Robust gain-scheduled control in web winding systems

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Abstract

The plant is a web transport system with winder and unwinder. Due to a wide-range variation of the radius and inertia of the rollers the system dynamics change considerably. Two different control strategies for web tension and linear transport velocity are presented. The first is an H_∞ robust control strategy with varying gains based on a particularity of the plant. The second is an LPV control strategy with smooth scheduling of controllers synthesized for different operating points. The quadratic stability and the quadratic performance of the closed loop system are analyzed. The LPV control strategy gives better results on an experimental setup, for the rejection of the disturbances introduced by velocity variations.

1 Introduction

The systems transporting paper, metal, polymers or textile are very common in the industry. The main goal is to increase as much as possible the web transport velocity while controlling the tension of the web. The main concern is the coupling existing between web velocity and tension. There exist many sources of disturbances on the velocity (roller non-circularity, web sliding). Due to the coupling introduced by the elastic web, these disturbances are transmitted to the web tension, resulting in a web break or fold. Several studies about control in the web handling domain can be found e.g. in [1], [2]. Multivariable control strategies have been recently proposed for industrial metal transport systems (see [3] and [4]), and an H_∞ robust control is presented in [5] concerning the decoupling of web tension and velocity. Another concern is the robustness to the radius and inertia changes during the winding process. As the synthesis of the multivariable controller is done for a particular operating point, the controller is not tuned during all the process.

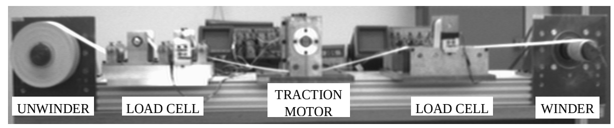

Figure 1: Experimental setup

2 Winding process

The system under study has three motors (cf. figure 1) and exhibits the inherent difficulties of elastic web transport systems.

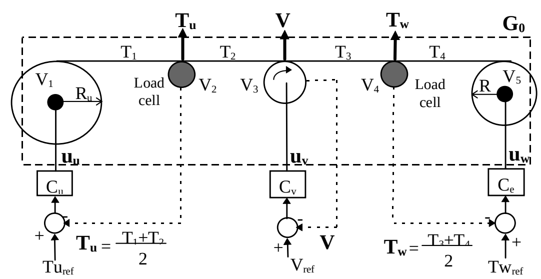

Figure 2: Decentralized control scheme

Figure 2 shows the different variables used in the model [5], [6], [7], the inputs (the control signals u_u, u_v and u_w) and the outputs (unwinding web tension T_u, traction motor's linear velocity V, and winding web tension T_w) of the system G_0 (defined by the dashed box). The control signals are the torque references of synchronous motors. The web velocity is imposed by the traction motor and the web tension is controlled by the unwinding and winding motors. The state space representation of the nominal model around an operating point can be expressed as (see [7]):

$$E(t)\dot{X} = A(t)X + B(t)U \qquad Y = CX$$

with

$$
A(t) = \begin{bmatrix}
-f_1(t) & R_u(t)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-E_0 & -V_0 & E_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -R_2^2 & -f_2 & R_2^2 & 0 & 0 & 0 & 0 & 0 \\
0 & V_0 & -E_0 & -V_0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -R_3^2 & -f_3 & R_3^2 & 0 & 0 & 0 \\
0 & 0 & 0 & V_0 & -E_0 & -V_0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -R_4^2 & -f_4 & R_4^2 & 0 \\
0 & 0 & 0 & 0 & 0 & V_0 & -E_0 & -V_0 & E_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -R_w(t)^2 & -f_5(t)
\end{bmatrix}
$$

$$
E(t) = \begin{bmatrix}
J_1(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & J_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & L_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & J_3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & L_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & J_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & L_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_5(t)
\end{bmatrix}
\qquad
B(t) = \begin{bmatrix}
-K_1 R_u(t) & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & K_3 R_3 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & K_5 R_w(t)
\end{bmatrix}
$$

$$
X^T = \begin{pmatrix} V_1 & T_1 & V_2 & T_2 & V_3 & T_3 & V_4 & T_4 & V_5 \end{pmatrix} \quad U^T = \begin{pmatrix} u_u & u_v & u_w \end{pmatrix} \quad Y^T = \begin{pmatrix} T_u & V_3 & T_w \end{pmatrix}
$$

where V_i, R_i, J_i and f_i are respectively the linear velocity, the radius, the inertia and the viscous friction coefficient of the roll i. T_i and L_i are the web tension and the web length respectively between the roll i and the roll $i+1$. K_1, K_3 and K_5 are the torque constants of each motor. V_0 is the nominal velocity and E_0 is a constant depending on the web elasticity.

