturbance waveform in advance, there is still the difficulty of choosing the phase of the counteracting signal appropriately. Luckily, there are forgiving disturbances in PC^R and we consider the constant vectors in particular. There are many others, but we shall find that some additional information will be needed.

A useful special case of this control law allows us to adaptively reject disturbances in certain cases while using a fixed gain controller.

Corollary 1: *Assume* u_d, f_d *are as defined in Theorem 2 and* span(G_p) $\subseteq$ span(B_p). *Furthermore, assume* (13) *represents a fixed gain control system such that the triple* $\left(A_p, B_p, C_p\right)$ *is SPR.. Then the adaptive gain law:*

$$\dot{H}_p = -y_p f_d^T \Phi \text{ , } \Phi \text{ positive definite}$$

along with the control law, $u_p = H_p f_d$ *, produce asymptotic regulation* ($\lim_{t\to\infty} y_p = 0$) *with uniformly bounded adaptive gain* H_p.

Proof: As in Theorem 2, we rewrite system (13) as

$$\dot{x}_p = A_p x_p + B_p(u_p - H_p^* f_d)\,.$$

Using the adaptive gain input, the entire set of equations has the following form,

$$\begin{gathered}\dot{x}_p = A_p x_p + B_p \Delta H_p f_d\,. \\ \Delta\dot{H} = \dot{H} = -y_p f_d \Phi\,,\end{gathered}$$

where $\Delta H = H_p - H_p^*$. A similar Lyapunov analysis of these equations proves the conclusion of the corollary.

We state a direct result, available through passivity theory, that becomes a consequence of Theorem 2 applied to CGT control. It is observed that a constant disturbance need not be present on the adaptive system in order to have asymptotic convergence of e_y to zero.

Corollary 2: *Theorem 1 also holds when*

$u_p(t) = S_{21}x_m + S_{22}u_m + G_p e_y + c - a\int_0^t e_y(t)\,dt$ *with c* Î

R^M *and scalar a > 0. Furthermore, we have* $\int_0^t e_y\,dt$ *is bounded for all t* ≥ 0.

Proof: Assume there exists a nonzero constant disturbance in (13) and take $\Gamma_p = 0$. Span(Γ_p) ⊆ span(B_p) so if we assume $f_1 = 1$, without loss of generality we can choose T = 0 which implies $H_p^* = 0$ as in the proof of Theorem 2. Take $a = \Delta_4$ and let the initial condition on H_p, $H_p(0)$, be equal to $c \in \mathrm{R}^M$. We have $H_p(t) = c + \int_0^t \dot{H}_p(t)\,dt$ is uniformly bounded from Theorem 2, so the integral must be bounded and the corollary follows from the definition of $\dot{H}_p$.

Any disturbance-free system (4) can be thought of in a similar manner, one can assume any "fictitious" $u_d \in \mathrm{PC}^R$ with a finite representation. Thus we conclude the following:

Theorem 3: *Theorem 1 is valid when*

$$u_p(t) = S_{21}x_m + S_{22}u_m + G_p e_y + \left(L - \int_0^t e_y f^T\, F\,dt \right) f \quad \textit{where}$$

f Î $\mathrm{PC}^{N'}$, F Î $\mathrm{R}^{N' \times N'}$ *a positive definite matrix, and a matrix operator* $L : \mathrm{PC}^{N'} \rightarrow \mathrm{R}^M$. *Furthermore, we have* $\int_0^t e_y f^T\,dt$ *is bounded for all t* ³ *0.*

Proof: Assume a nonzero disturbance and let $\Gamma_p = 0$. WLOG we can take T = 0. Set $F = \Delta_4$ as in the statement of Theorem 2 and we have

$$H_p(t) = H_p(0) + \int_0^t \dot{H}(t)\,dt = L - \int_0^t e_y f^T F\,dt$$

By Theorem 2, H_p is uniformly bounded and this implies $\int_0^t e_y f^T\,dt$ is bounded. The result follows using the definition of u_p in Theorem 2.

Theorem 3 gives us an alternative approach to CGT adaptive control since the addition of a disturbance rejection term will not destroy asymptotic stability. In fact, such a design may offer a broad range of possibilities for adjusting convergence rates in practice.

5. ALMOST PERIODIC REPRESENTATIONS

We have mentioned in the introduction that sinusoidal functions are solutions to the subset of differential equations which have non-repeated poles on the jω axis. These types of disturbances are in PC and it is not difficult to observe that we can extend them to vector functions in PC^R. Beginning with an illustrative example in PC, if the disturbance is sinusoidal with known frequency w_d, observe that we have a finite representation:

$$\begin{aligned} u_d(t) &= c \cdot \sin(w_d t + q) = a \cdot \cos(w_d t) + b \cdot \sin(w_d t) \\ &= a \cdot f_1(t) + b \cdot f_2(t) \end{aligned}$$

The unknown amplitude c and phase $0 \le q < 2\pi$ uniquely determine the real constants a and b. It is also noted that the functions $f_i(t)$ have the linear independence property stated in definition 2. Expanding this argument to R dimensions having P linearly independent sinusoidal disturbances (in PC) with known frequencies w_{d_i}, $1 \le i \le P$, observe that this becomes a sum of vector functions:

$$\begin{aligned} u_d(t) &= \sum_{i=1}^{P}\sum_{j=1}^{R}\left[a_{ij}e_j \cos(w_{d_i}t) + b_{ij}e_j \sin(w_{d_i}t) \right] \\ &= \sum_{i=1}^{P}\sum_{j=1}^{R}\left[a_{ij} f_{ij}^a + b_{ij} f_{ij}^b \right] \end{aligned}$$

where a_{ij} are scalars, e_j are unit vectors. The set $\{f_{ij}^a\} \cup \{f_{ij}^b\} \subset \mathrm{PC}^R$ is a linearly independent set of vectors that covers the space of all sinusoidal elements with frequencies $\{w_{d_i}\}_{i=1}^{P}$ in PC^R. We create a $2P$ length vector, f, of all of the cosines and sines and define a matrix E as:

$$f = \left[\cos(w_{d_1}t) \quad \sin(w_{d_1}t) \quad \cdots \quad \cos(w_{d_P}t) \quad \sin(w_{d_P}t) \right]^T$$

$$E = -\sum_{j=1}^{R}\left[a_{1j}e_j \quad b_{1j}e_j \quad \cdots \quad a_{Pj}e_j \quad b_{Pj}e_j \right]$$

If span(Γ_p) ⊆ span(B_p) in equation (13), we have a transformation, T, between the two matrices such that $\Gamma_p = B_p$ T. Hence $H_p^* = TE$ is our "ideal" matrix on which we base our disturbance rejecting adaptive gain law. In practice, this would require that one create $2P$ sinusoids for use in the adaptive loop, a relatively simple matter.

6. CONCLUSIONS

We have illustrated an adaptive technique which complements model reference adaptive control and guarantees asymptotically stable tracking in the presence of a disturbance for certain cases. Several results follow from the main theorem, notably that there exists a direct adaptive disturbance rejecting controller without model reference adaptation. Disturbances with almost periodic representations have been discussed as one of the simplest cases, but it is noted that these functions are only a subset of possible piecewise continuous functions.

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