Study of Hopf bifurcations in a simple power system model

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Abstract: This paper studies the occurrence of Hopf bifurcations in a simple single-machine-infinite-bus (SMIB) power system model. Our interest is primarily on the subcritical or supercritical nature of the Hopf bifurcations under variations in operating parameters. We observe that the Hopf bifurcations are mostly subcritical. When the exciter control is a fast high gain control and when the Thevenin equivalent transmission line impedance is high, the Hopf bifurcation can become supercritical leading to birth of stable limit cycles. Two degenerate bifurcations namely, one related to double zero eigenvalues, and the other related to Hopf normal form coefficient becoming zero, give rise to homoclinic orbits and nested limit cycles respectively.

Key words: power system dynamics, bifurcations, power system stability.

1. Introduction

Under parametric variations, the phase portrait of a dynamical system undergoes qualitative changes at bifurcation points. When the changes occur around equilibrium points, the bifurcations are local and there exist well-established tools in bifurcation theory for analyzing the local bifurcations in a hierarchical fashion. Roughly speaking, when all the qualitative changes in the phase portrait near the bifurcation point can be characterized by variation of one parameter, the bifurcation is said to be of codimension one bifurcation. On the other hand, when the qualitative changes in the phase portrait can only be characterized by variation of two parameters, the bifurcation is of codimension two.

For smooth systems, Hopf bifurcations and saddle node bifurcations are the "typically" encountered codimension one bifurcations. When the system Jacobian has a simple pair of purely imaginary eigenvalues and no other zero real part eigenvalues, a Hopf bifurcation (denoted B_H) occurs. Hopf bifurcation Theorem [1,2] then predicts the existence of limit cycles near the bifurcation point under some transversality conditions. The stability of the limit cycles depend on the nonlinear terms and specifically on the sign of a cubic normal form coefficient that is usually denoted "*a*" [1,2]. When *a* is negative, the Hopf bifurcation is supercritical and when *a* is positive, the bifurcation is subcritical. Supercritical and subcritical Hopf bifurcations lead to birth of stable and unstable limit cycles respectively. When the system Jacobian has a simple zero eigenvalue and no other zero real part eigenvalues, a saddle node bifurcation (denoted B_{SN}) occurs under certain transversality conditions. Saddle node Theorem then proves the annihilation of the equilibrium point at a saddle node bifurcation. In summary, Hopf bifurcations are used to locate birth of limit cycles in a nonlinear system and saddle node bifurcations stand as static limit type boundaries where equilibrium points disappear. Hopf and saddle node bifurcations have been studied extensively in the context of power system dynamics (e.g. [3,6]).

When the operating parameters are near a codimension two bifurcation, the phase portraits may be richer in nonlinear phenomena. In this paper, we study two different codimension two bifurcations by phase portraits. At the first denoted a double zero bifurcation (denoted B_{DZ}), the system Jacobian has couple of zero eigenvalues and no other zero real part eigenvalues. Double zero bifurcations arise at intersections of saddle node and Hopf bifurcations. Homoclinic orbit which is a closed trajectory connecting an equilibrium to itself, is of interest in the context of complicated system behavior. Homoclinic orbits are difficult to locate in general. Whereas, they are born at double zero bifurcations under certain technical conditions. Double zero bifurcations in power system examples have been analyzed in [6,7].

The other type of codimension two bifurcation studied is a degenerate form of Hopf bifurcation denoted B_H^0 where the cubic normal form coefficient *a* becomes zero. Bifurcation theory [1,2] predicts the existence of nested limit cycles in the vicinity of B_H^0 and we illustrate the same in the paper.

In this paper, we study the dynamics of a simple yet fundamental SMIB model through phase portraits and Hopf normal form analysis. Previously in [8], we studied a similar model and showed the existence of hardlimit induced stable limit cycles and hardlimit induced chaos in a similar model. In this paper, we study the problem of when stable limit cycles unrelated to hardlimits, may exist.

