Estimation of Parameters in State Equations via Multiple Observers

Eiichi MURAMATSU
Department of Aerospace Engineering
Osaka Prefecture University
Sakai, Osaka 599-8531, Japan
muramatu@aero.osakafu-u.ac.jp

Masao IKEDA
Department of Mechanical Engineering
Osaka University
Suita, Osaka 565-0871, Japan
ikeda@mech.eng.osaka-u.ac.jp

Abstract

A parameter estimation problem is considered for state equations of linear time-invariant systems. Under a certain condition, it is shown that the output of the system can be asymptotically described by a linear combination of output estimates which are generated by suitable multiple observers. An identification scheme for the weights of the linear combination is proposed. The obtained weights determine the parameter values to be estimated.

1 Introduction

State equations of linear time-invariant systems are usually derived from physical laws. Then, coefficient matrices of the state equation contain physical constants, quantities, and their combinations as parameters. In many practical cases, some of such parameters cannot be measured or computed from known constants or quantities. Therefore, their values need to be estimated in order to design control systems.

In this paper, we propose an estimation method for unknown parameters in state and input coefficient matrices of linear time-invariant state equations. We first rewrite the coefficient matrices as a linear combination of some virtual matrices, where the weights are related to the parameters to be estimated. Then, the given state equation is described by a linear combination of systems defined by the virtual matrices. We construct observers for individual virtual systems under a certain condition. It is shown that the output of the given state equation is asymptotically described by a linear combination of the output estimates generated by the multiple observers with the same weights as those employed to rewrite the coefficient matrices. Using this property, the weights of the linear combination are identified, and estimation formulae for the unknown parameters are presented.

Identification of state space models have been considered in literatures, e.g., [1]∼[3], where coefficient matrices are derived from input and output data. In these cases, the state space is not specified a priori. The problem formulation of the present paper is different. We suppose that the state space has been fixed by the derivation process, where the coefficient matrices contain unknown parameters as well as known constants. Then, we estimate the unknown parameters only, while the above identification methods intend to determine the whole coefficient matrices.

Multiple observers have been introduced in the areas of failure detection [4], [5] and switching control [6], where the multiple observers are designed for candidate plant models, one of which describes the functioning plant dynamics. In the estimation scheme of the present paper, the multiple observers are designed for virtual systems, each of which is not required to describe actual dynamics of the given plant.

2 Problem Formulation

In this section, we give a state equation whose coefficient matrices contain unknown parameters. Then, introducing a linear transformation of the unknown parameters, we rewrite the coefficient matrices as a linear combination of some virtual matrices.

2.1 State Equation with Unknown Parameters

Let us consider a state equation

$$\dot{x}(t) = A(a_1, a_2, \cdots, a_p)x(t) + B(b_1, b_2, \cdots, b_q)u(t) \quad (1)$$

where $x(t)$ is the state and $u(t)$ is the input. In the matrices A and B, there are unknown parameters a_1, $\cdots$, a_p and b_1, $\cdots$, b_q, which are assumed to appear affinely as

$$A(a_1, a_2, \cdots, a_p) = A_0 + \sum_{i=1}^{p} a_i A_i \quad (2)$$

$$B(b_1, b_2, \cdots, b_q) = B_0 + \sum_{j=1}^{q} b_j B_j. \quad (3)$$

Here, A_0 and B_0 are matrices obtained from $A(a_1, a_2, \cdots, a_p)$ and $B(b_1, b_2, \cdots, b_q)$ by substituting 0 for all a_i and b_j. The matrices A_i ($i = 1, \cdots, p$) and B_j ($j = 1, \cdots, q$) are obtained from $A(a_1, a_2, \cdots, a_p) - A_0$

and $B(b_1, b_2, \cdots, b_q) - B_0$ by setting the unknown a_i, b_j to be 1, respectively, and others to be 0.

The output of the system is described by

$$y(t) = Cx(t) \tag{4}$$

where the matrix C does not contain any unknown parameter. We assume that $(C,\ A(a_1,\ a_2,\ \cdots\ ,\ a_p))$ is detectable when $a_1, a_2, \cdots, a_p$ take their actual values. The objective of this paper is to estimate the unknown parameters a_i $(i = 1, \cdots, p)$, b_j $(j = 1, \cdots, q)$ from the input and output signals of the system described by (1) and (4).

