A Control of underactuated hopping gait systems :Acrobot example

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Abstract

While many researchers have been working on hopping gait systems, most of the previous works have focused on the mechanism designed by Raibert which has two actuators. In this paper, we consider the acrobot, which has only one actuator as our model of a hopping gait system. In spite of difficulty of realizing acrobot's hopping gait, it produces more interesting and attractive control problems than Raibert's model. Furthermore, the hopping principle of legged systems can be understood by treating the system which is difficult to control by intuition like acrobot.

1 Introduction

Numerous researches on animal's and human being's walk have done in various fields such as zoology[6], biomechanics, robotics[1, 7] etc. and it is known that there are some patterns for gaits of animals. In this research, we pay attentions to the gait form which is called hopping gait. An outstanding characteristic of this hopping gait is in which there exists the state that there is no support leg, that is, the body completely floats in the air, in a certain time under walk. The hopping gait system which has such a hopping gait is statically unstable, and its stabilizing control is indispensable when walking. Moreover, the design of the control system for stabilization is very difficult, since the balance between the motion of the center of gravity and walk problems is hard, and it has the situation that plurality differs. The hopping system has been researched in recent years, and many significant results have been obtained [7, 2, 3]. However, many of these results focused on the model designed by Raibert *et al* and models which have many actuators than Raibert's. Especially Raibert's model has attracted attention also in it. Raibert's one legged planar hopper has only two actuated joints: a springy prismatic leg and a revolute hip joint, and is capable of hopping at a fixed place, hopping at various desired rates, maintaining its balance when disturbed mechanically and leaping over small obstacles. With only one leg there is no need to coordinate several legs, so this difficult problem is avoid, whereas the need for active balance is central. Moreover, it can be said that it is easy to control, since the control problem is separable so that the hopping height is controlled by a prismatic leg and the body attitude is controlled by the revolute hip joints. Unlike with Raibert'system, it is intuitively difficult to realize the acrobot's hopping gait. The acrobot is a more attractive than Raibert's' model, and we think there is a room for the further research about the acrobot.

We recall the work of Berkemeier *et al.*'s research [1] for they are dealing with the hopping problem of the acrobot like our research. They showed that various gaits are realizable by choosing the output function which performs the simple harmonic motion and changing the multiplier of the output function, and a parameter of the model. However, they also showed that the direction of a lift-off, the hopping height and the posture at touchdown etc.can not be controlled well, thus a free hopping like Raibert's can not be realized. In this paper, we consider the hopping problem which attains control purposes at lift-off and touchdown which are needed to be realized the free continuous hopping, and realize the control system to it.

2 Dynamics and constraints

Various situations can be considered in the hopping gait. As stated previously, hopping motion is divided into stance phase and flight phase, by considering whether the foot is on the ground or not. Moreover, also in stance phase, the situation of a system differs clearly in takeoff and touchdown. The difference of these circumstances is not caused by the change of the mechanism itself, but by the constraints. In this section, we derive the equations of motion for the acrobot, constraints, and equations of motion involving the constraints on each phase below.

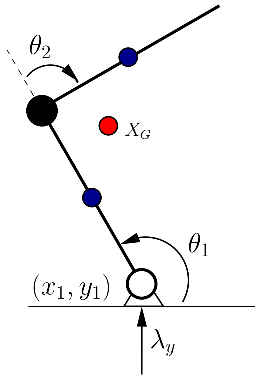

Figure 1: Coordinates for acrobot.

2.1 Acrobot model

[Plant]
We consider the *acrobot* which lies in the sagittal plane. Since it is not fixed, if the constraint force with the ground becomes negative, it will take off from the ground. Moreover suppose that *there doesn't occur any side-slip* , *inelastic impulsive impact* happens at the moment of touchdown.

The equations of motion for the acrobot are expressed by the following standard forms from Lagrangean approach.

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \begin{bmatrix} 0 \\ \tau \end{bmatrix} \qquad (1)$$

where $\theta = (\theta_1, \theta_2) \in S^1 \times S^1$ denote the generalized coordinates, $M(\theta) \in \Re^{2\times 2}$ is the inertia tensor, $C(\theta,\dot{\theta})\dot{\theta} \in \Re^2$ contains the Coriolis and centrifugal forces, $G(\theta) \in \Re^2$ contains the effects of gravity, and τ is the torque applied between the first and second links.