2. A simple power system model

We consider a single-machine-infinite-bus power system whose dynamics is represented by a simple model shown below that is commonly used in the literature:

$$\dot{d} = 2pf_0 w \tag{1}$$

$$M\dot{w} = -Dw + P_T - P_G \tag{2}$$

$$T_{d0}\dot{E}' = -(x_d + x)E'/(x'_d + x) + (x_d - x'_d)\cos d/(x'_d + x) + E_{fd} \tag{3}$$

$$T_A\dot{E}_{fd} = -K_A(V - V_{ref}) - (E_{fd} - E_{fd0}) \tag{4}$$

where

$$P_G = E_d I_d + E_q I_q \qquad V = \sqrt{E_d^2 + E_q^2}$$

$$E_d = \frac{X_q \sin \boldsymbol{d}}{X + X_q} \qquad E_q = \frac{XE' + X_d' \cos \boldsymbol{d}}{X + X_d'}$$

$$I_d = \frac{E' - E_q}{X_d'} \qquad I_q = \frac{E_d}{X_q}$$

In the model, equations (1) and (2) represent the electromechanics. Equation (3) simplifies the electromagnetics in the form of a single axis flux decay equation, and equation (4) is a first order delay type field voltage control.

Here V stands for the generator terminal bus voltage; P_G and Q_G are the real and reactive power outputs of the generator; $\boldsymbol{d}$ is the internal rotor angle; $\boldsymbol{w}$ is the speed deviation in per unit. The state space for the power system model now consists of four variables $\boldsymbol{d}$, $\boldsymbol{w}$, E', and E_{fd}. Among the parameters, the synchronous machine parameters M, X_d, X_q, and X_d' do not vary during system operation. So in our study, they are specified as $M = 2*H = 10$ sec., $X_d = X_q = 1.0$, and $X_d' = 0.2$. Also, we assume that the damping constant $D = 1$ pu.

Among the remaining parameters, we study the locus of Hopf bifurcations by varying two parameters at a time. The steady state real power output P_T is taken as one parameter while the other parameter is one of K_A, T_A, X, T_{d0}', and V_{ref}. The parameters P_T and V_{ref} are the real power and nominal bus voltage setting of the generator and these two are true operating parameters. K_A and T_A are exciter control parameters that depend on the type of control system used. Even though T_{d0}' is a machine parameter, we study variations in T_{d0}' since it is seen to be a sensitive parameter in determining the nonlinear properties of the Hopf bifurcation. Line reactance X models the static power transfer capacity of the SMIB power system. For a high gain fast exciter controls which are commonly used, the bus terminal voltage V will be close to the voltage reference setting V_{ref} from Equation (4). Therefore, as discussed in power system text books, the maximum real power that can be transferred from the generator to the infinite bus is limited by V_{ref} / X. The higher the values for reactance X, the more stressed the SMIB system is.

3. Study of Hopf bifurcations

In the following figures, dashed lines represent subcritical Hopf bifurcations while solid lines represents supercritical bifurcations. The Hopf coefficient $\boldsymbol{a}$ is computed along the Hopf bifurcation locus using a Matlab routine presented in [9]. B_H^0 is the transition point where the cubic normal Hopf coefficient $\boldsymbol{a}$ equals zero. It can be analyzed by computing the sign of a fifth order normal form coefficient a_5, and a brief discussion of the codimension two bifurcation B_H^0 follows later.

As stated earlier, our primary interest is on the study of supercritical Hopf bifurcations in the model. In our simulations, we found that the Hopf bifurcations are mostly subcritical. Supercritical Hopf bifurcations could be found only under the conditions of 1) high control gain K_A , 2) fast exciter control i.e. small T_A , and 3) large Thevenin reactance X, and small generator time-constant T_{d0}'. The parameters are varied accordingly to study transitions from and around supercritical Hopf bifurcations.

In all the plots, the Hopf bifurcation locus B_H divides the parameter space into two regions: 1) The region to the left of B_H is denoted "stable" where the equilibrium point is locally stable, and 2) the region to the right of the B_H is denoted "unstable" where the equilibrium is locally unstable.