2.2 Transformation of Parameters

For the purpose of estimation, we introduce a transformation

$$[\, a_1 \ \cdots \ a_p \ b_1 \ \cdots \ b_q \ 1 \,]^T = \tilde{T}[\, \theta_1 \ \theta_2 \ \cdots \ \theta_r \,]^T \tag{5}$$

which relates the parameters a_i $(i = 1, \cdots, p)$ and b_j $(j = 1, \cdots, q)$ to θ_k $(k = 1, \cdots, r)$, where $r = p + q + 1$. In (5), the matrix $\tilde{T}$ is defined as

$$\tilde{T} \triangleq \begin{bmatrix} & T & \\ 1 & \cdots & 1 \end{bmatrix} \tag{6}$$

where T is a $(p+q) \times (p+q+1)$ matrix such that $\tilde{T}$ is nonsingular.

From (5), the matrices in (2) and (3) are rewritten as

$$A(a_1, a_2, \cdots, a_p) = \sum_{k=1}^{r} \theta_k \bar{A}_k \tag{7}$$

$$B(b_1, b_2, \cdots, b_q) = \sum_{k=1}^{r} \theta_k \bar{B}_k \tag{8}$$

where $\bar{A}_k$ and $\bar{B}_k$ are matrices obtained from $A(a_1,\ a_2,\ \cdots\ ,\ a_p)$ and $B(b_1,\ b_2,\ \cdots\ ,\ b_q)$ by substituting the values of $a_1, \cdots, a_p, b_1, \cdots, b_q$ calculated by (5) with $\theta_k = 1$, $\theta_\ell = 0$ $(k \neq \ell)$. That is, they are expressed by

$$\begin{cases} \bar{A}_k = A_0 + \sum_{i=1}^{p} t_{ik} A_i \\ \bar{B}_k = B_0 + \sum_{j=1}^{q} t_{(p+j)k} B_j \end{cases}, \quad k = 1, \cdots, r \tag{9}$$

where t_{ik} is the (i, k) element of T in (6).

As shown in (7) and (8), the matrices of $A(a_1,\ a_2,\ \cdots,\ a_p)$ and $B(b_1,\ b_2,\ \cdots,\ b_q)$ are expressed by a linear combination of $\bar{A}_k$ and $\bar{B}_k$ weighted by the parameters θ_k $(k = 1, \cdots, r)$ which satisfy the bottom row of (5), that is,

$$\sum_{k=1}^{r} \theta_k = 1. \tag{10}$$

However, $\sum_{k=1}^{r} \theta_k \bar{A}_k$ and $\sum_{k=1}^{r} \theta_k \bar{B}_k$ are not convex in θ_k $(k = 1, \cdots, r)$, since the parameters θ_k $(k = 1, \cdots, r)$ are not required to be nonnegative.

By the descriptions (7) and (8), the state equation (1) and (4) are rewritten as

$$\begin{cases} \dot{x}(t) = \sum_{k=1}^{r} \theta_k \bar{A}_k x(t) + \sum_{k=1}^{r} \theta_k \bar{B}_k u(t) \\ y(t) = Cx(t). \end{cases} \tag{11}$$

We consider identification of θ_k $(k = 1, \cdots, r)$ in (11) which give the estimates of a_i $(i = 1, \cdots, p)$ and b_j $(j = 1, \cdots, q)$ through (5).

3 Multiple Observers

In this section, under a certain condition, we introduce observers for virtual systems which are obtained from the description (11). We then show that the output of the given state equation is asymptotically described by a linear combination of the output estimates generated by the multiple observers.

3.1 Output Described by Multiple Estimates

By setting $\theta_k = 1$ and $\theta_\ell = 0$ $(\ell \neq k)$, we consider virtual systems

$$\begin{cases} \dot{x}_k(t) = \bar{A}_k x_k(t) + \bar{B}_k u(t) \\ y(t) = Cx_k(t) \end{cases}, \quad k = 1, \cdots, r. \tag{12}$$

We assume that there exist matrices L_k $(k = 1, \cdots, r)$ which satisfy

$$\bar{A}_1 - L_1 C = \bar{A}_2 - L_2 C = \cdots = \bar{A}_r - L_r C \tag{13}$$

and $\bar{A}_k - L_k C$ $(k = 1, \cdots, r)$ are stable. The existence of such L_k $(k = 1, \cdots, r)$ will be discussed in the next subsection.

