Data Fusion based State Estimation of Nonlinear Discrete Systems

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Abstract

In this paper, we propose a geometric data fusion(GDF) method using Perception-Net which can provide error reducing, uncertainty management, and maintaining consistency. We propose a Perception-Net to design a new state estimator for dynamic systems and apply the proposed geometric data fusion method to obtain the optimal estimate, propagate uncertainties and utilize the system knowledge. We present comparisons between the proposed estimator and the conventional estimators. It is also shown that the additional priori information on the system can be easily utilized in the proposed estimator to improve the performance. Through illustrative examples, it is verified that the proposed estimator presents better performances than the existing filters and improves performances via utilizing system knowledge.

1 Introduction

The sensing and estimation problem often requires an integration of not only distributed data from logical and/or physical sensors but also historically sensed and estimated data. The key to a successful integration may be a system architecture that provides uncertainty management and adaptive error recovery through the interaction among processes such as feature transformation, data fusion, and consistency maintenance based on knowledge. To this end, data fusion methods have been proposed and utilized [1, 2, 3]. However, there are a number of problems that the conventional methods cannot handle such as uncertainty propagation in nonlinear feature transformation, maintaining system consistency and utilizing system knowledge. In order to overcome these drawbacks, we propose so-called Perception-Net which automatically reduces errors further and further as uncertainties become smaller and smaller with iterations. As a method to resolve the estimated variable and the corresponding uncertainties, we also propose a geometrical data fusion (GDF) method which unifies the framework for computing the forward and backward propagation of uncertainties and errors thorough the net.

The state estimation problem of nonlinear/linear dynamic systems is also a kind of the sensing and estimation problem where the geometric data fusion method can be applied. In the general dynamic systems, the sensor data is a function of multiple states and the system dynamics can be utilized as a knowledge of the system. This clarifies the application of the geometric data fusion method to the state estimation problem. For the state estimation of nonlinear dynamic systems, the Extended Kalman Filter (EKF) has been mostly used [4, 5, 6]. However, it is well known that the EKF often shows poor performance, diverges and is biased due to lack of optimality. Since the proposed geometric data fusion method can deal with nonlinear feature transformation while optimizing a performance criterion, we can guess that it can be easily applied to state estimation problems for nonlinear dynamic systems and it can show better performance than the conventional EKF. Moreover, in the case that some priori information on the system can be known, there have been no methods to utilize such information to improve the estimation performance. If such priori information can be formulated as equality and/or inequality constraints on states, they can be easily utilized in the proposed geometric data fusion method to improve the estimation performance.

In this paper, we propose a new geometric data fusion method based on Perception-Net as an effort to improve the performance of sensing and estimation system by providing uncertainty management, maintaining consistency, and utilizing the system knowledge. Then we propose a Perception-Net for state estimation of dynamic systems and apply the proposed geometric data fusion method to obtain the optimal estimate. In the proposed geometric data fusion method for state estimation, we involve the constraint that the predicted output from the system dynamics and the previously estimated state is equal to the measured output. The covariance matrices of the predicted and measured output are utilized as weighting matrices defining the corresponding ellipsoidal uncertainties. We propose a method to incoporate the constraint on the system such as physical limitations, operating range, and system characteristics as priori knowledge to improve the performance of the proposed estimator. We also present some comparisons with the conventional estimators such as Klaman filter or Extended Kalman filter. Through illustrative examples, we verify that the proposed estimator shows better performance than the conventional nonlinear filters and improves the performance by utilizing the priori system knowledge.

This paper is organized as follows. In Section 2, we propose a new geometric data fusion method in Perception-Net. In Section 3, we apply the proposed Perception-Net and data fusion method to the estimation problem for nonlinear/linear dynamic systems. In Section 4, we compare the proposed estimator to the conventional EKF and LKF. In Section 5, illustrative examples are presented. Finally, conclusions are followed in Section 6.