Further, we can write equations of motion for the acrobot in the affine form.

$$\frac{d}{dt}\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} \dot{\theta} \\ -M^{-1}(C\dot{\theta}+G) \end{bmatrix}}_{f(\theta)} + \underbrace{\begin{bmatrix} 0 \\ M^{-1}\left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right] \end{bmatrix}}_{g(\theta)} \tau \qquad (2)$$

The acrobot was studied by Hauser and Murray [4]. The following fundamental results are obtained.

- The acrobot is not exactly input-state linearizable.
- The Jacobian linearization about any of the inverted equilibrium states, where there center of gravity is directly above the support point, is completely controllable.

2.2 Redundant generalized coordinates

Since we consider the hopping problem of the acrobot, we need to specify the location of the acrobot to an inertial frame of reference. It follows, therefore, coordinates $x = (x_1, y_1)^T$ is taken at the grounding point of the ground and the first link, and the following q is defined as new generalized coordinates.

$$q = \begin{pmatrix} x^T & \theta^T \end{pmatrix}^T = \begin{pmatrix} x_1 & y_1 & \theta_1 & \theta_2 \end{pmatrix}^T \qquad (3)$$

As a result, the equations of motion for the acrobot on inertial coordinates are expressed as follows.

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \begin{bmatrix} 0 \\ \tau \end{bmatrix} \qquad (4)$$

where $M(q) \in \Re^{4\times 4}$ denotes the inertia tensor, $C(q,\dot{q})\dot{q} \in \Re^4$ contains the Coriolis and centrifugal forces, $G(q) \in \Re^4$ contains the effects of gravity.

2.3 Stance phase

The following holonomic constraint works in stance phase.

$$N(q) = \begin{bmatrix} x_1 & y_1 \end{bmatrix}^T = 0 \qquad (5)$$

Therefore the maneuver is controlled by equations(4), this constraint(5) and the equivalent nonholonomic constraint in stance phase.

$$\begin{cases} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q,\dot{q}) & = \tau - J^T(q)\lambda_r \\ J(q)\dot{q} & = 0 \end{cases} \qquad (6)$$

where $J(q) = \frac{\partial N(q)}{\partial q}$, $\lambda_r = (\lambda_x, \lambda_y) \in R^2$ denotes the constraint force which act on grounding point $O(x_1, y_1)$. If constraint force λ_y becomes negative, it will shift to flight phase from stance phase.

2.4 Flight phase

If the center of gravity X_G of the system is considered as an origin of an inertial frame in flight phase, the maneuver of the center of gravity and the problem of a posture can be decoupled, and the center of gravity will draw a parabola. On the other hand, since θ_1 acts as a cyclic coordinate, namely, θ_1 does not appear in the Lagrangian. Therefore, the following angular momentum conservation law is imposed.

$$K_1(\theta_2)\dot{\theta}_1 + K_2(\theta_2)\dot{\theta}_2 = L \qquad (7)$$

3 Lift-off control problem in stance phase

3.1 Problem statement

In stance phase, it is not possible to follow arbitrary paths in configuration space, since second-order nonholonomic constraint due to the assumption that a grounding point is underactuated is imposed. However, from the standpoint of the realization of the continuous hopping, we can see that it is indispensable to control the direction and the angular momentum at the moment of lift-off. Moreover, we must

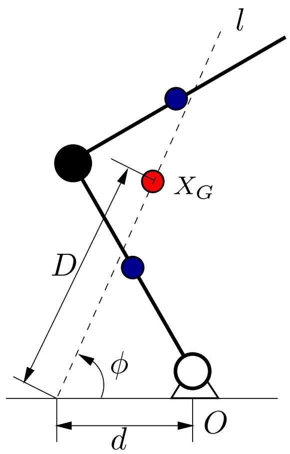

Figure 2: output function in stance phase $h(\theta, d) = \phi - \phi_d$

avoid taking off without attaining a control purpose, because it shift to flight phase once the constraint force becomes negative. Therefore below, the problem statement in view of above is mentioned and the control system which satisfies them is designed.

> **[problem statement : stance phase]**
> Given a initial state $\boldsymbol{x}_0 = (\theta_0, \dot{\theta}_0)$, we desire that the next goals are realized when the system will lift off.
>
> A. The acrobot has the desired speed direction $\phi_d = arg(v_d)$.
>
> B. The acrobot has the desirable angular momentum L_d to realize the desired landing posture $\boldsymbol{x}_{td} = (\theta_{td}, \dot{\theta}_{td})$.
>
> C. The duration of flight$t_f$becomes the biggest while goals for control A.B are attained.

