Non-Model-Based Robust Controller Design for Flexible Spacecraft

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Abstract

This paper presents a class of non-model-based position controllers for a kind of flexible spacecraft. A very basic system energy relationship of the flexible spacecraft is involved in the controller design and stability analysis instead of system dynamics themselves. With the controllers, one can achieve not only the closed-loop stability of the original distributed-parameter system, but also the asymptotic stability of the truncated system, which is obtained through representing the deflection of the appendage by an arbitrary finite number of flexible modes. Simulation results carried out on a kind of spacecraft with one flexible appendage justify the effectiveness of the proposed controller.

1 Introduction

Early attitude control for spacecraft was based on the rigid body assumption. However, with the advent of many advanced applications such as communications, space research, remote sensoring, spacecraft is of significantly large and flexible structures for the purpose of pointing, tracking, etc. Large-angle rapid rotational maneuvers of such spacecraft are essential to meet mission requirements. Thus, the dynamic effects of flexible parts have to be taken into account in controller design. Many proposed applications require large-angle rapid maneuvers between two configurations [1]. In [2][3][4], a kind of planar single-axis rest-to-rest rotational maneuvers have been investigated. This problem is linear for the rigid spacecraft, but for the flexible case, even a very small elastic deformation will lead to a nonlinear system with infinite dimension. Challenges in the area of spacecraft control lie in the need for simultaneous maneuvering control and vibration suppression.

Many attempts have been made to the controller design of flexible spacecraft maneuvering problem. Research on optimal control has been done in [2][8]. Control laws based on linearization and nonlinear inversion were presented in [5][6]. Singular perturbation technique has been used for control system design in [7]. Recently, boundary control of flexible systems has drawn a great deal of attention [3][9][11]. The control approaches mentioned above are all based on linear or truncated models. Other problems include: (i) accurate knowledge about system dynamics is required but very difficult in practice, (ii) a relatively high order controller and high demand for real-time computing are often necessary, (iii) control and observation spillovers may occur due to the ignored high frequency dynamics.

The motion of flexible spacecraft is described by a set of coupled ordinary and partial differential equations (PDEs). Such hybrid systems is actually of infinite dimension and many control strategies based on rigid body or linear model are no longer applicable. In [10], an effective control strategy called energy-based robust control was proposed. The control design only utilizes a very basic energy relationship of the system and requires no information of the system dynamics. Furthermore, the deflection of the beam has also been taken account into the control law, which provides direct control effort on vibration suppression. In this paper, this control strategy is extended for flexible spacecraft rotational maneuver control as well as vibration suppression of the flexible appendages. The asymptotic stability of the truncated system with an arbitrary finite number of flexible modes can also be achieved.

The paper is organized as follows: In Section 2, the spacecraft system with flexible appendage is briefly introduced. The non-model-based robust controller design is presented in Section 3. Computer simulations are carried out on a rigid central body attached with a flexible beam to verify the effectiveness of the controller in 4, followed by the conclusion in Section 5.

2 Spacecraft Systems with Flexible Appendage

Consider a flexible spacecraft as with a rigid central body rotating freely in inertial space, to which a flexible beam-like appendage is attached. We assume that the flexible beam performs only planar motion as shown in Fig. 1.

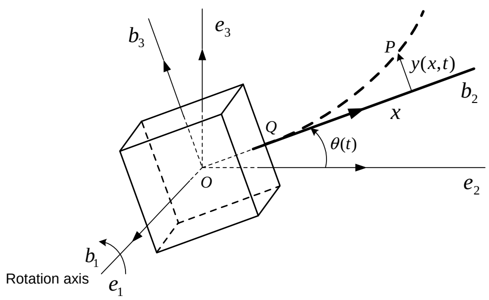

Figure 1: Configuration of a flexible spacecraft

In Fig. 1, $Oe_1e_2e_3$ and $Ob_1b_2b_3$ denotes an inertial frame and a frame fixed in the rigid body respectively, where O is also the center of mass of the rigid body and b_1, b_2, b_3 are along the principal axes of the rigid body. The beam is clamped to the rigid body at the point Q. θ is the angle of rotation of the rigid body. We assume that the mass of the rigid body is larger than that of the appendage, hence the center of mass of the rigid body is approximately that of the whole configuration, and the center of mass of the rigid body is fixed in the inertial frame. These assumptions are only used for the convenience of dynamic modeling but not for controller design, since the controller is a model free one which will be shown later. For clarity, some basic notations are listed below.