For the virtual systems (12), we introduce observers

$$\mathcal{O}_k \begin{cases} \dot{\hat{x}}_k(t) = (\bar{A}_k - L_k C)\hat{x}_k(t) + [L_k \ \ \bar{B}_k] \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} \\ \hat{y}_k(t) = C\hat{x}_k(t) \end{cases}$$
$$k = 1, \cdots, r \tag{14}$$

where the inputs of these observers are $u(t)$ and $y(t)$ which are the input and output signals of the given system described by (1) and (4).

The following relation holds between the outputs $\hat{y}_k(t)$ $(k = 1, \cdots, r)$ of (14) and $y(t)$ of (4).

Theorem 1 Let us define

$$\hat{y}_\theta(t) \triangleq \theta_1 \hat{y}_1(t) + \theta_2 \hat{y}_2(t) + \cdots + \theta_r \hat{y}_r(t).$$

Then,

$$\hat{y}_\theta(t) \to y(t), \quad t \to \infty. \tag{15}$$

This theorem means that in Fig.1, the signal $\hat{y}_\theta(t)$ becomes identical with the output $y(t)$ of the system when

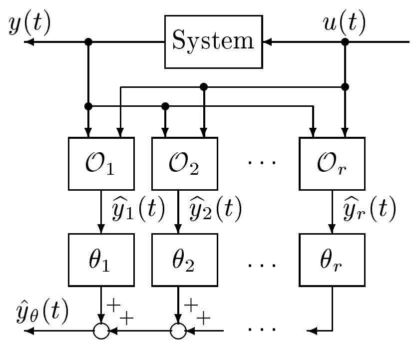

Fig.1 Outputs of the system and multiple observers

$t \to \infty$. Using this fact, an identification scheme for θ_k $(k = 1, \cdots, r)$ is proposed in Section 4.

Proof of Theorem 1 : We consider the description (11) instead of the state equation of (1) and (4). From (10) and (13), (11) can be transformed to

$$\begin{aligned}\dot{x}(t) &= \sum_{k=1}^{r} \theta_k (\bar{A}_k - L_k C) x(t) \\ &\quad + \sum_{k=1}^{r} \theta_k L_k C x(t) + \sum_{k=1}^{r} \theta_k \bar{B}_k u(t) \\ &= A_s x(t) + \sum_{k=1}^{r} \theta_k \{ L_k y(t) + \bar{B}_k u(t) \} \qquad (16)\end{aligned}$$

where

$$A_s \triangleq \bar{A}_k - L_k C, \quad k = 1, \cdots, r. \qquad (17)$$

Then, it is seen from (14), (16) and (17) that

$$e(t) \triangleq \sum_{k=1}^{r} \theta_k \hat{x}_k(t) - x(t)$$

satisfies

$$\dot{e}(t) = A_s e(t).$$

Since A_s is stable, $e(t) \to 0$ as $t \to \infty$. Thus,

$$\begin{aligned}\hat{y}_\theta(t) - y(t) &= \sum_{k=1}^{r} \theta_k C \hat{x}_k(t) - Cx(t) \\ &= Ce(t) \to 0, \quad t \to \infty.\end{aligned}$$

3.2 Existence Condition for Multiple Observers

To construct the observers $\mathcal{O}_k$ $(k = 1, \cdots, r)$, the matrices L_k $(k = 1, \cdots, r)$ which satisfy (13) need to exist. In this subsection, we consider the choice of T in (6), and clarify the condition on $A(a_1, a_2, \cdots, a_p)$ and C for the existence of L_k $(k = 1, \cdots, r)$.

We note that detectability of $(C, \bar{A}_k)$ is necessary for $\bar{A}_k - L_k C$ $(k = 1, \cdots, r)$ to be stable. Since the matrices $\bar{A}_k$ $(k = 1, \cdots, r)$ are dependent on t_{ik} $(i = 1, \cdots, p)$ as in (9), the matrix T in (6) should be chosen so that $(C, \bar{A}_k)$ $(k = 1, \cdots, r)$ are detectable. The existence of such a T is guaranteed by the assumption that $(C, A(a_1, \cdots, a_p))$ is detectable for actual $a_1, \cdots, a_p$. For example, if t_{ik} $(i = 1, \cdots, p)$ are in the neighborhood of the actual values of the unknown parameters a_i $(i = 1, \cdots, p)$, then $\bar{A}_k$ is close to $A(a_1, \cdots, a_p)$ and $(C, \bar{A}_k)$ is expected to be detectable, as $(C, A(a_1, \cdots, a_p))$ is so.