All the remaining parameters assume the parameters $T_{d0}' = 6$, $T_A = 0.01$, $K_A = 1000$, and $V_{ref} = 1.05$ unless as stated otherwise. Figure 1 shows the Hopf bifurcation locus for (P_T, K_A) variation. There are two different types of behavior, for small gain values roughly, K_A less than 100, and larger gain K_A values. For small K_A , the Hopf bifurcation occurs at lower P_T values when K_A increases. On the other hand, for gains over 100, the Hopf bifurcation occurs at higher P_T values when K_A is increased and the equilibrium is small-signal stable over larger values of P_T . Moreover, the bifurcation is supercritical only for large K_A and large X. We can expect that stable limit cycles will be present surrounding the unstable equilibria for parameter values near the Hopf bifurcation locus in Figure 1.

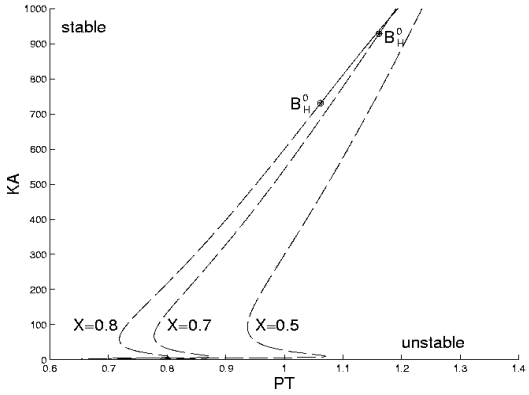

Figure1. Hopf bifurcation locus of (P_T, K_A)

Figure 2 shows Hopf bifurcation loci when P_T and T_A are varied. As T_A becomes smaller, the exciter control is faster and the equilibrium becomes remains stable over larger values of P_T. We can see from Figure 2 that Hopf bifurcation is supercritical only when X is large and the time-constant T_A is small.

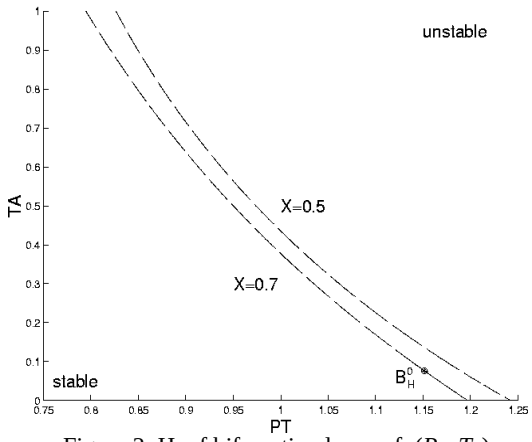

Figure 2. Hopf bifurcation locus of (P_T, T_A)

Figure 3 shows Hopf bifurcations as we vary P_T and V_{ref}. The Hopf bifurcations occurs later in P_T, when V_{ref} becomes larger. Moreover, V_{ref} is seen to be insensitive to the sign of the Hopf coefficient $\boldsymbol{a}$.

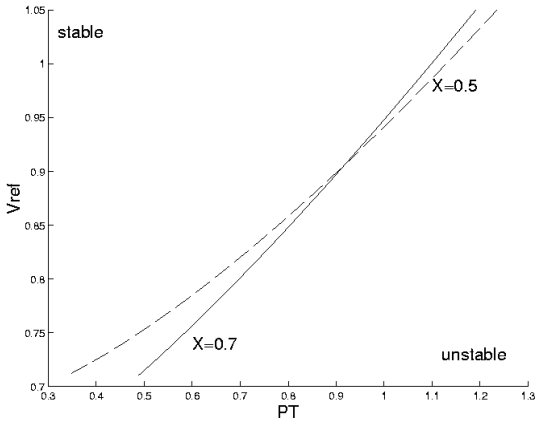

Figure 3. Hopf bifurcation locus of (P_T, V_{ref})

Figure 4 studies variations in P_T and T_{d0}'. Hopf bifurcation being a dynamic bifurcation is sensitive to time-constants both in terms of when it occurs and in determining the nonlinear nature of the bifurcation.