2 Geometric Data Fusion in Perception-Net

2.1 Basic Concept of Perception-Net

The Perception-Net connects logical sensors or features of various levels of abstraction that can be identified by the sensor system. The Perception-Net is formed by the interconnection of logical and physical sensors with three types of modules: feature transformation module (FTM), data fusion module (DFM), and constraint satisfaction module (CSM). The uncertainties propagate in the Perception-Net through the input-output relationships of FTM and DFM modules, as well as through the constraints defined by CSM modules. Through the bidirectional (forward and backward) state update process, the net provides not only the reduction of uncertainties but also the monitoring of errors and faults.

2.2 Forward propagation

In the forward propagation relation, we assume that the input/output relationship is given by

$$y = f(x, n), \tag{1}$$

where x is the input vector, y is the corresponding output vector, and n is the white noise induced from measurement, process noise, and modeling error.

As a method to represent the confidence of a variable corrupted by noise or uncertainties, the error covariance has been widely used. Consider a random variable x, its estimate $\hat{x}$, and the corresponding error covariance $P_x = E\{(x-\hat{x})(x-\hat{x})'\}$. In this subsection, we will describe how to propagate the error covariance of the variable through the forward propagation.

We denote that $\hat{x}$ is the variable which will be propagated. Then, the propagated result $\hat{y}$ and the corresponding covariance be obtained by

$$\hat{y} = f(\hat{x}, 0) \tag{2}$$

$$E\{(y-\hat{y})(y-\hat{y})'\} = \frac{\partial f}{\partial x} P_x \frac{\partial f'}{\partial x} + \frac{\partial f}{\partial n} nn' \frac{\partial f'}{\partial n} \tag{3}$$

under the assumption that $E\{(x-\hat{x})n'\} = 0$. Now, for the easy use of the geometric data fusion method in the following backward propagation module, we consider a hyper-ellipsoid defined by $(x-\hat{x})' {P_x}^{-1} (x-\hat{x}) \leq 1$. We regard this hyper-ellipsoid as the uncertainty region involved in the variable x. From now on, the notation $W_x = {P_x}^{-1}$ will be used as the uncertainty matrix representing the hyper-ellipsoidal uncertainty region of the variable x.

2.3 Backward propagation

Consider two variables x_m and y_m which are obtained from the forward propagation module and/or logical/physical sensors. We assume that the corresponding uncertain matrices are W_{x_m} and W_{y_m} which may be defined from the corresponding error covariance matrices. The backward propagation starts from the CSM where all the variables are estimated by minimizing the weighted distance from $(x_m,\ y_m)$ while satisfying given constraints. The optimization problem for minimizing the weighted distance is defined by

$$\min_{x,y} \ \frac{1}{2}[||x-x_m||^2_{W_{x_m}} + ||y-y_m||^2_{W_{y_m}}]$$
$$\text{subject to} \ \ f_C(x,y) = 0 \text{ and } g_c(x,y) \leq 0,$$

where the function f_C denotes the equality constraint function and g_C denotes the inequality constraint function associated with the variables. Let (x^*, y^*) denote the optimal solution of the above optimization problem. Then the back propagated variable is given by (x^*, y^*). Now, we present how to propagate the uncertain weighting matrices based on the backward propagated variable. Consider the following linearized constraint manifold at the optimized point (x^*, y^*): $X'x + Y'y = 0$, where $X = \frac{\partial f_C}{\partial x}\Big|_{x=x^*}$ and $Y = \frac{\partial f_C}{\partial y}\Big|_{y=y^*}$. Then, the propagated uncertainty bounds through CSM can be obtained by projecting them to this linearized constraint manifold : $W_{x^*} = W_{x_m} + XY^{-1}W_{y_m}Y^{-1}X'$ and $W_{y^*} = W_{y_m} + YX^{-1}W_{x_m}X^{-1}Y'$. In the above uncertainty propagation equation, we observe that the uncertainty bound is decreased, which is so reasonable because the variables are estimated so as to satisfy the given constraint. However, it is noted that this propagation equation itself has some uncertainties induced from the linearization of the constraint manifold.