Here, we assume that T has been chosen so that $(C, \bar{A}_k)$ $(k = 1, \cdots, r)$ are detectable and the matrix L_1 has been determined so that $\bar{A}_1 - L_1 C$ is stable. We then consider the condition (13), that is, the existence of L_k $(k = 2, \cdots, r)$ which satisfy

$$\bar{A}_k - L_k C = \bar{A}_1 - L_1 C, \quad k = 2, \cdots, r. \qquad (18)$$

By (9), this condition is reduced to

$$\sum_{i=1}^{p} (t_{i1} - t_{ik}) A_i = (L_1 - L_k) C, \quad k = 2, \cdots, r. \qquad (19)$$

This means that L_k $(k = 1, \cdots, r)$ exists, which satisfies (19) and then (18), if and only if

$$\mathrm{Ker}\{\sum_{i=1}^{p} (t_{i1} - t_{ik}) A_i\} \supseteq \mathrm{Ker} C, \quad k = 2, \cdots, r,$$

that is,

$$\bigcap_{k=2}^{r} \mathrm{Ker}\{\sum_{i=1}^{p} (t_{i1} - t_{ik}) A_i\} \supseteq \mathrm{Ker} C. \qquad (20)$$

Since the left side of (20) is equal to $\bigcap_{i=1}^{p} \mathrm{Ker} A_i$ as shown in Appendix, we can give the following lemma.

Lemma 1 For detectable pairs $(C, \bar{A}_k)$ $(k = 1, \cdots, r)$, there exist L_k $(k = 1, \cdots, r)$ which satisfy (13) if and only if

$$\mathrm{Ker} A_i \supseteq \mathrm{Ker} C, \quad i = 1, \cdots, p.$$

For example, let us consider

$$A(a_1, a_2, a_3) = \begin{bmatrix} \circ & \circ & \circ \\ a_1 & \circ & a_2 \\ \circ & \circ & a_3 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\circ$ denotes the elements whose values are known. Then

$$A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and from Lemma 1, there do not exist L_k $(k = 1, 2, 3)$ which satisfy (13), since $\mathrm{Ker}A_1 \not\supset \mathrm{Ker}C$. However, if

$$A(a_1, a_2, a_3) = \begin{bmatrix} \circ & \circ & \circ \\ \circ & a_1 & a_2 \\ \circ & \circ & a_3 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

then there exist L_k $(k = 1, 2, 3)$ which satisfy (13).

4 Parameter Estimation

In this section, we present an identification scheme for a_i and b_j through θ_k based on the relation (15) in Theorem 1.

4.1 Estimation under No Measurement Noise

We define a parameter vector

$$\Theta = [\ \theta_1 \ \ \theta_2 \ \ \cdots \ \ \theta_r\ ]^T$$

and matrices

$$\hat{Y}(t) = \begin{bmatrix} \hat{y}_1(t) & \hat{y}_2(t) & \cdots & \hat{y}_r(t) \\ \hat{y}_1(t-\tau_1) & \hat{y}_2(t-\tau_1) & \cdots & \hat{y}_r(t-\tau_1) \\ \vdots & \vdots & \cdots & \vdots \\ \hat{y}_1(t-\tau_\ell) & \hat{y}_2(t-\tau_\ell) & \cdots & \hat{y}_r(t-\tau_\ell) \end{bmatrix}$$

$$Y(t) = [\ y^T(t) \ \ y^T(t-\tau_1) \ \ \cdots \ \ y^T(t-\tau_\ell)\ ]^T$$

where $0 < \tau_1 < \tau_2 < \cdots < \tau_\ell$. From (10) and (15), we see that

$$\begin{bmatrix} f \\ \hat{Y}(t) \end{bmatrix} \Theta \to \begin{bmatrix} 1 \\ Y(t) \end{bmatrix}, \quad t \to \infty$$

where

$$f = [\ 1 \ \ 1 \ \ \cdots \ \ 1\ ].$$

If $[\ f^T \ \ \hat{Y}^T(t)\ ]^T$ has a column full rank, then

$$\hat{\Theta}(t) \triangleq \left([\ f^T \ \ \hat{Y}^T(t)\ ] \begin{bmatrix} f \\ \hat{Y}(t) \end{bmatrix} \right)^{-1} [\ f^T \ \ \hat{Y}^T(t)\ ] \begin{bmatrix} 1 \\ Y(t) \end{bmatrix}$$

