A STOCHASTIC APPROACH TO THE FLOOD CONTROL PROBLEM[1]

Marinho G. Andrade [2] ; Marcelo D. Fragoso[3] ; Adriano A.F.M. Carneiro[4]

Abstract

We propose a stochastic model in conjunction with reliability analysis concepts to improve estimates for the protection volume that should be allocated in a reservoir to control a flood wave. In this approach, the inflow that reaches the reservoir during a flood is considered to be a load, and the reservoir capacity to control this flood is considered to be the resistance that the reservoir offers against the propagation of the flood. Here, the load and the resistance are modeled as a diffusion stochastic process, and the protection volume is determined via Ito formula. In this scenario, an explicit formula for the failure risk is derived. The parameter inference is carried out by a Bayesian approach for a time discret version of the load, and the estimates are obtained by using Monte Carlo Markov Chain Algorithms (MCMC). The maximum likelihood estimators are used in the comparison. The record utilized comprises nine years of daily inflow rates during flood periods that come to the Chavantes Hydroelectric Power Plant (CHPP) in Southeast Brazil. The protection volumes estimated through the proposed model are compared to the volumes obtained by other existing methods.

Keywords: Diffusion process, flood control,reliability analysis, Monte Carlo Markov Chain, Bayesian inference.

1 Introduction

The Brazilian Hydrothermal generation system is predominantly hydro and most of the hydroelectric system reservoirs were designed and built with the sole objective of generating electric power, disregarding the regulation of the water flows. In order to control the floods, it is necessary to predict the availability of empty reservoir volumes, capable of absorbing some inflow parcels, to avoid or reduce the damage caused to the downstream area. However, keeping protection volumes in the reservoirs leads to a reduction in the energy availability of the hydroelectric power plants, increasing the thermal power generation which is much more expensive. Therefore, from an energy standpoint, it is desirable to allocate the smallest possible protection volume, while from a flood control standpoint, it is desirable to have reliable estimate of the possibility of the reservoir failing to control a flood when a given protection volume is allocated. Minimize this conflict is tantamount to allocate the smallest possible protection volume, while minimizing the probability of the reservoir failing to contol a flood.

Among the methods used in the calculation of the protection volume and failure risk, the most traditional method is the Volume x Duration Curve Method (VDC) (see, e.g., [1]). The main difficulties this method are the sampling variations that cause the (VDC) to be nonconcave, as required, and the choice of which probability distribution should be fitted to the data. Another difficulty is that this method provides only one protection volume for the whole wet season disregarding the flood potential variation. Since there are conflicting interests, this solution is not efficient.

In an attempt to weed out the difficulties of the (VDC) method, a method named Critical Path (CP) was proposed by Kelman in [2]. This method allows the determination, for every wet season day, of the protection volume associated to a previously established risk and ensures, with some risk, that the discharge constraints will not be violated if the hydrological situation observed in the past is repeated. However, the method is not appropriate for the data records usually available. For instance, assuming a size record of $n = 30$ years and a risk $r = 0.02$ (50 year return period), the number of unprotected critical paths in this case would be $k = rn = 0.6 < 1$. Therefore, the use of this methodology is only viable when series of significant length are utilized, which can only be obtained from synthetic series of daily average streamflow.

We propose in this work a stochastic model(a stochastic differential equation) along with a set of reliability analysis concepts. Reliability approach has been reviewed by Yen in [3], and more recently by Meon in [4]. In these models, the inflow that reaches the reservoir during a flood is considered as a load,and the reservoir capacity

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[2] SCE - ICMC - USP, CP. 668 - 13560-970 - São Carlos - SP, Brazil, e-mail: marinho@icmc.sc.usp.br

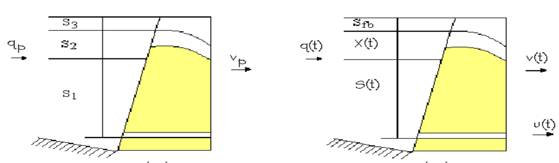
[3] National Laboratory for Scientific Computing - LNCC/CNPq, Av. Getulio Vargas 333, Quitandinha, CEP 25651-070, Petrópolis - Rio de Janeiro- Brazil, e-mail frag@lncc.br