3.2 Control strategy

3.2.1 Output zeroing control: At first, we aim to move the center of gravity in the goal direction. So, we define the output function $h_{lo}(\theta, d)$ as follows.

$$h_{lo}(\theta, d) = \phi(\theta, d) - \phi_d \tag{8}$$

where $\phi(\theta, d)$ is the angle of elevation when the center of gravity X_G is seen from the place which is d away from the connection point O, and ϕ_d is the desired value.

Since the system(2)(8) has relative degree 2, we define the coordinate transformation as follows.

$$\chi = \begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} h_{lo} \\ L_f h_{lo} \\ \theta_2 \\ M_{11}\dot{\theta}_1 + M_{12}\dot{\theta}_2 \end{bmatrix} \tag{9}$$

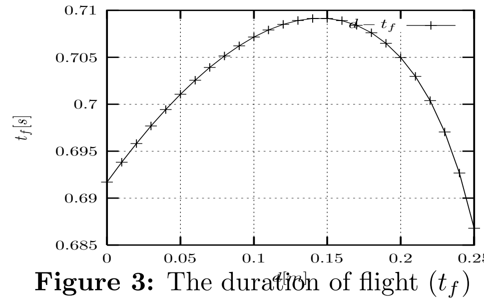

Figure 3: The duration of flight (t_f)

where $\xi \in S^1 \times \Re$ is the part which is linear in the new coordinate, $\eta \in S^1 \times \Re$ is the part which produces *zero dynamics*. Using this transformation χ, we can get the following canonical form.

$$\begin{cases} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= b(\xi, \eta, d) + a(\xi, \eta, d)\tau \\ \dot{\eta} &= \zeta(\xi, \eta, d) \\ y &= \xi_1 \end{cases} \tag{10}$$

The input $c_{lo}(\theta, d)$ which makes the output zero becomes the next equation.

$$c_{lo}(\theta, d) = -\frac{b(\xi, \eta, d)}{a(\xi, \eta, d)} = -\frac{L_f^2 h_{lo}}{L_g L_f h_{lo}}(\theta, d) \tag{11}$$

Here, when $\xi = 0$ is realized as an initial state, $\xi = 0$ will be filled whenever output zeroing input (11) is imposed. Moreover, regardless of the value of d, zero dynamics of this system becomes non-minimum phase.

The movement of the acrobot after output zeroing has the following meanings from the above.

1. The center of gravity of the system moves on the straight line l. That is, a system maneuvers, maintaining the desired rate direction.
2. Since zero dynamics of the system is unstable, the acrobot is performing the diffusive oscillating motion and, thereby, obtains energy and takes off.Moreover, the motion at that time is controlled with the following secondary nonlinear system.
$$\dot{w} = s(w, d) \tag{12}$$
where $w = (\theta_2, \dot{\theta}_2)$.

3.2.2 The design of the free parameter d: When we formulate the situation of attaining control purpose A.B., we can get the next equations.

$$\begin{cases} \xi &= 0 \\ \lambda_y(\xi, \eta, d) &= 0 \\ L(\xi, \eta, d) &= L_d(\xi, \eta, \theta_{1_{td}}, \theta_{2_{td}}) \end{cases} \tag{13}$$

where $\theta_{1_{td}}, \theta_{2_{td}}$ show the desired postures at touchdown, L_d shows a desirable angular momentum to touchdown with a purpose posture. If the free parameter d is fixed, (13) can be solved numerically and we obtain the state $\boldsymbol{x}_{lo} = (\theta_{1_{lo}}, \theta_{2_{lo}}, \dot{\theta}_{1_{lo}}, \dot{\theta}_{2_{lo}})$ which attains the control purpose A.B. to each d since they are four nonlinear equations of 4th variable (ξ, η). Moreover, we can achieve the control purpose C. because we can choose the free parameter d which attains a control purpose C using Fig.3. Therefore, the state x_{lo} and the free parameter d which attain above the control purposes at lift-off are obtained. Moreover, the set of the state U_{d_C} that purposes in stance phase can surely be attained at lift-off will be obtained.

$$U_{d_C} = \{(\xi, \eta) | \xi = 0, \dot{\eta} = -\zeta(0, \eta, d_C), \lambda_y > 0\} \quad (14)$$

As a consequence, we can get the exosystem which attains the control purposes A.B.C. in stance phase as follows.

$$\dot{w} = s(w, d_C) \quad w(0) \in U_{d_C} \quad (15)$$

The duration of flight t_f used here and the desirable angular momentum L_d are calculated as follows.