can be computed and it converges, that is,

$$\hat{\Theta}(t) \to \Theta, \quad t \to \infty.$$

Thus, an estimate $\hat{\Theta}$ is obtained as the steady state value of $\hat{\Theta}(t)$. From the relation of (5), the parameters in the state equation (1) are given by

$$[\ a_1 \ \ \cdots \ \ a_p \ \ b_1 \ \ \cdots \ \ b_q\ ]^T = T\hat{\Theta}. \tag{21}$$

Remark 1 If the input $u(t)$ of the system excite all the system dynamics, and the number ℓ of the past data sets satisfies $(\ell+1)n_y + 1 \geq r$ where n_y denotes the dimension of $y(t)$, then $[\ f^T \ \ \hat{Y}^T(t)\ ]^T$ generally has a column full rank. Thus, if $n_y \geq r - 1 (= p + q)$, any past data are not needed for the estimation of the parameters $a_1, \cdots, a_p$ and $b_1, \cdots, b_q$.

4.2 Estimation under Measurement Noise

In this subsection, we suppose that the measurement signal is disturbed by a noise $w(t)$ with zero mean value. Then, the output equation in (11) is replaced with

$$\tilde{y}(t) = Cx(t) + w(t)$$

and $y(t)$ in (14) is replaced with $\tilde{y}(t)$. Therefore, the relation corresponding to (15) reduces to

$$\hat{y}_\theta(t) \to \tilde{y}(t) + \tilde{w}(t), \quad t \to \infty$$

where $\tilde{w}(t)$ is output of the system

$$\begin{cases} \dot{z}(t) = A_s z(t) + \sum_{k=1}^{r} \theta_k L_k w(t) \\ \tilde{w}(t) = Cz(t) - w(t) \end{cases}$$

with $z(0) = 0$. That is, for a sufficiently large t, we obtain

$$\hat{y}_\theta(t) = \tilde{y}(t) + \tilde{w}(t).$$

Using the data at the times of $t_1 < t_2 < \cdots < t_N$, this equation and (10) imply

$$\begin{bmatrix} f \\ \hat{Y} \end{bmatrix} \Theta = \begin{bmatrix} 1 \\ \tilde{Y} \end{bmatrix} + \begin{bmatrix} 0 \\ W \end{bmatrix}$$

$$\hat{Y} = \begin{bmatrix} \hat{y}_1(t_1) & \hat{y}_2(t_1) & \cdots & \hat{y}_r(t_1) \\ \hat{y}_1(t_2) & \hat{y}_2(t_2) & \cdots & \hat{y}_r(t_2) \\ \vdots & \vdots & \cdots & \vdots \\ \hat{y}_1(t_N) & \hat{y}_2(t_N) & \cdots & \hat{y}_r(t_N) \end{bmatrix}$$

$$\tilde{Y} = [\ \tilde{y}^T(t_1) \ \ \tilde{y}^T(t_2) \ \ \cdots \ \ \tilde{y}^T(t_N)\ ]^T$$

$$W = [\ \tilde{w}^T(t_1) \ \ \tilde{w}^T(t_2) \ \ \cdots \ \ \tilde{w}^T(t_N)\ ]^T.$$

Since the value of W is not known except that the mean value is zero, we apply the least square method minimizing

$$\left\| \begin{bmatrix} f \\ \hat{Y} \end{bmatrix} \Theta - \begin{bmatrix} 1 \\ \tilde{Y} \end{bmatrix} \right\|^2$$

to obtain the estimate

$$\hat{\Theta} = \left([\ f^T \ \ \hat{Y}^T\ ] \begin{bmatrix} f \\ \hat{Y} \end{bmatrix} \right)^{-1} [\ f^T \ \ \hat{Y}^T\ ] \begin{bmatrix} 1 \\ \tilde{Y} \end{bmatrix}. \tag{22}$$

The estimates of the unknown parameters in the state equation (1) are computed as (21).