3 Gain-scheduled control

Considering the unwinder and the winder separately, if the time varying parameters were constant, the transfer function between the control signals and the web tensions are inversely proportional to the radius:

$$
\lim_{s \to 0} \frac{T_1(s)}{u_u(s)} \propto \frac{1}{R_u} \qquad \lim_{s \to 0} \frac{T_4(s)}{u_w(s)} \propto \frac{1}{R_w} \tag{1}
$$

A new plant G_R is obtained by multiplying the con-

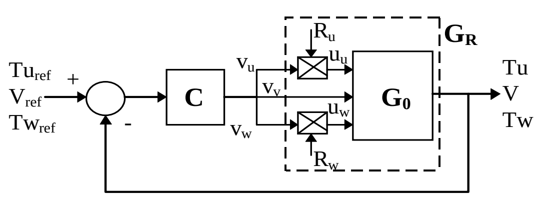

Figure 3: Modified system

troller output signals v_u and v_w by the radius R_u et R_w respectively. This plant has the advantage of making the gain at low frequency less dependent of the radius and inertia. See figure 4, which shows the maximum singular values of the system G_0 and G_R at different operating points with different radius an inertia. The synthesis of a multivariable controller is done using the model G_R which includes the varying gains. The robust H_∞ controller is synthesized using the mixed sensitivity method [8]. The order of the resulting controller is 15. It has been implemented on the experimental setup in state space representation with a sampling period of 10 ms.

An H_∞ controller has also been synthesized in a similar way considering the nominal system G_0. The operating point used for this controller synthesis corresponds to the starting phase. The main reason is that, web tension and velocity must be well controlled during the starting phase to avoid problem later.

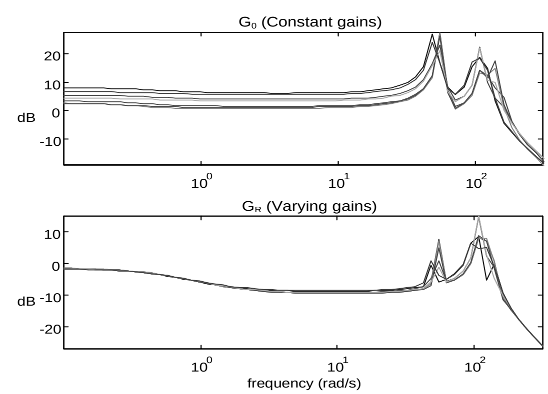

Figure 4: Singular values for different radius

We can see on figure 5 that the controller with varying gains keeps sensibly the same performance for reference tracking of the web tension and the same response to the perturbation introduced by the velocity variations, during all the process operation.

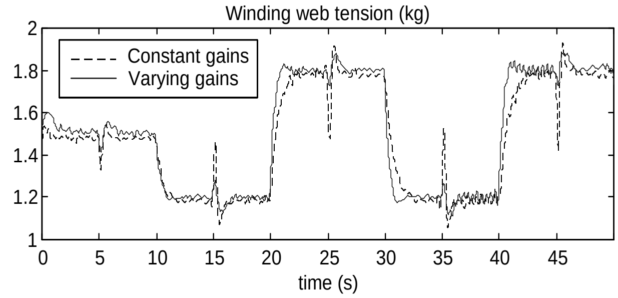

Figure 5: Comparison of winding web tensions

4 LPV control

The next step is to look for a controller which guarantees exactly the same performances during all the process run. Since the system is linear with varying parameters, the solution to this problem consists into finding a controller for different operating points and doing a smooth scheduling between these controllers. We consider the plant, $P(\theta)$, a parameter-dependent system, where the parameter $\theta^T = [R_1(t)\ R_5(t)]$ is time varying. We assume that the other time varying parameters (depending on the inertia and friction), appearing in the matrices of the state space representation can be expressed approximately affinely in function of the radius R_1 and R_5. The plant $P(\theta)$ is in a polytopic set $\mathcal{P}_\theta$, i.e.:

$$\theta(t) \in \mathcal{P}_\theta := Co\{\theta_1, \theta_2, \theta_3, \theta_4\}, \qquad (2)$$

the notation $Co\{.\}$ stands for the convex hull of the set $\{.\}$. Given any convex decomposition of the parameter θ, i.e.:

$$\theta(t) = \alpha_1\Pi_1 + \alpha_2\Pi_2 + \alpha_3\Pi_3 + \alpha_4\Pi_4$$
$$\alpha_i \geq 0 \text{ for } i \in [1,..,4], \qquad \sum_{i=1}^{i=4} \alpha_i = 1 \qquad (3)$$

where the Π_i, for $i \in [1,..,4]$, correspond to the corners of the parameter box as shown in figure 6 (the radius of each roller varies always between a R_{min} and a R_{max} values).
A parameter-dependent controller is sought with the same vertex property:

$$K(\theta) = \alpha_1 K(\Pi_1) + \alpha_2 K(\Pi_2) + \alpha_3 K(\Pi_3) + \alpha_4 K(\Pi_4)$$

For our application the synthesis of such a controller using LMI resolution (cf. [9]) is not feasible. Since, the controller can not be computed directly by the resolution of an LMI problem our approach consists in synthesizing several H_∞ controllers, then, analyzing the quadratic stability and the quadratic H_∞ performances of the resulting controller. The controllers $K(\Pi_i)$, for $i \in [1,..,4]$, are computed separately for 4 H_∞-like problems corresponding to the corners of the parameter box. The same weighting functions are used in the synthesis of each controller. The resulting controller, $K(\theta)$, is obtained with :

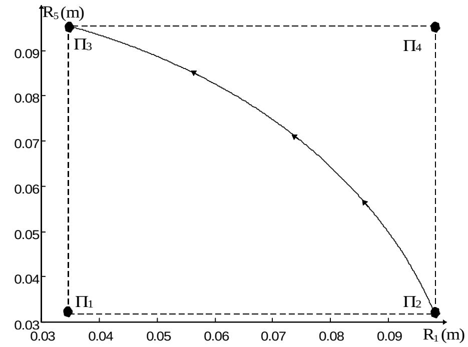

Figure 6: Parameter trajectory

$$\alpha_1 = (1-\rho_u)(1-\rho_w) \quad \alpha_2 = \rho_u(1-\rho_w)$$
$$\alpha_3 = (1-\rho_u)\rho_w \quad \alpha_4 = \rho_u\rho_w$$

where

$$\rho_u = \frac{R_u(t) - R_{min}}{R_{max} - R_{min}} \qquad \rho_w = \frac{R_w(t) - R_{min}}{R_{max} - R_{min}}$$

4.1 Study of the stability

The stability of the controller is satisfied on the corners of the parameter box, but need to be verified for all admissible parameter trajectories. A sufficient condition for the asymptotic stability is the existence of a positive-definite quadratic Lyapunov function $V(x_{cl})$ such that (with $P = P^T > 0$)

$$V(x_{cl}) = x_{cl}^T P x_{cl} \quad \text{and} \quad \frac{dV(x_{cl})}{dt} < 0$$

along all state trajectories. This is equivalent to the existence of a matrix $Q = P^{-1}$ (cf. [9]):

$$A_{cl}(t)Q + QA_{cl}(t)^T < 0 \qquad \forall t \qquad (4)$$

The LMI formulation of the stability for a polytopic system $A_{cl}(t) \in Co\{A_{cl1}, ..., A_{cln}\}$ is the following [10]: The considered polytopic system is quadratically stable if there exist a symmetric matrix Q and scalars $t_{ij} = t_{ji}$ such that for $i, j \in \{1, ..., n\}$

$$\begin{array}{rcl} A_{cli}Q + QA_{cli}^T + A_{clj}Q + QA_{clj}^T & < & 2t_{ij}I \\ Q & > & I \\ \begin{bmatrix} t_{11} & \cdots & t_{1n} \\ \vdots & \ddots & \vdots \\ t_{1n} & \cdots & t_{nn} \end{bmatrix} & < & 0 \end{array} \qquad (5)$$