[desirable angular moment : L_d]
The desirable angular momentum L_d at the takeoff is given to touch down with the desired landing posture with the next equation in approximation.

$$L_d = \frac{K_1(\theta_{td})}{t_f} \left\{ (\theta_{1_{td}} - \theta_{1_{lo}}) - (J_{td} - J_{lo}) \right\} \quad (16)$$

where J denotes the indefinite integral of $-\frac{K_2}{K_1}\dot{\theta}_2$.

3.3 Control law

The exosystem (15) which can attain the control purposes above at takeoff is obtained. So, we can use output regulation control law and backstepping to realize the control purposes.

3.4 Numerical example

The control purpose in stance phase is attaining control purpose A.B.C to the given initial state at the takeoff. ($\phi_d = 60°, \theta_{td} = (120°, -30°)$) It turns out that purpose A.B. is satisfied from Fig.4, Fig.5. The reason why there is the discontinuity of L_d in Fig.5 is that we use only first term when the inside of root of (16) is negative. On the other hand, since we choose the free parameter d which satisfy the control purpose C. using Fig.3 ($d_C = 0.15$), all control purpose in stance phase are achieved. The motion of the acrobot at this time becomes like Fig.6.

4 Attitude control in flight phase

4.1 Problem statement

In flight phase, it is very difficult to change the angle of the body and the posture arbitrarily, because an angular momentum conservation law is imposed. Moreover, since the angular momentum conserved is generally not zero, a hopping system cannot avoid a natural change of the body angle while the center of gravity draws a parabola trajectory. However, it is necessary to fill some postures and the angle of the body at the time of the landing to realize continuous hopping gaits. Therefore, the problem which fills the conditions at touchdown under such a restraint is considered. the problem statement is shown below.

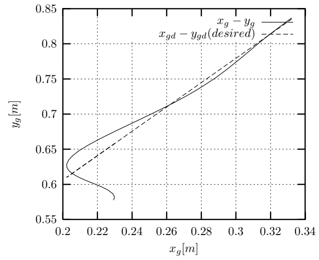

Figure 4: X_G

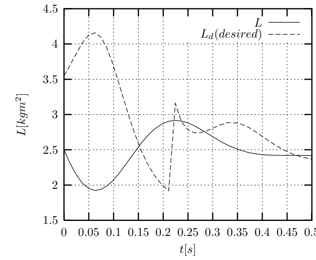

Figure 5: L, L_d

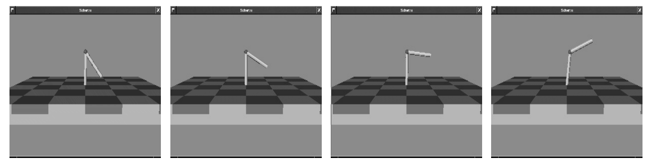

Figure 6: frames for lift-off

[problem statement : flight phase]
Assume that the initial angular momentum is L and the initial state at lift-off is $x_{lo} = (\theta_{lo}, \dot{\theta}_{lo})$. Our purpose is realizing the control system which reaches the desired posture $\theta_{td} = (\theta_{1_{td}}, \theta_{2_{td}})$ in landing time t_f in order to realize the stable continuous hopping gait.

4.2 Control strategy

What is necessary is to stabilize the system to the following trajectory which is feasible for dynamics (2) and constraint (7), in order to make desired posture $\theta_{td} = (\theta_{1_{td}}, \theta_{2_{td}})^T$ realized at time t_f.

$$\begin{bmatrix} \theta_d(t) \\ \dot{\theta}_d(t) \end{bmatrix} = \begin{bmatrix} \theta_{1d}(t) \\ \theta_{2d}(t) \\ \dot{\theta}_{1d}(t) \\ \dot{\theta}_{2d}(t) \end{bmatrix} = \begin{bmatrix} \theta_{1_{td}} + \frac{L}{K_1(\theta_{td})} \times (t - t_f) \\ \theta_{2_{td}} \\ \frac{L}{K_1(\theta_{td})} \\ 0 \end{bmatrix} \quad (17)$$

This trajectory has the following characteristics.