x:	the distance of any point on the appendage to Q;
L:	the length of the appendage;
EI:	the uniform flexural rigidity of the appendage;
ρ:	the uniform mass per unit length of the appendage;
I_z:	the inertia tensor of the rigid body along z axis;
b:	the distance between the system center of mass to the point of attachment and
$y(x,t)$:	the elastic deformation of the appendage at time t and distance x.

The total kinetic energy E_k and potential energy E_p can be calculated by

$$E_k = \frac{1}{2}I_z\dot{\theta}^2 + \frac{1}{2}\rho\int_0^L \left\{[(x+b)\dot{\theta}+\dot{y}]^2 + (y\dot{\theta})^2\right\}dx \quad (1)$$

$$E_p = \frac{1}{2}EI\int_0^L [y^{''}(x,t)]^2 dx \quad (2)$$

According to the extended Hamilton's Principles [15], we can obtain the following system dynamics:

$$I_z\ddot{\theta} + \rho\int_0^L y^2dx\ddot{\theta} + 2\rho\int_0^L y\dot{y}dx\dot{\theta} + \rho\int_0^L (x+b)ydx\dot{\theta}^2$$

$$= \tau + EI\left[y^{''}(0,t) - by^{'''}(0,t)\right] \quad (3)$$

$$\rho\ddot{y} + \rho(x+b)\ddot{\theta} - \rho y\dot{\theta}^2 = -EIy^{''''}(x,t) \quad (4)$$

where the dots and primes denote the derivatives with respect to time and space, respectively. By recalling that the spacecraft is operated without gravitational influence, we can conclude that the total change in system energy must be equal to the work done by the actuator torques, i.e.,

$$E_k - E_{k0} + E_p - E_{p0} = \int_0^t \tau\dot{\theta}(t)dt \quad (5)$$

where E_{k0} and E_{p0} are kinetic energy and potential energy of the system at the initial moment, τ is the torque generated th by actuator. Differentiating with respect to time on both sides of (5) yields

$$\dot{E}_k + \dot{E}_p = \tau\dot{\theta}(t) \quad (6)$$

3 Non-Model-Based Controller Design

In this section, we present an non-model-based controller design approach for the flexible spacecraft described in Section 2. The control objective is to drive the central body to a pre-defined position quickly, smoothly and accurately. Although the pure PD control can stabilize the flexible spacecraft system, generally the system performance is not satisfactory because the elastic vibrations cannot be effectively suppressed. In this paper, in addition to the pure PD control, some feedbacks related to the bending of the flexible appendage will be introduced into the controller, and thus provide direct control effort on the elastic vibrations.

3.1 Normal Non-Model-Based Controller

The controller is given by

$$\tau = -k_p[\theta(t) - \theta_f] - k_d\dot{\theta}(t) - k_f f(x_s,t)\int_0^t \dot{\theta}(\sigma)f(x_s,\sigma)d\sigma \quad (7)$$

where k_p, k_d are positive constants; $k_f \geq 0$; $0 \leq x_s \leq L$; θ_f is the constant final position of the central body; $\dot{\theta}$ is the time derivative of angular position θ; $f(x,t)$ can be variables related to the bending of the beam, or any combination of bending variables of the beam. The condition on choosing $f(t)$ is that it must be zero when the beam is static and undergoes no deformation. The stability of the closed-loop system is stated by the following theorem.

Theorem 1 *The closed-loop flexible spacecraft system described by (3), (4) and (7) is stable [13].*

3.2 Improved Energy-Based Robust Controller

Note that only the closed-loop stability is claimed in Theorem 1. It is difficult to prove the asymptotic stability due to the infinite dimensionality of the system, where the LaSalle's Theorem [17] used to prove the asymptotic stability of finite dimensional systems is not applicable. In this section, we shall present a modified controller. The modification allows us to (i) prove the close-loop stability using the same energy relationship (6); and (ii) achieve asymptotic stability of the truncated system with arbitrary finite number of flexible modes. In addition, the restriction on $f(x,t)$ can be removed, thus great freedom in the choice of feedback signals is allowed.