3 State Estimation for Nonlinear/Linear Dynamic Systems

In this section, we apply the proposed geometric data fusion method to the state estimation problem for dynamic systems. First, we consider the following nonlinear discrete time system:

$$
\begin{aligned}
x_k &= f_{k-1}(x_{k-1}, v_{k-1}) && (4)\\
y_k &= h_k(x_k) && (5)\\
y_k^m &= y_k + w_k, && (6)
\end{aligned}
$$

where x_k denotes the state, y_k denotes the system output, and y_k^m denotes the measured output from sensors. We assume that v_k and w_k denote the process white noise and measurement white noise, and have the covariance matrices Q_k and R_k, respectively.

The state estimation problem can be divided to two procedures. The first one is the state prediction procedure where the state in the current time step is predicted from the estimated state in the previous time step using the system dynamics. The other one is the measurement update procedure where the state is estimated from the predicted state and the measured output. In this paper, we apply the proposed Perception-Net to the state estimation problem. In this Perception-Net, we utilize the forward propagation module for the state prediction procedure and the backward propagation module for the measurement update procedure.

Denote the predicted state as x_k^- and the estimated state as x_k^+. The state dynamic equation (4) can be regarded as an input output relationship (1) where the input variable is the estimated state x_{k-1}^+ at the previous time step, the noise is v_{k-1}, and the output variable is the predicted state x_k^- at the current time step. Then, from (2) and (3), the predicted state and the corresponding error covariance P_k^- are given by

$$
\begin{aligned}
x_k^- &= f_{k-1}(x_{k-1}^+, 0). && (7)\\
P_k^- &= F_{k-1}' P_{k-1}^+ F_{k-1} + Q_{k-1}, && (8)
\end{aligned}
$$

where $F_{k-1} = \frac{\partial f_{k-1}}{\partial x_{k-1}}\Big|_{x_{k-1}=x_{k-1}^+}$

The measurement update and the corresponding covariance update is done through the CSM, where the predicted state and the measurement output can be regarded as sensor outputs from the logical sensor and the physical sensor, respectively. In CSM for the measurement update, we regard $y_k = h_k(x_k)$ as the constraint function $f_c(x,y) = c$, which says that the measured output and the predicted output should be equal to each other. Then the optimization problem for the measurement update is described by

$$
\begin{aligned}
&\underset{x,y}{\text{Min}} \quad ||x - x_k^-||^2_{(P_k^-)^{-1}} + ||y - y_k^m||^2_{R^{-1}}. && (9)\\
&\text{subject to} \quad y = h_k(x). && (10)
\end{aligned}
$$

Let x^* and y^* denote the optimal solution of (9). Then the estimated state is given by $x_k^+ = x^*$. The uncertainty matrix induced from the error covariance of the estimated state is given by the propagation equation for the uncertainty in the previous section:

$$
W_k^+ = W_k^- + H_k' R^{-1} H_k \qquad (11)
$$

where $W_k^+ = (P_k^+)^{-1}$, $W_k^- = (P_k^-)^{-1}$, and $H_k = \frac{\partial h_k}{\partial x_k}\Big|_{x_k=x_k^+}$.

Remark 1 *If we can know the constraint on the state, it can be used as an additional constraint to (10). Generally, the state may have certain operating range due to system characteristics or physical limitations. If such operating range can be known priori, we can represent it as an inequality constraint on states such as $g_C(x) \le 0$. Then this constraint can be added to (10). In Section 5, we will present an example to give more detailed insight utilizing such constraints.*