5 Example

Let us consider the following system.

$$\dot{x}(t) = \begin{bmatrix} -7 & 4 & -3 \\ -1 & a_1 & a_2 \\ 0 & -8 & a_3 \end{bmatrix} x(t) + \begin{bmatrix} 1 & b_1 \\ 0 & 2 \\ 3 & -1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(t)$$

Here, a_i $(i = 1,2,3)$ and b_1 are unknown parameters to be estimated, and let their actual values be

$$a_1 = -3\ ,\quad a_2 = 2\ ,\quad a_3 = -1\ ,\quad b_1 = 3. \qquad (23)$$

In this case, $p = 3$, $q = 1$, $r = p + q + 1 = 5$ and the matrices A_0, A_i $(i = 1,2,3)$, B_0, B_1 in (2) and (3) are

$$A_0 = \begin{bmatrix} -7 & 4 & -3 \\ -1 & 0 & 0 \\ 0 & -8 & 0 \end{bmatrix},\quad A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},\quad A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$B_0 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & -1 \end{bmatrix},\quad B_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

For the transformation of (5), we choose T as

$$T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \qquad (24)$$

There are infinite number of T which satisfy the constraints of nonsinglarity of $\tilde{T}$ and detectability of $(C, \bar{A}_k)$. Among them, we choose (24).

Then, the matrices of $\bar{A}_k$, $\bar{B}_k$ $(k = 1, \cdots, 5)$ in (7) and (8) are determined as

$$\begin{aligned} &\bar{A}_1 = A_0 + A_1,\quad \bar{A}_2 = A_0 + A_1 + A_2 \qquad (25)\\ &\bar{A}_3 = A_0 + A_3,\quad \bar{A}_4 = \bar{A}_5 = A_0 \\ &\bar{B}_1 = \bar{B}_2 = \bar{B}_3 = \bar{B}_5 = B_0,\quad \bar{B}_4 = B_0 + B_1. \end{aligned}$$

We note that $(C, \bar{A}_k)$ $(k = 1, \cdots, 5)$ are all detectable.

From Lemma 1, there exists L_k $(k = 1, \cdots, 5)$ which satisfy (13). Such L_k $(k = 1, \cdots, 5)$ can be obtained by the following procedure. Fisrt, let L_1 be

$$L_1 = \begin{bmatrix} -13 & -1.5 & -4 \\ -3 & 0 & 3 \end{bmatrix}^T$$

so that $\bar{A}_1 - L_1 C$ is stable (its eigenvalues are -1, -2, and -3). Next, let us compute L_2 which satisfies

$$\bar{A}_1 - L_1 C = \bar{A}_2 - L_2 C.$$

From (25), this equation becomes

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} = (L_1 - L_2) \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and the solution is

$$L_1 - L_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}^T.$$

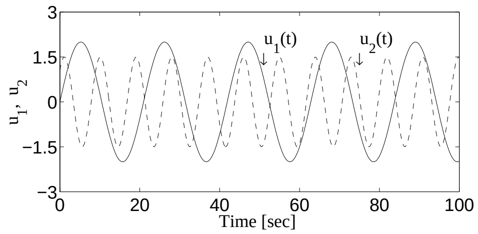

Fig.2 Input signals

Then, L_2 is obtained as

$$L_2 = L_1 - \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}^T.$$

Similarly,

$$L_3 = L_1 - \begin{bmatrix} 0 & 0.5 & 0 \\ 0 & 0 & -1 \end{bmatrix}^T$$
$$L_4 = L_5 = L_1 - \begin{bmatrix} 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T$$

are obtained. Using these matrices, the observers $\mathcal{O}_k$ $(k = 1, \cdots, 5)$ of (14) are constructed.

Suppose that the input of the system is

$$u(t) = \begin{bmatrix} 20\sin 0.3t \\ 15\sin(0.7t + \pi/4) \end{bmatrix}$$

as shown in Fig.2. When there is no noise, the measurement signals are obtained as Fig.3. We estimate the unknown parameters by the method presented in Subsection 4.1 using the data at $t = 51.4$, $\tau_1 = 0.3$, and $\tau_2 = 0.6$. In this case, the estimates computed by (21) are identical to (23).

When there is a measurement noise which is white and Gaussian with the variance 0.01, the measurement signal is obtained as Fig.4. In this case, we compute (22) using the 20 sets of data sampled in the period $t \in [47, 54]$. The estimates computed by (22) and (21) are

$$a_1 = -3.00,\quad a_2 = 1.99,\quad a_3 = -0.99,\quad b_1 = 3.01,$$

which are close enough to the actual valeus of (23).