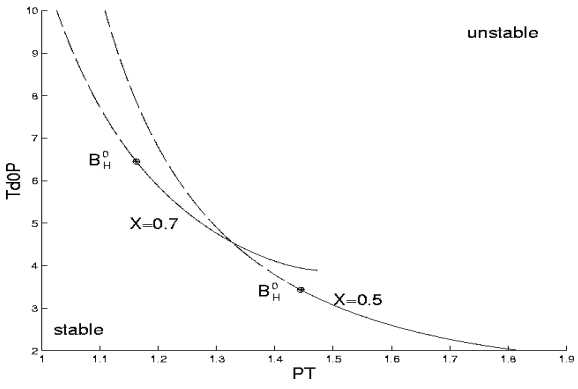

Figure 4. Hopf bifurcation locus of (P_T, T_{d0}')

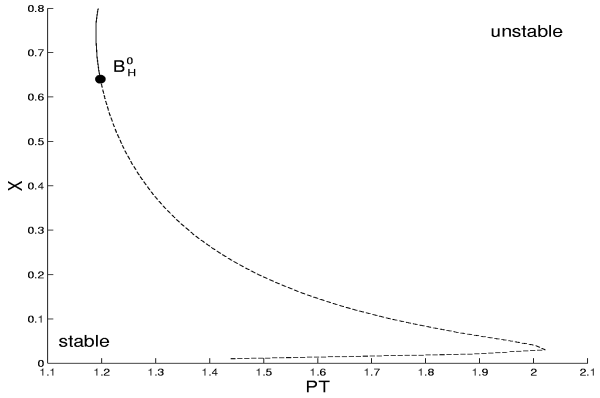

Figure 5. Hopf bifurcation locus of (P_T, X)

Figure 5 shows Hopf bifurcations for variations in P_T and X. Excepting for very small values of X, the Hopf bifurcation occurs at smaller P_T values when X increases. As stated earlier, the saddle node bifurcation or the static limit that occurs approximately at V_{ref}/X, is inversely proportional to X. For larger X values above 0.8 (not shown in Figure 5), Hopf bifurcation locus intersects the saddle node bifurcation locus at a codimension two, double zero bifurcation B_{DZ}. We will study the qualitative system features near the codimension two bifurcations, namely, the degenerate Hopf bifurcation B_H^0, and the double zero bifurcation B_{DZ} in the next section.

In summary, we observe that the Hopf bifurcations in the simple SMIB model (1)-(4) are mostly subcritical. Supercritical bifurcations and stable limit cycles do exist when the exciter control is a fast high gain control and when the system is stressed in the form of large line reactance X.

4. Study of Codimension two bifurcations

4.1 Double Zero bifurcation B_{DZ}:

Hopf bifurcation locus B_H intersects the saddle node bifurcation locus B_{SN} at the B_{DZ} bifurcation in Figure 6. The system Jacobian has a double zero eigenvalue at the B_{DZ} bifurcation, and the remaining two eigenvalues have negative real parts. By analyzing the quadratic normal form of the vector field at the B_{DZ} bifurcation point, the presence of homoclinic orbits along a saddle connection bifurcation locus B_{SC} can be proved [1,2]. Figure 6 shows the bifurcation loci computed numerically for our model. In Figure 6, stable limit cycles born from the supercritical Hopf bifurcations along B_H are annihilated along the saddle connection bifurcation locus B_{SC} by colliding with a saddle point that is nearby. The existence of such homoclinic orbits which are attracting trajectories from the unstable focus inside the homclinic orbit is interesting in the context of complicated nonlinear phenomena. It is interesting to note that the homoclinic orbits along B_{SC} exist over a rather large range of P_T and X values. The implications of homoclinic orbits will be studied in a future paper.