The proof of the quadratic stability guarantees arbitrary fast time variations, which may be too conservative if the system order is too high. The order of the matrix A_{cl} of our application is 72. To avoid this, the polytopic closed loop system is computed from the polytopic open loop system $L_i = K_i G_i$ for $i \in \{1, ..., 4\}$. The resulting order of the closed loop system is now 27. Its quadratic stability is verified using the LMI toolbox of Matlab.

4.2 Study of the performance

The performance of a control is characterized by the L_2 norm of the operator connecting w to z. For a given γ, with $\gamma > 0$ the inequality:

$$\|z\|_{L_2} < \gamma \|w\|_{L_2} \tag{6}$$

is satisfied for all bounded w if there exists a positive-definite Lyapunov function $V(x_{cl}) = x_{cl}^T P x_{cl}$, $P = P^T$, such that (cf. [10]):

$$\frac{dV(x_{cl})}{dt} + z^T z - \gamma^2 w^T w < 0 \tag{7}$$

This is equivalent to the following family of matrix ("Bounded Real Lemma") inequalities with $Q = P^{-1}$:

$$\begin{pmatrix} A_{cl}Q + QA_{cl}^T & B_{cl} & QCz_{cl}^T \\ B_{cl}^T & -\gamma I & D_{11}^T \\ Cz_{cl}Q & D_{11} & -\gamma I \end{pmatrix} < 0 \tag{8}$$

The quadratic H_∞ performance is defined by the smallest γ for which a Lyapunov function exists.
A LMI formulation of the problem (8) is for a polytopic system can be found in [10]. The quadratic performance computed for the polytopic system defined by the vertices corresponding to the corners Π_i, for $i \in [1, .., 4]$, (see figure 6) are compared with the H_∞ norm of the closed loop system T_{cl} computed in the corners:

$$\| T_{cl}(\Pi_1) \|_\infty = 3,39 \qquad \| T_{cl}(\Pi_2) \|_\infty = 2,24$$
$$\| T_{cl}(\Pi_3) \|_\infty = 2,18 \qquad \| T_{cl}(\Pi_4) \|_\infty = 3,59$$

$$Qperf(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 5,39$$

This high value of the quadratique performance explain why the synthesis of an LPV controller controller by LMI resolution was not satisfaying.

4.3 Results on the experimental setup

The synthesized LPV controller is compared with the varying gain H_∞ controller on the experimental setup, see figure 7. Remind that the main concern is to control the web tension during all the process. We observe an improvement of the web tension with the LPV controller, which gives the same response of web tension for the rejection of periodical velocity variations. The maximum web tension variation ($\Delta T/T$) is reduced from 9 % (with varying gain H_∞ controller) to 5% (with LPV controller). Notice that this variation is about 30 % with constant gain H_∞ controller and 75 % with decentralized PID control [5].

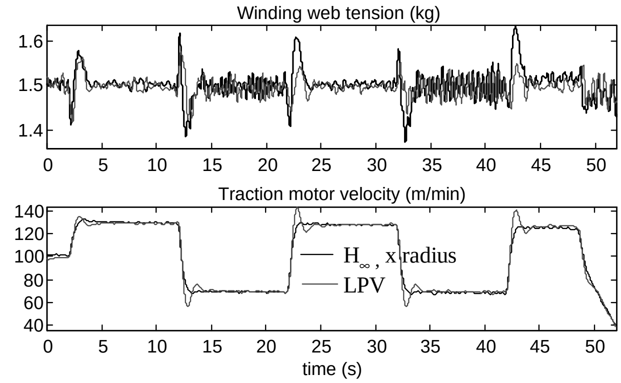

Figure 7: Web tension for periodical velocity variations

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