[4] SEL - EESC - USP, CP. 359 - 13560 - 270 - São Carlos -SP, Brazil, e-mail: adriano@sel.eesc.sc.usp.br

of controlling this flood is considered as the reservoir resistance. In [3] and [4], the load and the resistance are considered as independent time-invariant random variables, which simplifies the problem. However, this is quite inadequate because the load and the resistance are related by the dynamic equation of the reservoir as it appears in this work. Regarding modelling via stochastic differential equation, Unny and Karmeshu [5], Bodo and Unny [6], Lin and Wang [7] has employed this class of models in rainfall-runoff studies. In their work, the input to the hydrologic system is modeled as a diffusion stochastic process(a Itô equation). Here the load and the resistance are modeled as a diffusion stochastic process. Assuming that the processes are homogeneous for short periods of time within the wet season and using Itô formula [8], we can formulate the boundary value problem, whose solution expresses the failure risk as a function of the reservoir resistance. The proposed method allows the consideration of the flood potential variation during the wet season and the estimation of the protection volumes, associated to a given risk, for different periods. With this method it is possible to allocate dynamically the protection volume without incurring in the difficulties found in the Critical Paths Method.

2 The Load Versus Resistance

In order to formulate the problem of the reservoir reliability in the flood control, we shall consider a reservoir designed to operate with a maximum volume S_1 and a protection volume S_2 to control a project flood q_p, assuming a project discharge v_p, as shown in Figure 1(a). The volume associated to the freeboard dimension S_3, for the retention of wind-induced waves, is not considered in this work. The maximum reservoir volume is given by $S_{max} = S_1 + S_2$ and the level associated to this volume is called max-maximorum.



(a) (b)

Figure 1(a)-Representation of the project volumes. (b) - Representation of the volumes at time t.

Figure 1(b) represents the reservoir operating with a stored volume $S(t)$ at a time $t \in [0, T]$ when a flood wave $q(t)$ arrives at the reservoir. The power plant discharged and spilled water at this time are denoted by $u(t)$ and $v(t)$ respectively, and the total discharge as $r(t) = u(t) + v(t)$. In flood control problems this outflow $r(t)$ is limited by the reference discharge q_r, which represents the maximum discharge that the reservoir can release without damage to the downstream area. Then the protection value (the resistance) $X(t)$ of the reservoir is given by:

$$X(t) = S_{max} - S(t) \tag{1}$$

Considering the time interval $[t, t + \Delta t]$, the resistance variation in this interval is given by:

$$X(t + \Delta t) - X(t) = -[S(t + \Delta t) - S(t)] \tag{2}$$

We know that in order to be able to control a flood, the maximum outflow that the reservoir can release is the reference discharge q_r. Thus, the continuity equation of the reservoir in the interval $[t\,, t + \Delta t]$ is given by:

$$S(t + \Delta t) = S(t) + \int_t^{t+\Delta t} q(s)ds - \int_t^{t+\Delta t} q_r ds \tag{3}$$

The first integral in (3) represents the load that reaches the reservoir in the interval $[t, t + \Delta t]$. This integral is written as:

$$Y(t + \Delta t) = Y(t) + \int_t^{t+\Delta t} q(s)ds \tag{4}$$

Then we can write the resistance variation by substituting (3) and (4) in (2) as follows:

$$\Delta X(t) = -\Delta Y(t) + q_r \Delta t \tag{5}$$

Equation (5) shows that the resistance variation is equal to the load variation with opposite signal plus the outflow volume in this interval, when a constant reference discharge is adopted.

3 Stochastic Model for $X(t)$ and $Y(t)$

Bearing in mind that the load depends on uncertain meteorological conditions such as rain, temperature and soil conditions, it is certainly adequate to model it as a random variable(for each time t). Here we propose a diffusion model for the load and, as a by-product, the resistance model is derived from Equation (5). It turns out to be also a diffusion.

3.1 Modeling the Load $Y(t)$

It is a *fait accompli* that, in general, we can model the inflow $q(t)$ that input to the reservoir as a stochastic process, expressed in term of its mean and zero-mean pertubation, i.e:

$$q(t) = \alpha(t) + \sigma(t)a(t)$$

where $\alpha(t)$ (the drift coefficient) is a purely deterministic function that represents the mean and the periodicity of the inflow, $\sigma(t)$ (the diffusion coefficient) represents the variance of the inflow, and $a(t)$ refers to the pertubation, a zero-mean stochastic process. The $a(t)$ is often regarded as a Gaussian white noise. Because the formal derivative of a Wiener process has the same statistical properties of a Gaussian white noise [8], we can further write $dB(t) = a(t)dt$, where $\{B(t), 0 \leq t \leq T\}$ is a Wiener process. Hence, the load that reaches the reservoir in the interval $[t, t+\Delta t]$ may be modeled by the stochastic equation:

$$Y(t+\Delta t) - Y(t) = \int_t^{t+\Delta t} \alpha(s)ds + \int_t^{t+\Delta t} \sigma(s)dB(s) \tag{6}$$

