Passivity Properties of Neuro Identifier

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Abstract

In this paper the passivity approach is applied to access several stability properties of neuro identifier. A dynamic neural network is used for nonlinear system no-line identification. By using a simple gradient learning law, the conditions for passivity, stability, asymptotic stable and input-to-state stability are established. We get a very interesting result: gradient algorithm is robust with respect to all kinds of bounded uncertainties for neuro identifier.

1 INTRODUCTION

In many application, black-box identification using neural networks has emerged as a viable tool for unknown nonlinear systems. This model-free approach uses the nice features of neural networks, but the lack of model makes hard to obtain theoretical results on the stability and performance of neuro identifier. For the engineers it is very important to assure the stability in theory before they apply it to a real system.

There are not many results on stability analysis of neural networks in spite of their successful applications. The global asymptotic stable (GAS) of dynamic neural networks has been developed during the last decade. Diagonal stability [7] and negative semi-definiteness [8] of the interconnection matrix may make Hopfield-Tank neuro circuit GAS. Multilayer perceptron (MLP) and recurrent neural networks can be related to the Lur'e systems, the absolute stabilities were developed by [16] and [10]. Input-to-state stable (ISS) analysis method [15] is an effective tool for dynamic neural networks, [14] stated that if the weights are small enough, neural networks are ISS and GAS with zero input. Many publishes investigate the stability of identification error and tracking error of neural networks. [6] studied the stability conditions when multilayer perceptrons are used to identify and control a nonlinear system. Lyapunov-like analysis is suitable for dynamic neural network, the signal-layer case were discussed in [13] and [18], the high-order networks and multilayer networks may be found in [9] and [11].

Another stability analysis tool is passivity theory, it may deal with the poor define nonlinear systems, usually by means of sector bounds. But it offers elegant solutions for the proof of absolute stable. A promising approach to stability analysis of neuro systems may be the passivity framework, because it can lead to general conclusions on the stability using only input-output characteristics. The passivity properties of MLP were examined in [2]. By means of analyzing the interconnected of error models, they derived the relationship between passivity and closed-loop stable. To the best of our knowledge, open loop analysis based on the passivity method for dynamic neural networks has not yet been established in the literature.

In this paper, the passivity method is used to develop the stability properties of neuro identifier. It is shown that a gradient-like learning law will make the identification error stable, asymptotic stable and input-to-state stable. A simulation for vehicle idle speed identification shows the effective of gradient algorithm.

2 Preliminaries

Consider a class of nonlinear systems given by

$$\begin{array}{l} \dot{x}_t = f(x_t, u_t) \\ y_t = h(x_t, u_t) \end{array} \tag{1}$$

where $x_t \in \Re^n$ is the state, $u_t \in \Re^m$ is the input vector, $y_t \in \Re^m$ is the output vector. $f : \Re^n \times \Re^m \to \Re^n$ is locally Lipschitz, $h : \Re^n \times \Re^m \to \Re^m$ is continuous. It is also assumed that for any $x^0 = x_0 \in \Re^n$, the output $y_t = h(\Phi(t, x^0, u))$ of system (1) is such that $\int_0^t \mid u_s^T y_s \mid ds < \infty$, for all $t \geq 0$, *i.e.* the energy stored in system (1) is bounded. Following to [1] and [3], let us now recall some passivity properties as well as some stability properties of passive systems.