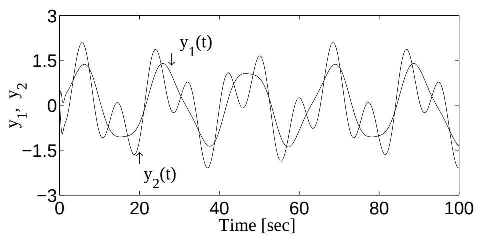

Fig.3 Measurement signals with no noise

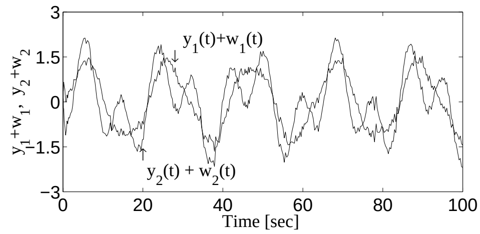

Fig.4 Measurement signals disturbed by noise

6 Conclusion

We have presented an estimation method for unknown parameters in linear time-invariant state equations, using multiple observers for virtual systems whose linear combination describes a given system. We have derived a condition for such observers to exist. An illustrative example has demonstated the usefulness of the proposed idea.

The proposed estimation scheme can be used in real time and may be applied for adaptive control when the parameter change in the plant is sufficiently slow. The details will be discussed in a future paper.

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Appendix

We prove

$$\bigcap_{k=2}^{r} \mathrm{Ker}\{\sum_{i=1}^{p}(t_{i1}-t_{ik})A_i\} = \bigcap_{i=1}^{p} \mathrm{Ker}A_i.$$

We first note that if a vector η satisfies $\eta \in \bigcap_{i=1}^{p} \mathrm{Ker}A_i$, that is, $A_i\eta = 0$ $(i = 1, \cdots p)$, then obviously $\{\sum_{i=1}^{p}(t_{i1} - t_{ik})A_i\}\eta = 0$ $(k = 2, \cdots, r)$. Therefore,

$$\bigcap_{i=1}^{p} \mathrm{Ker}A_i \subseteq \bigcap_{k=2}^{r} \mathrm{Ker}\{\sum_{i=1}^{p}(t_{i1}-t_{ik})A_i\}.$$

We next show

$$\bigcap_{k=2}^{r} \mathrm{Ker}\{\sum_{i=1}^{p}(t_{i1}-t_{ik})A_i\} \subseteq \bigcap_{i=1}^{p} \mathrm{Ker}A_i \tag{26}$$

by contradiction. Let us suppose that there exists a vector η such that $\eta \in \bigcap_{k=2}^{r} \mathrm{Ker}\{\sum_{i=1}^{p}(t_{i1}-t_{ik})A_i\}$, but $\eta \notin \bigcap_{i=1}^{p} \mathrm{Ker}A_i$. Then,

$$\begin{aligned} &\sum_{i=1}^{p}(t_{i1}-t_{ik})A_i\eta \\ &= [\ A_1\eta \ \ A_2\eta \ \ \cdots \ \ A_p\eta\ ] \begin{bmatrix} t_{11}-t_{1k} \\ t_{21}-t_{2k} \\ \vdots \\ t_{p1}-t_{pk} \end{bmatrix} = 0 \\ &\qquad k = 2, \cdots, r, \end{aligned}$$

which implies

$$\begin{aligned} &[\ \xi \ \ \xi \ \ \cdots \ \ \xi\ ] \\ &= [\ A_1\eta \ \ A_2\eta \cdots \ \ A_p\eta\ ] \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1r} \\ t_{21} & t_{22} & \cdots & t_{2r} \\ \vdots & \vdots & \vdots & \vdots \\ t_{p1} & t_{p2} & \cdots & t_{pr} \end{bmatrix} \end{aligned} \tag{27}$$

where

$$\xi \triangleq [\ A_1\eta \ \ A_2\eta \cdots \ \ A_p\eta\ ] \begin{bmatrix} t_{11} \\ t_{12} \\ \vdots \\ t_{p1} \end{bmatrix}.$$

If ξ has a zero element, then (27) means that the first p rows of the matrix $\tilde{T}$ of (6) are not independent. If ξ has a nonzero element, then the bottom row of $\tilde{T}$ depends on the first p rows of $\tilde{T}$. In both cases, $\tilde{T}$ is singular. This contradicts the definition of $\tilde{T}$ and concludes (26). The proof is completed.