- The desired posture$(\theta_{1_{td}}, \theta_{2_{td}})$is satisfied at t_f.
- From the physical interpretation, this is the trajectory maneuvering for inverse time with

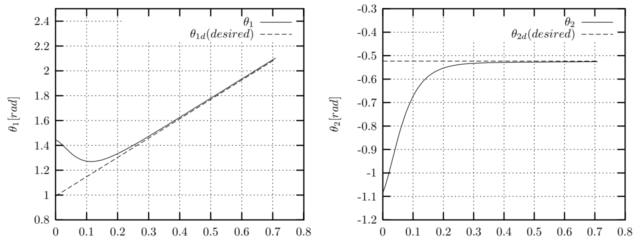

Figure 7: Configuration variables in flight phase

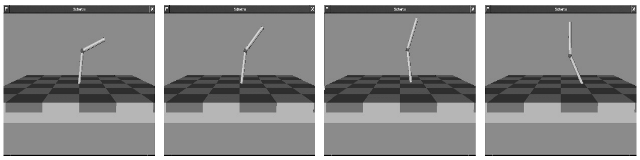

Figure 8: frames in flight phase

the initial state $(\theta_{1_{td}}, \theta_{2_{td}}, \frac{L}{K_1}, 0)$ and the angular momentum L.

- (17) is the trajectory which is produced by the zero dynamics which is made the output $h_{fl}(\theta) = \theta_2 - \theta_{2_{td}}$ zero by the output zeroing input

$$c_{fl}(\theta) = -\frac{L_f^2 h_{fl}}{L_g L_f h_{fl}}(\theta) \qquad (18)$$

and initial state $(\theta_{1_{td}}, \theta_{2_{td}}, \frac{L}{K_1}, 0)$. Moreover, if $L \neq 0$, zero dynamics will become unstable. In the view of physical implication, it implies that a body angle carries out the natural variation under the affection of a drift term when the second joint is fixed.

Flight phase is also finalized by the limited time t_f. Therefore, output regulation control law and backstepping are used for the control law for the stabilization to the trajectory like the foregoing paragraph.

4.3 Numerical example

Although the control purpose in flight phase is attaining the desired landing posture to the given initial state at the time of landing, we observe that a control purpose is fulfilled from Fig.7 while the center of gravity draws a parabola trajectory . Moreover, a motion of the acrobot becomes like Fig.8 $(L = 2.41[kgm^2], t_f = 0.71[s], \theta_{td} = (120°, -30°))$

5 Phase shift problem after touchdown

5.1 Problem statement

We need to shift to the state in which a desired takeoff is possible after touchdown in order to realize a continuous gait. However, the path planning problem is hard in this phase, because arbitrary trajectories do not exist on state space due to the second-order nonholonomic constraint like section 3. Then we notice that the acrobot has not an isolated equilibrium point but a manifold of equilibrium, divide the control problem into two phases and we avoid the above problem. Namely, we plan the trajectory from the state from which we can take off to the state inside of a manifold of equilibrium. It is reasonable to suppose that the trajectory exists, since this is the path planning problem from a certain point to a certain set, not between 2 certain points. On the other hand, the control system from a landing state to a certain equilibrium point already exists. Therefore, we think that it is controllable to the state from which it can take off by separating a control problem into two in this way, and show problem statement below.

[problem statement : after touchdown]
Suppose the system have the state $\boldsymbol{x}_{td} = (\theta_{td}, \dot{\theta}_{td})$ after impact of touchdown. Our purposes are realizing the following two objectives.

A. we realize the control system reaches equilibrium $\boldsymbol{x}_{eq} = (\theta_{eq}, 0)$ from any initial state $\boldsymbol{x}_{td}$ without making the constraint force negative.

B. The control system which reaches to the state $\boldsymbol{x}_{pl} = (\theta_{pl}, \dot{\theta}_{pl})$ from which it can take off from the equilibrium $\boldsymbol{x}_{eq}$ which reached by control system A is realized.

5.2 Control strategy

Although the control system is designed in this section, we design in the order of phase B and phase A so that intelligibly flow of design.

5.2.1 Phase B: We deal with the following path planning problem from an equilibrium to the state from which it can take off here.