In case of linear systems or linear measurement equation, the equation (5) is given by $y_k = C_k x_k$. Hence, the constraint function $f_c(x,y) = c$ in the CSM is represented as $y = C_k x$. Then the optimization problem for the backward propagation is described by

$$\underset{x,y}{\text{Min}} \quad ||x - x_k^-||^2_{(P_k^-)^{-1}} + ||y - y_k^m||^2_{R^{-1}} \quad (12)$$

$$\text{subject to} \quad y = Cx \quad (13)$$

While the solution for (9) is obtained via a numerical nonlinear programming, the solution of (12) is obtained in an explicit form and given by

$$x_k^+ = x^* = ((P_k^-)^{-1} + C'R^{-1}C)^{-1}((P_k^-)^{-1}x_k^- + C'R^{-1}y_k^m). \quad (14)$$

From the uncertainty propagation equation (11) in the nonlinear case, the covariance update equation in the linear case is given by

$$P_k^+ \quad = \quad ((P_k^-)^{-1} + C'R^{-1}C)^{-1}. \quad (15)$$

4 Comparisons with the conventional filters

In this section, we compare the proposed estimator in the previous section with the conventional estimators such as the Kalman filter and the extended Kalman filter. First, we consider the linear case, where the system dynamics is give by $x_k = A_{k-1}x_{k-1} + v_{k-1}$. For the simplicity of the notations, we will skip the time index k in the system matrices such as A_k and C_k. Now, we present the Kalman filter in the linear case. First, time update equations are given by $x_k^- = Ax_{k-1}^+ \text{and} P_k^- = A'P_{k-1}^+A + Q$ And measurement update equations are given by $x_k^+ = x_k^- + K_k(y_k^m - Cx_k^-)\text{and}P_k^+ = (I - K_kC)P_k^-$ where $K_k = P_k^-C'(CP_k^-C' + R)^{-1}$. We can easily verify that the time update equations in the Kalman filter are equivalent to prediction equations (7) and (8) which are obtained through the forward propagation module. Now we will show that the measurement update equations in the Kalman filter are also equivalent to those from the backward propagation module.

Theorem 1 *The measurement update equations in the Kalman filter are equivalent to (14) and (15).*

Proof: See the reference [7] ■

Now, we turn to the nonlinear system (4)-(6). As a representative nonlinear filter, we present the extended Kalman filter(EKF). First, time update equations are given by $x_k^- = f_{k-1}(x_{k-1}^+)\text{and}P_k^- = F_{k-1}'P_{k-1}^+F_{k-1} + Q$ And measurement update equations are given by $x_k^+ = x_k^- + E_k(y_k^m - H_kx_k^-)\text{and}P_k^+ = (I - E_kH_k)P_k^-$ where $E_k = P_k^-H_k'(H_kP_k^-H_k' + R)^{-1}$ and $H_k = \frac{\partial h_k}{\partial x_k}\Big|_{x_k = x_k^-}$. From the result of Theorem 1, we can regard that the measurement update in the EKF is the optimal solution of the constrained optimization problem with the constraint $y = H_kx$ which is a linear approximation of the real constraint $y = h_k(x)$ at $x = x_k^-$. A graphical interpretation of the measurement update from the EKF is represented in Fig.1 comparing with the optimal solution of (9). From this figure, we observe that the EKF is suboptimal in the context of the optimization problem in the proposed geometric data fusion method. Indeed the optimal solution of (9) is maximum posteriori estimate, if the the state and output has a Gaussian distribution. Hence, we can guess that the estimate from geometric data fusion will show better performance than the EKF, even if the Gaussian property is not guaranteed due to nonlinearity in the measurement equation. We will verify this through the illustrative example which will be followed in the next section.