The modified controller is given by

$$\begin{aligned} \tau &= -k_p[\theta(t)-\theta_f] - k_d\dot{\theta}(t) \\ &\quad -k_f f(x_s,t)\mathrm{sgn}(\dot{\theta}) \int_0^t |\dot{\theta}(\sigma)| f(x_s,\sigma) d\sigma \end{aligned} \quad (8)$$

where k_p, k_d, k_f are defined as before, and the signum function is defined by

$$\mathrm{sgn}(\dot{\theta}_i) = \begin{cases} 1, & \dot{\theta}_i > 0 \\ 0, & \dot{\theta}_i = 0 \\ -1, & \dot{\theta}_i < 0 \end{cases}$$

The stability of the system is stated in the following two theorems.

Theorem 2 *The closed-loop system described by (3), (4) and (8) is stable in the sense of Lyapunov.*

Proof: Considering the following Lyapunov candidate:

$$V = E_k + E_p + \frac{1}{2}k_p[\theta(t)-\theta_f]^2 + \frac{1}{2}k_f\left[\int_0^t |\dot{\theta}(\sigma)| f(x_s,\sigma)d\sigma\right]^2 \quad (9)$$

Taking time derivative to (9), and invoking the energy relationship (6), $\dot{V}(t) = -k_d\dot{\theta}^2$ directly follows. ■

Theorem 3 *The controller given by equation (8) can guarantee the asymptotic stability of the truncated system, which is obtained through representing the deflection of the beam by an arbitrary finite number of flexible modes.*

Proof: See Appendix A. ■

4 Simulation Study

In this section, we consider a slewing maneuver example of spacecraft to demonstrate the applicability of the results presented in Section 3. The flexible beam is simulated by assumed modes method in which the first four modes are retained in the evaluation model. The system parameters and maneuver specifications are listed in Table 1. One torque actuator located on the rigid central body is used to control the maneuvering.

Table 1: System Parameters and Maneuver Specifications

Distance between O and Q	b	0.40 m
Central body inertia of z axis	I_z	13.35 Kgm^2
Beam length	L	1.80 m
Beam flexural rigidity	EI	15.52 Nm^2
Beam material density	ρ	0.81 Kg/m
Total slewing angle	θ_f	20.00 deg
Maximum available torque	T_{max}	150.00 Nm

4.1 Pure PD Control

Firstly, we shall give the simulation results of the following pure PD controller

$$\tau_{PD} = -k_p(\theta-\theta_f) - k_d\dot{\theta} \quad (10)$$

Readers can refer to [13] for more details. In what follows the performance of this PD controller with $k_p = 145.869$, $k_d = 97.246$ will be plotted by dashed lines in figures for comparison with the non-model-based controllers.

4.2 Normal Non-Model-Based Control

From (7), one can see that in addition to pure PD control effort, a nonlinear term has been introduced to explicitly control the elastic vibrations of the flexible appendage. As stated in Section 3, the introduction of the term into the controller allows great freedom in feedback design according to actual instrumentation. In this paper, two possible bending variables have been considered to represent $f(x,t)$ in (7). They are the tip deflection $y(L,t)$ and the base strain $y''(0,t)$. The control parameter k_p, k_d is chosen as the same as in the PD control and k_f is 10000 and 8500 for tip deflection feedback (denoted by Case 1) and base strain feedback (denoted by Case 2) respectively.

Fig. 2 shows the rotation motion in the above two cases denoted by solid, dash-dotted lines respectively. Tip trajectories are shown in Fig. 3. Compared with the results of pure PD control (dashed lines), one can see that the controller can suppress the elastic deflections more effectively without slowing down the rotation motion. Furthermore, the rotation motion exhibits less vibration and overshoot and are smoother that that of the pure PD control. Finally, the corresponding control effort is given in Fig. 4 for completeness.