The first integral on the right-hand side of (6) is a Riemann integral and the second is a stochastic integral. When $\Delta t \to 0$, we say symbolically that the load $Y(t)$ satisfies the following stochastic differential equation:

$$dY(t) = \alpha(t)dt + \sigma(t)dB(t) \tag{7}$$

3.2 Modeling the Resistance $X(t)$

The stochastic model for the reservoir resistance may be derived substituting Equation (6) into (5):

$$\Delta X(t) = \int_t^{t+\Delta t} (q_r - \alpha(s))ds + \int_t^{t+\Delta t} \sigma(s)dB(s) \tag{8}$$

and making $\Delta t \to 0$, we get that the resistance $X(t)$ satisfies the following stochastic differential equation:

$$dX(t) = (q_r - \alpha(t))dt - \sigma(t)dB(t) \tag{9}$$

Remark 1. In the flood control problem the inflows are considered in a daily time intervals basis. Hence the flood period $[0, T]$ may be represented as the union of intervals $I_i = (t_i, t_i + \Delta t]$, such that $\Delta t = 1$ means that each interval I_i represents one day of the flood period. Thus $[0, T] = \cup_{i=1}^{N} I_i$, where N represents the number of days of the wet season. We will consider here that the parameters $\alpha(t)$ and $\sigma^2(t)$ are constant in I_i.

Taking into account the above considerations, the stochastic differential equation for the resistance, Equation (9), may be expressed as:

$$dX(t) = (q_r - \alpha_i)dt - \sigma_i dB(t) \quad t \in I_i \tag{10}$$

With Equation (10) we may estimate the failure risk of the reservoir for every day of the interval $[0, T]$.

4 Daily Failure Risk

A reservoir fails to control a flood wave when the reservoir outflow exceeds the reference discharge q_r. This occurs if the load $Y(t)$ is greater than the resistance $X(t)$ at any time, obliging the operator to release more than what is allowed, in order to avoid the reservoir overtopping. Thus, the failure risk is defined as a function of the process $\{X(t), t \geq 0\}$, which represents the reservoir resistance (protection volume), i.e., we may say that the reservoir fails when a flood wave reaching the reservoir takes up all the protection volume, nullifying the reservoir resistance. The estimation of the failure risk may be carried out by defining the time of the failure occurrence:

$$\tau_i = \inf_{t \in I_i} \{t \geq 0 | X(t) \leq 0\}. \tag{11}$$

The failure risk is defined then as:

$$u_i(X(t)) = P\{\tau_i \leq t_i + \Delta t | X(t) \geq 0\} \quad t_i < t \leq t_i + \Delta t$$

$$\rho_i(X(t)) = E_{X(t)}\{u_i(X(s))\} \quad t_i < t \leq s \tag{12}$$

In order to assess the failure risk by means of Equation (12), the load $X(t)$ is considered as being limited by the reservoir capacity: $X(t) \in R_X = [0, S_{max}]$. Now, if the failure risk is assumed to be a function $\rho_i(X(t)) \in C^2(R_X)$ (the class of real functions that are continuous and have continuous derivatives up to the order 2), then it may be estimated using the following result:

Proposition 1 *Let $\{X(t), t \in I_i\}$ be a Markov diffusion process in the probability space $(R_X, \mathcal{F}, P)$, satisfying the stochastic differential equation (10). Then the failure risk $\rho_i(X(t))$ defined by Equation (12) is given by*

$$\rho_i(X(t)) = \frac{1 - \exp\{-2(q_r - \alpha_i)(S_{max} - X(t))/\sigma_i^2\}}{1 - \exp\{-2(q_r - \alpha_i)S_{max}/\sigma_i^2\}}, \quad 0 \leq X(t) \leq S_{max} \tag{13}$$

Proof: see [9]