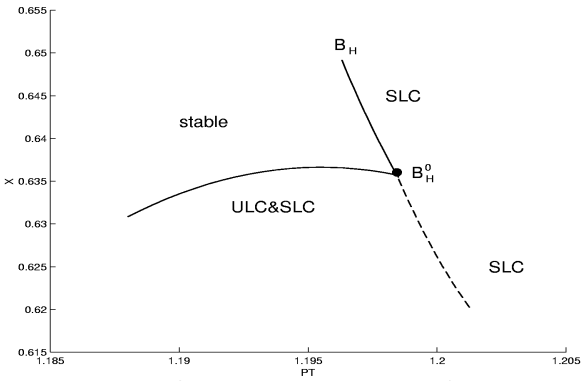

Figure 6. Bifurcation diagram near B_{DZ} for parameter variations in P_T and X

4.2 Degenerate Hopf bifurcation B_H^0:

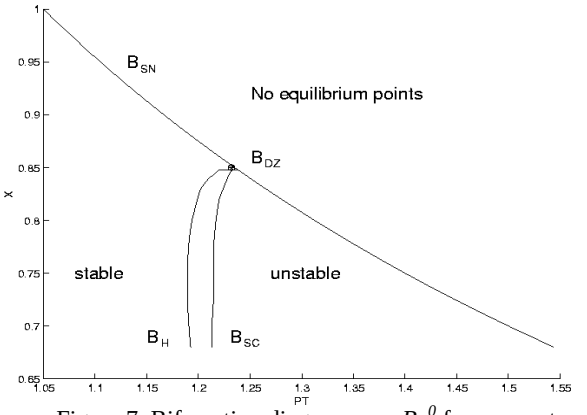

Figure 7. Bifurcation diagram near B_H^0 for parameter variations in P_T and X

The Hopf bifurcation changes from supercritical Hopf to subcritical Hopf at the B_H^0 bifurcation point in Figure 7. In other words, the Hopf cubic normal form coefficient $\boldsymbol{a}$ is zero at B_H^0 . By analyzing the fifth order normal form at the bifurcation point, it can be proved that nested limit cycles are present for some parameter values near B_H^0 and the sign of the fifth order normal form coefficient $\boldsymbol{a_5}$ will determine whether the stable limit cycles surround and dominate over the unstable limit cycles (when $\boldsymbol{a_5} < 0$) or vice versa. Figure 7 shows the actual bifurcation diagram for our model that has been computed numerically.

From Figure 7, the fifth order coefficient $\boldsymbol{a_5}$ appears to be negative. Accordingly, when the Hopf bifurcation is supercritical (B_H above B_H^0 in Figure 7), stable limit cycles are born at B_H when P_T increases above B_H . For a normal subcritical Hopf bifurcation, we expect unstable limit cycles (ULC's) to exist surrounding the stable equilibrium for P_T near B_H^0 . On the other hand, for parameters near B_H^0, when $\boldsymbol{a_5} < 0$, both stable and unstable limit cyles exist near B_H for P_T less than B_H values**.** Stable and unstable limit cycles are born along the global bifurcations called the cyclic fold bifurcations B_F, and the unstable limit cycles are annihilated at the subcritical Hopf bifurcations. The stable limit cycles (SLC's) persist on both sides of the subcritical Hopf bifurcation locus for B_H below B_H^0 in Figure 7. Specifically, in the region bounded by the cyclic fold bifurcations B_F and B_H in Figure 7, SLC's of larger size surround ULC's of smaller size. The stable manifold of the ULC separates the regions of attractions of the stable equilibrium point and of the SLC. Therefore, for parameter values in this region, if a postfault initial condition is outside the stability boundary of the stable equilibrium anchored by ULC, then the transient may still converge to the SLC leading to sustained oscillations. An example of the stable limit cycle (SLC) surrounding the unstable limit cycle (ULC) is shown in Figure 8.

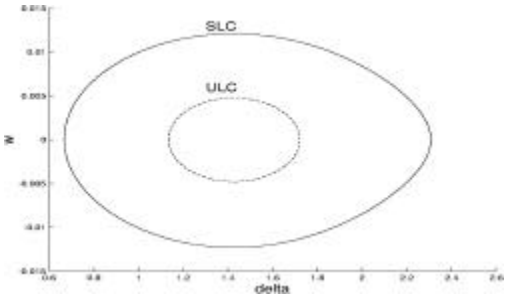

Figure 8. Example of nested limit cycles at P_T = 1.195 and X = 0.635.

5. Conclusions

The paper analyzes the nonlinear nature of Hopf bifurcations in a simple power system model. The paper identifies parametric conditions when the Hopf bifurcation is supercritical. Occurrence and implications of two degenerate bifurcations are studied. Presence of homoclinic orbits and nested limit cycles in the model are observed.

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