4.3 Improved Base Strain Feedback

From (8), the modified controller for improving the base strain feedback is given by

$$\tau = \tau_{PD} - 8500y''(0,t)\mathrm{sgn}(\dot{\theta}) \int_0^t |\dot{\theta}(s)| y''(0,s) ds \quad (11)$$

As we can see, the parameters of the controller is the same as those in Case 2. The rotation motion and tip

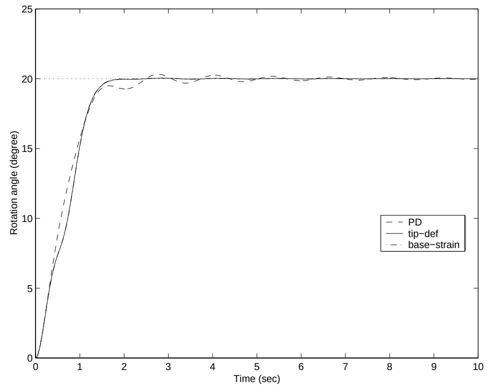

Figure 2: Rotation angle in Cases 1 and 2

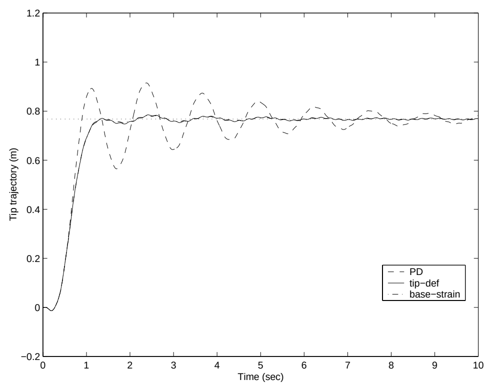

Figure 3: Tip trajectory in Cases 1 and 2

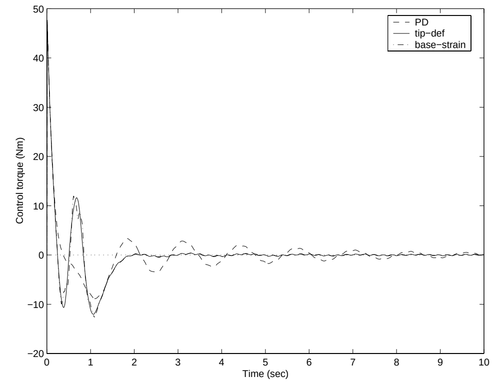

Figure 4: Control torque in Cases 1 and 2

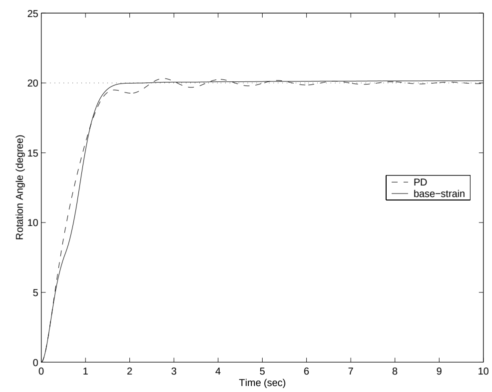

Figure 5: Rotation angle in Case 3

trajectory are plotted in Fig. 5 and Fig. 6. The performance is also greatly improved compared to pure PD control. For completeness, the control torques are also plotted in Fig. 7. One can see that $\dot{\theta}$ is positive (such that $\mathrm{sgn}(\dot{\theta}) = 1$) for most of the time in the process of operation. It is noted that if the sign of $\dot{\theta}$ is kept unchanged in the whole control process, the controller (8) will be equivalent to (7).

In the above, through simulations, we have shown the effectiveness of the controller in (7) and (8). It should be pointed out that the approaches presented in this paper are not complete, and other kinds of feedbacks and/or other kinds of combinations of feedbacks can also be considered depending on the available sensor facilities, since controller (8) actually allows great freedom of feedback design.

5 Conclusion

In this paper, a non-model-based controller design approach for a kind of flexible spacecraft has been presented. The controller is independent of system parameters and subsequently possesses stability robustness to parameters variations. Furthermore, the controller is very simple and very flexible in its form and can be easily implemented according to actual instrumentation. Numerical simulations have showed that the central body's position converges fast along smooth trajectories with negligible overshoots and the elastic vibration of the appendage is effectively suppressed as well.