5 Parameter Estimation

5.1 Maximum Likelihood Estimators

In order to estimate these coefficients we will consider a discrete version of the process $Y(t)$ constructed from the process observations at the points $t_i \in I_i$, of the set $[0, T] = \cup_{i=1}^{N} I_i$, so that almost surely a path of $Y(t)$ is completely specified when the $Y_i = Y(t_i)$ are known for every i Considering the time interval $\Delta t = 1$ day, each path with N days corresponds to one flood period in a year. Provided that there is only one flood period per year, each path corresponds to a different year. Therefore, the data records consist of M paths (years) with N observations (days) on each path and we denote these data as $\{Y_{j,i}, i = 0, 1, \ldots, N$ and $j = 1, \ldots, M\}$. The conditional probability density for each path j of the process $\{Y_{j,i}\,, i = 0, 1, \ldots N\}$ is given by:

$$f_j(Y_{j,i}|Y_{j,i-1}, \alpha_i, \sigma_i^2) = (2\pi\sigma_i)^{-1/2} \exp\{-\frac{1}{2\sigma_i^2}(Y_{j,i} - Y_{j,i-1} - \alpha_i)^2\} \tag{14}$$

since $\Delta t = 1$ day, then $q_{j,i} = Y_{j,i} - Y_{j,i-1}$ is the daily average inflow that reaches the reservoir on the day i of the year j. Denoting the drift coefficient vector as $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_N)'$ and the $N \times N$ diffusion matrix as $\mathbf{\Sigma} = diag(\sigma_1^2 \dots \sigma_N^2)$, the likelihood function for the path $\boldsymbol{D}_j = \{Y_{j,i},\, i = 0, 1, \dots, N\}$ may be written as:

$$L_j(\boldsymbol{\alpha}, \mathbf{\Sigma}\,|\boldsymbol{D}_j) \propto \prod_{i=1}^{N} (\frac{1}{\sigma_i^2})^{\frac{1}{2}} \exp\{-\frac{1}{2\sigma_i^2}\,(q_{j,i} - \alpha_i)^2\}$$

Considering now M paths of the process $\{Y_{j,i}\ i = 0, 1, \dots N$ and $j = 1, \dots, M\}$, then $L_j(\boldsymbol{\alpha}, \mathbf{\Sigma}\,|\boldsymbol{D}_j)$ represents the contribution of a path to the complete likelihood function, which is given by:

$$L(\boldsymbol{\alpha}, \mathbf{\Sigma}|\boldsymbol{D}) \propto \prod_{j=1}^{M} \Big(\prod_{i=1}^{N} (\frac{1}{\sigma_i^2})^{\frac{1}{2}} \exp\{-\frac{1}{2\sigma_i^2}\,(q_{j,i} - \alpha_i)^2\} \Big)$$

where we denote the set of all M paths as $\boldsymbol{D} = \{\boldsymbol{D}_j\,,\, j = 1, \dots, M\}$. Rearranging the terms in $L(\boldsymbol{\alpha}, \mathbf{\Sigma}|\boldsymbol{D})$:

$$L(\boldsymbol{\alpha}, \mathbf{\Sigma}|\boldsymbol{D}) \propto |\mathbf{\Sigma}|^{-M/2} \exp\{-\frac{1}{2}\,\mathrm{tr}(\mathbf{\Sigma}^{-1}\boldsymbol{S})\} \qquad (15)$$

where $|\mathbf{\Sigma}|$ is the determinant of the matrix $\mathbf{\Sigma}$ and $\boldsymbol{S}$ is a diagonal matrix of the sum of the squares related to $\boldsymbol{\alpha}$, with the diagonal elements given by

$$s_{ii} = \sum_{j=1}^{M} (q_{j,i} - \alpha_i)^2 \qquad i = 1 \dots N \qquad (16)$$

In order to calculate the maximum likelihood estimators, we shall consider the relation:

$$\bar{q_i} = \frac{1}{M} \sum_{j=1}^{M} q_{j,i} \qquad S_i^2 = \frac{1}{(M-1)} \sum_{j=1}^{M} (q_{j,i} - \bar{q_i})^2$$

Differentiating the logarithm of Equation (15) in relation to α_i and σ_i^2, we have the following maximum likelihood estimators:

$$\widehat{\alpha}_i = \bar{q_i} \qquad \widehat{\sigma}_i^2 = \frac{M-1}{M} S_i^2 \quad i = 1 \dots N \quad (17)$$