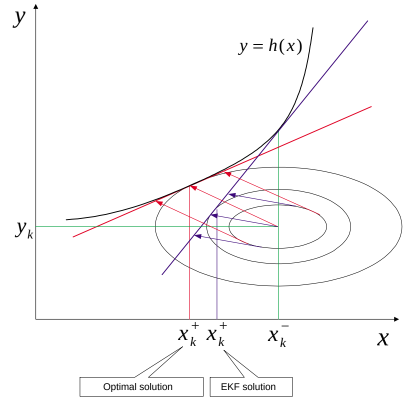

Figure 1: Estimate from geometric data fusion and extended Kalman filter

5 Illustrative Examples

5.1 Nonlinear estimation

In this subsection, we present some illustrative examples for verification of the performance of the proposed estimator based on the geometric data fusion(GDF) and comparison with the conventional EKF and the iterated Kalman filter (IKF) which was proposed as an effort to improve the EKF by iterating the measurement updates. For detailed definition of the IKF, see the reference [4]. We consider the following discrete linear dynamic system with the nonlinear measurement equation: $x_{k+1} = \begin{bmatrix} 1 & 0.1 \\ 0.1 & 0.9 \end{bmatrix} x_k + v_k, \;\; y_k = h(x_k) + w_k$ where the covariance matrices of v_k and w_k are given by $Q = 0.05 I_{2\times 2}$ and $R = 0.05$, respectively, and $h(x_k)$ will be specified later. For initial information, we assume that $x_0^+ = [0\ 0]'$ and $P^+(0) = I_{2\times 2}$.

We consider the case that $h(x)$ is given by $h(x) = [1/2((x(1)+1)^2 + x(2)^2)\ 1/2((x(1)-1)^2 + x(2)^2)]$ and the initial state is given by $x_0 = [0.5\ \ -0.5]'$. In this simulation, we fix the iteration number for GDF and IKF to be 2. The error norm trajectories are shown in Fig 2, where we observe that the GDF shows the best performance. Now, we consider the case that $x_0 = [3\ \ -5]'$, where the initial error is much larger than the previous case. In this case, both of the IKF and EKF diverge, while the GDF converges. Moreover, in Fig 3, we observe that the GDF shows better performances as the maximum iteration number increases. Through the example presented in this subsection, we observe that the proposed GDF estimator shows excellent performances over the conventional nonlinear estimators such as the IKF and EKF for the highly nonliear systems and large initial errors.

5.2 Constrained estimation

In this subsection, we apply the proposed estimator(GDF) to a constrained system where the state always stays in a bounded region, and compare it with the conventional linear Kalman filter(LKF). Since we showed that the LKF is equivalent to the proposed estimator in the linear case, we can regard the LKF as the proposed estimator not utilizing the constraint. Consider the following linear system: $x_{k+1} = \begin{bmatrix} 0.9950 & 0.0998 \\ -0.0998 & 0.9950 \end{bmatrix} x_k + v_k, \;\; y_k = Cx_k + w_k$, where C will be specified later and $x_0 = [10\ \ -10]'$. We assume that the initial state is not known priori and instead the following information on the initial state is known priori: $x_0^{low} \le x_0 \le x_0^{up}$. Since the poles of this system are located on the unit circle, the state oscillates and is always bounded by $||x_k|| \le ||x_0|| \le ||x_0^{lim}||$ where the ith element of x_0^{lim} is defined by $x_0^{lim}(i) = max\{x_0^{up}(i),\ -x_0^{low}(i)\}$. Now, we represent the above inequality in a conservative way as $-x^{lim} \le x_k \le x^{lim}$ where $x^{lim} = ||x_0^{lim}|| \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. In order to investigate the effects from the initial covariance assumption, we set the initial covariance as $P_0^+ = p_0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ where p_0 will be used as a design parameter. For the backward propagation in the proposed GDF, we solve the following constrained optimization problem:

$$\begin{aligned} &\underset{x,y}{\text{Minimize}} \quad ||x - x_k^-||^2_{(P_k^-)^{-1}} + ||y - y_k||^2_{R^{-1}} \\ &\text{subject to} \;\; y = Cx, \;\; -x^{lim} \le x \le x^{lim} \end{aligned}$$

which can be easily solved by the Quadratic Programming [8].