Appendix A Proof of Theorem 3

It can be easily proved by following the procedures in [14]. It is given here for completeness and easy reference. Using the same Lyapunov candidate

$$V = E_k + E_p + \frac{1}{2}k_p[\theta(t) - \theta_f]^2 + \frac{1}{2}k_f\left[\int_0^t |\dot{\theta}(\sigma)| f(x_s, \sigma) d\sigma\right]^2$$

We consider the motion of the system in the largest invariant set in the set $Z := \{(\theta, y) | \dot{V} = 0\}$. When $\dot{V} \equiv 0$, we have $\dot{\theta} \equiv 0$, and hence $\ddot{\theta} = 0$. The controller becomes

$$\tau = -k_p[\theta - \theta_f]$$

which is a constant.

With the aid of the notable Hamilton's principle, considering the motion of system in $\dot{V} = 0$, we have the

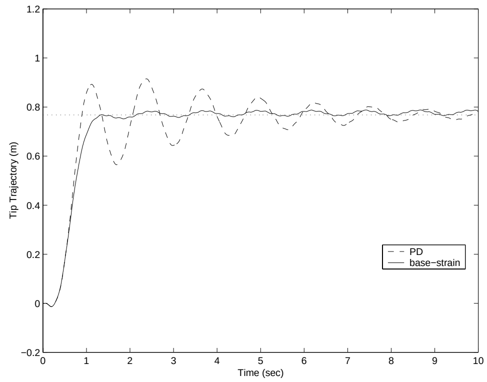

Figure 6: Tip trajectory in Case 3

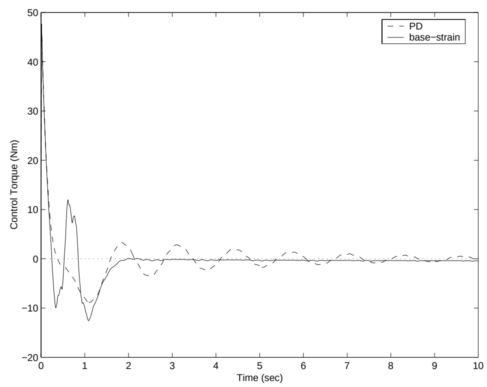

Figure 7: Control torque in Case 3

following PDEs of the appendage

$$k_p[\theta - \theta_f] = EI\left[y^{''}(0,t) - by^{'''}(0,t)\right] \quad (A.1)$$

$$\rho\ddot{y}(x,t) = -EIy^{''''}(x,t), \quad 0 < x < L \quad (A.2)$$

and boundary conditions (BCs)

$$y(0,t) = y^{'}(0,t) = 0 \quad (A.3)$$

$$y^{''}(L,t) = y^{'''}(L,t) = 0 \quad (A.4)$$

It is noted that the right-hand side of equation (A.1) is a function of time, while the left-hand side is constant. Let us firstly assumed that the constant is zero. It is shown later that the constant being non-zero leads to invalid solutions.

From the method of separating variables [18], we assume that the solution of (A.2) is of the form $y(x,t) = \Phi(x)Q(t)$, and equation (A.2) becomes

$$\frac{\Phi^{''''}(x)}{\Phi(x)} \cdot \frac{EI}{\rho} = -\frac{\ddot{Q}(t)}{Q(t)} \quad (A.5)$$

where primes denote the derivatives of x and dots denote the derivatives of t. It is clear that the left hand side of equation (A.5) is a pure function of x while the right hand side depends on t only. Therefore, both sides of equation (A.5) should be equal to a constant. If k is used to denote the constant, the PDE (A.5) can be reduced into two ordinary differential equations (ODEs), namely

$$\ddot{Q}(t) = -kQ(t) \quad (A.6)$$

$$\Phi^{''''}(x) = \frac{\rho}{EI}k\Phi(x) \quad (A.7)$$

The BCs become

$$\Phi(0) = \Phi^{'}(0) = 0$$

$$\Phi^{''}(L) = \Phi^{'''}(L) = 0 \quad (A.8)$$

We will consider equation (A.7) and conditions (A.8) with regard to different values of the constant k.