The estimations of α_i and σ_i^2 given by (17) may be substituted in (13) to calculate the failure risk, or by fixing a failure risk ρ_i, we may calculate the protection volume X_i associated to this risk, for every day $i = 1, \dots, N$, using the relation:

$$X_i = \frac{\widehat{\sigma}_i^2}{2(q_r - \widehat{\alpha}_i)} \ln\Big[\frac{1 - \rho_i\Big(1 - \exp\{-2(q_r - \widehat{\alpha}_i)S_{max}/\widehat{\sigma}_i^2\}\Big)}{\exp\{-2(q_r - \widehat{\alpha}_i)S_{max}/\widehat{\sigma}_i^2\}}\Big], \quad i = 1, \dots, N \quad (18)$$

5.2 Bayesian Estimators

In the Bayesian approach the parameters α_i and σ_i^2 are considered random variables and the joint prior probability densitiy $\pi_0(\boldsymbol{\alpha}, \mathbf{\Sigma})$ is assumed to be known. Combining these prior densities with the likelihood function through Bayes theorem, we have the joint posterior density of the parameters $\pi(\boldsymbol{\alpha}, \mathbf{\Sigma})$. The Bayesian parameter estimators are the expected values of this posterior density. In order to find the Bayesian estimators for the parameters in Equation (10), we shall assume that α_i and σ_i^2, $i = 1, \dots, N$ are independent and adopt the Jeffreys non-informative prior density ,[10], as prior density. Under these hypothesis of independence, the joint prior density is given by

$$\pi_0(\boldsymbol{\alpha}, \mathbf{\Sigma}) = \prod_{i=1}^{N} \Big| -\frac{1}{N} \frac{\partial^2 L(\boldsymbol{\alpha}, \mathbf{\Sigma}|\boldsymbol{D})}{\partial \sigma_i^2} \Big|^{\frac{1}{2}} = \prod_{i=1}^{N} \sigma_i^2 \quad (19)$$

The posterior density is given by $\pi(\boldsymbol{\alpha}, \mathbf{\Sigma}|\boldsymbol{D}) \propto L(\boldsymbol{\alpha}, \mathbf{\Sigma}|\boldsymbol{D})\pi_0(\boldsymbol{\alpha}, \mathbf{\Sigma})$, which results in:

$$\pi(\boldsymbol{\alpha}, \mathbf{\Sigma}|\boldsymbol{D}) \propto \prod_{i=1}^{N} (\frac{1}{\sigma_i^2})^{\frac{M}{2}-1} \exp\{-\frac{1}{2\sigma_i^2}[(M-1)S_i^2 + M(\alpha_i - \bar{q_i})^2]\} \quad (20)$$

let $\boldsymbol{\alpha_{-i}} = (\alpha_1, \dots, \alpha_{i-1}, \alpha_{i+1}, \dots, \alpha_N)$ and $\boldsymbol{\sigma}^2_{-i} = (\sigma_1^2, \dots, \sigma_{i-1}^2, \sigma_{i+1}^2, \dots, \sigma_N^2)$ the values that contain all the components of the vectors $\boldsymbol{\alpha}$ and $\mathbf{\Sigma} = diag(\sigma_i^2, \dots, \sigma_N^2)$, except for the components α_i and σ_i^2. Then we have the conditional densities:

$$f_i(\alpha_i|\boldsymbol{\alpha_{-i}}, \mathbf{\Sigma}, \boldsymbol{D}) \sim Normal(\bar{q_i}, \sigma_i^2/M)$$

$$g_i(\sigma_i^{-2}\,|\,\boldsymbol{\alpha}, \boldsymbol{\sigma}^2_{-i}, \boldsymbol{D}) \sim Gamma\Big(\frac{M}{2}, \frac{[(M-1)S_i^2 + M(\alpha_i - \bar{q_i})^2]}{2}\Big)$$

In the Bayesian context, $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_N)$ and $\mathbf{\Sigma} = diag\{\sigma_1^2, \dots \sigma_N^2\}$ are random variables, so for a fixed risk ρ the protection volume given in (18) is regarded as a function of these parameters. Thus, we calculate the protection volume associated to the risk ρ_i by taking the expected value in (18) in relation to these parameters. A simple way to assess the protection volume given in (18) is through Monte Carlo Markov Chain simulation, using Gibbs sampling algorithm (see, e.g.[11]). This algorithm is used to generate a sample $\{\boldsymbol{\alpha}^{(k)}, \mathbf{\Sigma}^{(k)},\, k = 1, 2, \dots K\}$ of the posterior density $\pi(\boldsymbol{\alpha}, \mathbf{\Sigma}|\boldsymbol{D})$. With this sample, the Monte Carlo estimator for the protection volume is given by:

$$\widehat{X}_i = \frac{1}{K} \sum_{k=1}^{K} \frac{(\sigma_i^2)^{(k)}}{2(q_r - \alpha_i^{(k)})} \ln\Big[\frac{1 - \rho_i\Big(1 - \exp\{-2(q_r - \alpha_i^{(k)})S_{max}/(\sigma_i^2)^{(k)}\}\Big)}{\exp\{-2(q_r - \alpha_i^{(k)})S_{max}/(\sigma_i^2)^{(k)}\}}\Big], \quad i = 1, \dots, N \quad (21)$$

6 Application

The flood control model developed in this work was applied to the reservoir of Chavantes Hydroelectric Power

Plant (CHPP) in Southeast Brazil. This reservoir is located on Paranapanema River, an affluent of Parana River, on the border of São Paulo and Parana States. It is part of a large complex of 35 hydroelectric power plants hydraulically interlinked, known as the Great Parana Basin, being the Itaipu power plant downstream to them.

The CHPP is part of a cascade of 6 hydroelectric power plants, 3 of them with reservoirs, including the CHPP, and 3 run-of-river power plants. The installed capacity is $416MW$ with the minimum downstream constraints equal to $100m^3/s$ and the maximum downstream constraints (reference discharges) is $1800m^3/s$. The maximum outflows through the turbines is $648m^3/s$. The designed protection volume is $S_2 = 615km^3$. The maximum reservoir volume is $S_{max} = 3041km^3$.

The paths $\boldsymbol{D_j}, j = 1, \ldots 9$, are a nine-year period (1981 to 1989) of daily inflows corresponding to 181 days of the wet period. We may notice in these data a much greater dispersion on the wet days (between days 60th and 80th) than on the initial and final days of the flood period. These data were used to estimate the drift and the diffusion coefficients of the load $Y(t)$.

Figures 2 show the estimation of the drift and diffusion coefficients obtained by the Bayesian inference methods. We notice that the coefficients estimated by the maximum likelihood method are more poor than those by the Bayesian method. This may be explained by the fact that the sample of years is smal to maximum likelihood estimation.

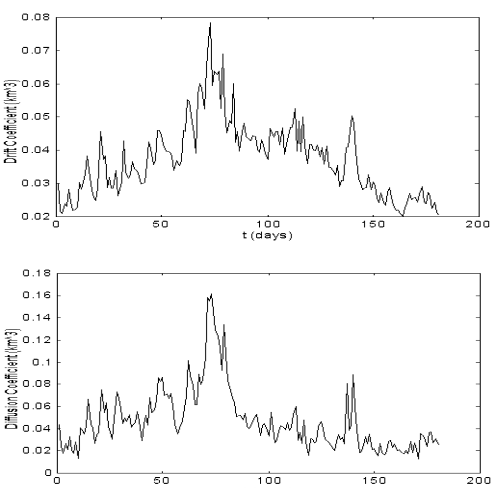


Figure 2: Bayesian estimation of drift coefficients α_i and diffusion coefficients σ_i^2 for $i = 1, \ldots 181$ days.

Bayesian estimations was obtained using (MCMC) algorithms (Gibbs Samples) and we monitored the convergence of Gibbs samples using Gelman and Rubin method (see [12]) . In this case the considered number of iterations are sufficient for approximate convergence if the estimated potencial scale reduction $\sqrt{\widehat{R}} < 1.1$.

Figure 3(a) shows the risk curve obtained with the maximum likelihood estimates, and Figure 3(b) shows the risk curve obtained when the Bayesian approach is adopted. It may be noticed in the latter figure that the resistance (protection volume) is greater than that obtained with the maximum likelihood estimations for the same risk value. This may be especially attributed to the fact that the Bayesian estimations of the diffusion coefficient are much greater than those of likelihood on these days.