We find a coefficient a, a function $s(t)$ and T_b which minimize the cost function
cost function : $J(\boldsymbol{x}(-T_b)) = ||x_g(-T_b)|| + ||\dot{\theta}(-T_b)|| + P$
when we impose output zeroing input during $-T_b$ under the next output function and the initial state for the acrobot(2)
output function : $h_{td_b}(\theta) = a\theta_1 + \theta_2 - s(t)$
initial state : $\boldsymbol{x}_{pl} = (\theta_{pl}, \dot{\theta}_{pl})$.
where a is a multiplier which shows the ratio of the increase in $\dot{\theta}_1$ and $\dot{\theta}_2$, $s(t)$ denotes any polynomial and P is the penalty function set to ∞ when the constraint force becomes negative. The gradient method is used as the approach of minimization.

We can get the trajectory which achieves the control purpose in phase A by solving above path planning problem. θ which minimizes the cost function J as a result of this optimization turns into the equilibrium θ_{eq} in phase A. We use output regulation and backstepping as a control law for the stabilization to the trajectory, since output regulation problem is solvable about this trajectory.

5.2.2 Phase A: That the acrobot is in an equilibrium state is the case where the center of gravity of a system is on a y shaft. Then, the following functions are considered as the output function $h_{tb_a}(\theta) = x_g$. Moreover, let the desired trajectory of θ_2 at that time be the the secondary curve which connect the state $\theta_{2_{td}}$ at touchdown and the purpose $\theta_{2_{eq}}$ which is an equilibrium with time T_a. The following thing can be said about the motion of the acrobot when it fulfill the output function and the trajectory. Namely, the control system always moves in a y axis and achieves the desired equilibrium x_{eq} in the desired time T_a. Moreover, the constraint force does not become negative when the system follows this trajectory. We formulates the control law by output regulation and backstepping, since output regulation problem is solvable also to this trajectory.

5.3 Numerical example

Solving the above path planning problem leads to get the desired trajectory etc. $T_a = 4.0[s]$, $T_b = 1.4$, $\theta_{eq} = (70.2°, 248°)$, $a = 2.65$ *and* $s(t) = 1.1 + 1.9t + 0.6t^2 - 1.1t^4 + 0.6t^5$. Though our purpose in this phase is to follow the desired trajectory to the given initial state, it has been satisfied, the motion of acrobot at this time becomes like Fig.9.

Finally, using the control system designed above, we demonstrate continuous hopping gaits in Fig.10. The acrobot advance a total of around 1.0 m during this cycle of the hopping gait and, the average speed for this gait is about 0.18 m/s. Also, there are 2 peaks in the motion of the center of gravity during one cycle. Though this speed and the motion of the center of gravity can be adjusted by the landing posture and T_a, T_b etc., we can not understand how to adjust this parameters.

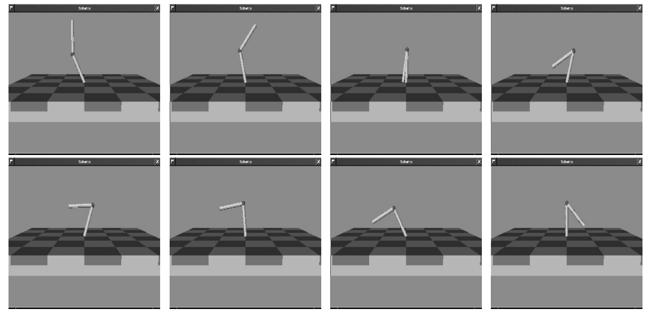

Figure 9: frames

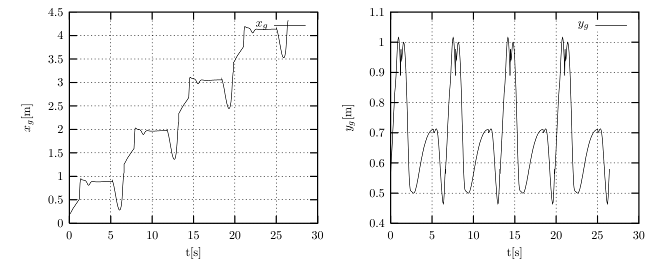

Figure 10: Center of mass for continuous hopping gait.

6 Conclusion

In this paper, we focused on the underactuated and variable constraint which makes realization of the hopping gait of legged systems difficult. Also we dealt with the following three problems and designed the control law.

A Lift off control problem in stance phase.
B Attitude control problem in flight phase.
C Phase shift problem after touchdown.

Finally, numerical simulations were presented and we showed that the acrobot could achieve continuous hopping gaits. A further direction of this study will be to consider the friction cone and to realize various continuous hopping gaits by changing the direction of the lift-off and the landing posture etc.

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