Definition 1 *A system (1) is said to be passive if there exists a* C^r *nonnegative function* $S(x_t) : \Re^n \to \Re$, *called storage function, such that, for all u_t, all initial conditions x^0 and all $t \geq 0$ the following inequality holds:*

$$\dot{S}\,(x_t) \leq u_t^T y_t - \varepsilon u_t^T u_t - \delta y_t^T y_t - \rho\psi\,(x_t)$$
$$(x_t, u_t) \in \Re^n \times \Re^m$$

where ε, δ and ρ are nonnegative constants, $\psi\,(x_t)$ is positive semidefinite function of x_t such that $\psi\,(0) = 0$. $\rho\psi\,(x_t)$ is called state dissipation rate. Furthermore, the system is said to be

- *lossless if $\varepsilon = \delta = \rho = 0$ and $\dot{S}\,(x_t) = u_t^T y_t$;*
- *input strictly passive if $\varepsilon > 0$*
- *output strictly passive if $\delta > 0$*
- *state strictly passive if $\rho > 0$*
- *strictly passive if there exists a positive definite function $V(x_t) : \Re^n \to \Re$ such that $\dot{S}\,(x_t) = u_t^T y_t - V(x_t)$*

Property 1. If the storage function $S(x_t)$ is differentiable and the dynamic system is passive, storage function $S(x_t)$ satisfies

$$\dot{S}\,(x_t) \leq u_t^T y_t$$

Definition 2 *A system (1) is said to be globally input-to-state stable if there exists a $\mathcal{K}$-function $\gamma(s)$ (continuous and strictly increasing $\gamma(0) = 0$) and $\mathcal{KL}$ -function $\beta\,(s,t)$ ($\mathcal{K}$-function and for each fixed $s_0 \geq 0$, $\lim_{t\to\infty} \beta\,(s_0, t) = 0$), such that, for each $u \in L_\infty$ $(\sup\{\|u(t)\|, t \geq 0\} < \infty)$ and each initial state $x^0 \in R^n$, it holds that*

$$\left\|x(t, x^0, u_t)\right\| \leq \beta\,(s,t)\left(\left\|x^0\right\|, t\right) + \gamma(s)\left\|u_t\right\|$$

for each $t \geq 0$.

Property 2. If a system is input-to-state stable, the behavior of the system should remain bounded when its inputs are bounded.

3 Neuro Identification via Passivity Technique

The nonlinear system to be identified is given as:

$$\dot{x}_t = f(x_t, u_t, t), \quad x_t \in \Re^n,\, u_t \in \Re^m \tag{2}$$

We construct the following dynamic neural network:

$$\dot{\widehat{x}}_t = A\widehat{x}_t + W_{1,t}\sigma(\widehat{x}_t) + W_{2,t}\phi(\widehat{x}_t)\gamma(u_t) \tag{3}$$

where $\widehat{x}_t \in \Re^n$ is the state of the neural network, $A \in \Re^{n\times n}$ is a stable matrix, $W_{1,t} \in \Re^{n\times n}$, $W_{2,t} \in \Re^{n\times n}$. The vector functions $\sigma(x_t) \in \Re^n$ is assumed to be $n-$dimensional with the elements increasing monotonically. The matrix function $\phi(\cdot)$ is assumed to be $\Re^{n\times m}$ diagonal: $\phi(\widehat{x}_t) = diag(\phi_1(\widehat{x}_1)\cdots\phi_n(\widehat{x}_n))$. $\gamma(u_t) \in \Re^m$, u_t is the control input of the plant (2). Function $\gamma(\cdot)$ is selected as $\|\gamma(u_t)\|^2 \leq \overline{u}$. The typical presentation of the elements $\sigma_i(\cdot)$ and $\phi_i(.)$ are as sigmoid functions, i.e.

$$\sigma_i(x_i) = \frac{a_i}{1 + e^{-b_i x_i}} - c_i. \tag{4}$$

Remark 1 *The neural networks have been discussed by many authors, for example [13], [9], [11] and [18]. One may see that Hopfield model [4] is the special case of this networks with $A = diag\{a_i\}$, $a_i := -1/R_iC_i$, $R_i > 0$ and $C_i > 0$. R_i and C_i are the resistance and capacitance at the ith node of the network respectively.*

Let us define identification error as

$$\Delta_t := \widehat{x}_t - x_t \tag{5}$$

Because $\sigma(\cdot)$ and $\phi(\cdot)$ are chosen as sigmoid functions, clearly they satisfy the following assumption.