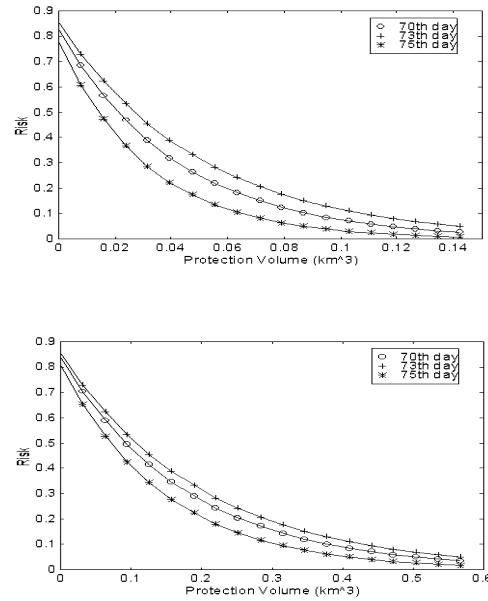


Figure 3: Risk curves for the days $i = 70, 73$ and 75, Maximum likelihood and Bayesian estimation.

Figures 4 show the protection volume estimated through the Bayesian method, for a failure risk fixed at $\rho_i = 5\%$ for every days in the rainy period. We notice in this plot that the maximum protection volume is estimated for the 73th day of the wet period.

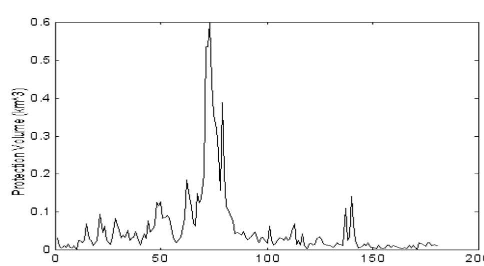


Figure 4: Protection volume estimated by Bayesian methods.

The maximum protection volume estimated by the maximum likelihood method is $0.15km^3$, whereas the Bayesian estimate is $0.58km^3$. Table 1 shows the volume values estimated through other renowned methods used in the solution of the flood control such as the (VDC) method and the (CP) method.

7 Comparisons and Conclusions

The comparison between the proposed method and the (VDC) and (CP) methods is done here by analyzing the simulated operation results for a period of daily inflows, during a 9-year periods of floods (1981-1989). Despite the fact that these data reflect an actual reservoir simulation, this period is too short for simulation purposes and for the conclusion reached here to be considered definitive. However, these data allow us to assess the main characteristics of the methods. The analyzed variables are shown in Table 1.

Table 1: Comparison between the proposed methods and other methods.

Methods	MLE	Bayes.	VDC	CP
Vol.(km^3)	0.150	0.585	0.498	0.065
Failure days	11	9	6	19
Stored Vol.(%)	96.3	95.5	83.6	99.8
Loss(GW-day)	0.352	0.811	4.188	0.011

The first variable in this analysis is the maximum protection volume allocated by each method. It may be noticed that the constant protection volume (adopted for the 181 days of the rainy season) determined by the Volume x Duration Curve Method is quite close to the maximum volume obtained by the proposed method. It should be emphasized here the first advantage of the proposed method: the volumes of this order are allocated only for a few days of the flood period (see and Figure 4). In the case of the Critical Path Method, the maximum protection volume is much smaller than the other ones. Nevertheless, the validity of this value may be questioned because it was determined using the recorded data only, considering that this method demands the simulation of a large number of generated synthetic series.

The number of days that the reservoir fails when each method is adopted is a variable used in the comparison. It may be noticed in Table 1 that when the proposed method and the (VDC) method are applied, they lead to approximately the same number of days that the reservoir fails, all of which occurred in 1989. In the case of the (CP) method, a larger number of failure days was found, a total of 19 days, distributed as follows: 3 days in 1982, 1 day in 1984, 2 days in 1988 and 13 days in 1989.

Another characteristic that should be observed is that the (VDC) method allocates a constant protection volume for the whole wet season, whereas the proposed method and the (CP) method estimate a different volume for every day of the same period. This characteristic enables a better use of the storage capacity of the reservoir, expressed in terms of the percentage used of the usable volume. Table 1 shows that the (VDC) method presents an average unoccupied storage capacity equal to 16.4% of the usable volume, i.e., it uses only 83.6% of the reservoir capacity, whereas the proposed method uses 96% on average, when it is adopted. This result has a direct effect on the production of electric power at the (CHPP), as shown by the values of wasted energy when each method is utilized, i.e., the (VDC) method wastes at least 5 times more energy than the other methods.

Comparing the Bayesian and the maximum likelihood approaches, Table 1 shows that, despite the fact that the protection volume allocated by the Bayesian method is slightly larger than the classical one, the Bayesian method is more valuable at the number of failure days, certainly due to the excellent use of the usable volume obtained through the Bayesian method.

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