As in the examples in the previous subsection, we also assume that $v_k = w_k = 0$ for clear verification and comparison and $x_0^+ = [0\ 0]'$. First, we consider the case that C is given by $C = [1\ 1]$. Fig 4 shows the results of this case. When we set $p_0 = 1$, the GDF shows slightly better performance than the LKF. However, when we set $p_0 = 100$, the GDF shows much better performance than the LKF. Especially, we can observe that the GDF shows no oscillation while the LKF does. Now, we consider the case that C is given by $C = [1\ \ -1]$. Fig 5 shows the results of this case. In this example, we can observe the superiority of the GDF to the LKF.

Throughout the examples in this subsection, we can conclude that the constraints on the system can be utilized to enhance the performance of the proposed estimator.

6 Conclusions

In this paper, we propose a new geometric data fusion based on the Perception-Net. The proposed method can improve the performance of the sensing and estimation system by providing uncertainty management, maintaining consistency, and utilizing system knowledge. The proposed data fusion method is applied to the state estimation problem for nonlinear/linear dynamic system and a numerical algorithm for obtaining the estimation solution is introduced. Illustrative examples are presented to verify the performance of the proposed estimator.

The proposed estimator is obtained from an optimization problem, while the EKF is a suboptimal solution in the optimization problem. Hence, the proposed estimator can show better performance than the EKF. Since the proposed estimator can incorporate the system knowledge, it can improve the performance comparing with the conventional Kalman filter not utilizing the system knowledge. In the case of linear systems, it is shown that the proposed estimator is equivalent to the linear Kalman filter. Through illustrative examples, we verified that the proposed estimator shows better performance than the conventional nonlinear filters such as EKF and IKF. We also verified that the proposed estimator can improve the performance by incorporating the system knowledge.

Future works regarding the proposed estimator will be to find conditions under which asymptotic convergence is guaranteed and to improve the computational efficiency.

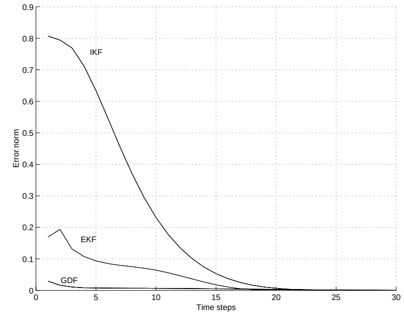

Figure 2: Estimation error for $x_0 = [0.5 \ -0.5]'$

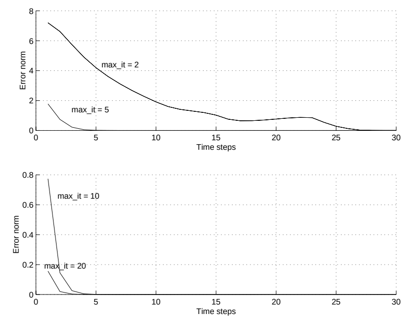

Figure 3: Estimation error with GDF and the corresponding maximum allowed iteration number for $x_0 = [3 \ -5]'$

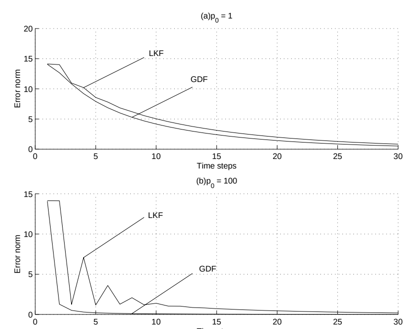

Figure 4: Estimation error for $y = h(x) = [1 \ 1]x$

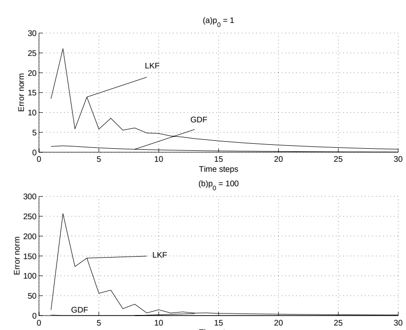

Figure 5: Estimation error for $y = h(x) = [1 \ -1]x$

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