It can be proved that when $k = 0$ and $k < 0$, the solution to equation (A.7) is trivial. Then we just need to consider the case for $k > 0$.

Letting $k = \omega^2 < 0$, equation (A.7) can be rewritten as

$$\Phi^{''''}(x) = (\frac{\beta}{L})^4\Phi(x) \quad (A.9)$$

with

$$(\frac{\beta}{L})^4 = \frac{\rho}{EI}\omega^2 \quad (A.10)$$

The general solution to equation (A.7) is of the form

$$\Phi(x) = C_1\cos\frac{\beta x}{L} + C_2\cosh\frac{\beta x}{L} + C_3\sin\frac{\beta x}{L} + C_4\sinh\frac{\beta x}{L} \quad (A.11)$$

From BCs (A.8), a set of equations are obtained

$$\begin{cases} C_1 + C_2 = 0 \\ C_3 + C_4 = 0 \\ -C_1\cos\beta + C_2\cosh\beta - C_3\sin\beta + C_4\sinh\beta = 0 \\ C_1\sin\beta + C_2\sinh\beta - C_3\cos\beta + C_4\cosh\beta = 0 \end{cases} \quad (A.12)$$

To obtain nontrivial solutions, the determinant of the coefficient matrix of equations (A.12) must be zero, i.e.,

$$1 + \cosh\beta\cos\beta = 0 \quad (A.13)$$

which may be satisfied by an infinite number of β. Consider only positive β_i, an infinite number of solutions to the boundary value problem can be given by

$$\Phi_i(x) = C_1^i\cos\frac{\beta_i x}{L} + C_2^i\cosh\frac{\beta_i x}{L} + C_3^i\sin\frac{\beta_i x}{L} + C_4^i\sinh\frac{\beta_i x}{L}$$

where $C_1^i \sim C_4^i$ denote the solution to equation (A.12) corresponding to β_i.

When $k = \omega^2$ with ω being non-zero number, the solution to equation (A.6) can be obtained

$$q_i(t) = D_1^i e^{\omega_i t} + D_2^i e^{-\omega_i t} \quad (A.14)$$

where D_1^i and D_2^i are related to the initial conditions of $y(x,t)$. Note that the "initial" moment t_0 should denote the moment when the system motion enters the invariant set, rather than the initial operating moment since we are considering the motion of the system in the largest invariant set in the set $\dot{V} = 0$. Then from the Superposition or Linearity Principle [18], a solution $y(x,t)$ can be given by

$$y(x,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t) \qquad (A.15)$$

Note that the ω_i in equation (A.14) can be either positive or negative, without loss of generality, if $\omega_i \geq 0$. This leads to $D_1^i = 0$ as follows. If $D_1^i \neq 0$, then $\lim_{t\to\infty} q_i(t) \to \infty$ and $\lim_{t\to\infty} \dot{q}_i(t) \to \infty$, and hence $\lim_{t\to\infty} \dot{y}(x,t) \to \infty$. This implies the kinetic energy E_k of the system approaches infinity, which contradicts the fact the V in equation (9) is actually bounded. Therefore, D_1^i must be zero. Consequently, when $k > 0$, the solution (A.15) approaches zero as time approaches infinity.

In summary, $y(x,t) = 0$ provided that the system motion is in the largest invariant set $\dot{V} = 0$. Moreover, recalling that we already have $\theta = \theta_f$, we further conclude that if the system motion is in the largest invariant set $\dot{V} = 0$, the appendage must stop in the final position described by $\theta = \theta_f$ and $y(x,t) = 0$.

Now, let us consider the case when the left-hand side of equation (A.1) is a non-zero constant. If this is true, then from $y(x,t) = \Phi(x)Q(t)$, Q(t) and hence $y(x,t)$ must be constant. This means that the beam is static, which leads to $y^{''}(0,t) = 0$ and $y^{'''}(0,t) = 0$ and implies the above assumption is trivial.

Now invoking the truncation assumption, the elastic deflection of the appendage is assumed to be described by a finite number of flexible modes, and subsequently the system is of only finite dimensions. For this truncated system, because it has been proven already that the largest invariant set $\dot{V} = 0$ is the final equilibrium position, the asymptotic stability directly follows the LaSalle's theorem. (QED)

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