A1: *The function $\sigma(\cdot)$ and $\phi(\cdot)$ fulfill generalized Lipshitz condition*

$$\widetilde{\sigma}^T\Lambda_1\widetilde{\sigma} \leq \Delta_t^T D_\sigma \Delta_t, \quad \left(\widetilde{\phi}_t u_t\right)^T \Lambda_2 \left(\widetilde{\phi}_t u_t\right) \leq \overline{u}\Delta_t^T D_\phi \Delta_t,$$

where

$$\widetilde{W}_{1,t} := W_{1,t} - W_1^*, \quad \widetilde{W}_{2,t} := W_{2,t} - W_2^*$$
$$\widetilde{\sigma} := \sigma(\widehat{x}_t) - \sigma(x_t), \quad \widetilde{\phi} := \phi(\widehat{x}_t) - \phi(x_t)$$

Λ_1, Λ_2, D_σ and D_ϕ are known positive constants, nonlinear.

Generally, dynamic neural network (3) cannot follow the nonlinear system (2) exactly, it may be written as

$$\dot{x}_t = Ax_t + W_1^*\sigma(x_t) + W_2^*\phi(x_t)\gamma(u_t) - \widetilde{f}_t \tag{6}$$

where W_1^* and W_2^* are bounded unknown matrices

$$W_1^*\Lambda_1^{-1}W_1^{*T} \leq \overline{W}_1, \quad W_2^*\Lambda_2^{-1}W_2^{*T} \leq \overline{W}_2 \tag{7}$$

$\overline{W}_1$ and $\overline{W}_2$ are priory known matrices, vector function $\widetilde{f}_t$ can be regarded as modelling error and disturbances. The error dynamic is obtained from (3) and (6)

$$\begin{aligned}\dot{\Delta}_t = A\Delta_t + \widetilde{W}_{1,t}\sigma(\widehat{x}_t) + \widetilde{W}_{2,t}\phi(\widehat{x}_t)\gamma(u_t) \\ + W_1^*\widetilde{\sigma} + W_1^*\widetilde{\phi}\gamma(u_t) + \widetilde{f}_t\end{aligned} \tag{8}$$

If we define

$$R := \overline{W}_1 + \overline{W}_2, \quad Q := D_\sigma + \overline{u}D_\phi + Q_0 \tag{9}$$

and the matrices A and Q_0 are selected to fulfill the following conditions:

(1) the pair $(A, R^{1/2})$ is controllable, the pair $(Q^{1/2}, A)$ is observable,

(2) local frequency condition [17] satisfies

$$A^T R^{-1} A - Q \geq \frac{1}{4}\left[A^T R^{-1} - R^{-1} A\right] R \left[A^T R^{-1} - R^{-1} A\right]^T \tag{10}$$

then the following assumption can be established:

A2: There exist a stable matrix A and a strictly positive defined matrix Q_0 such that the matrix Riccati equation

$$A^T P + PA + PRP + Q = 0 \tag{11}$$

has a positive solution $P = P^T > 0$.

This conditions is easily fulfilled if we select A as stable diagonal matrix. Next Theorem states the learning procedure of neuro identifier.

Theorem 1 *If the weights $W_{1,t}$ and $W_{2,t}$ are updated as*

$$\begin{array}{l} \dot{W}_{1,t} = -K_1 P \Delta_t \sigma^T(\widehat{x}_t) \\ \dot{W}_{2,t} = -K_2 P \phi(\widehat{x}_t)\gamma(u_t)\Delta_t^T \end{array} \tag{12}$$

where P is the solution of Riccati equation (11), then the dynamic of identification error (8) is strictly passive from $\widetilde{f}_t$ to the identification error $2P\Delta_t$

Proof: Select a Lyapunov function (storage function) as

$$S_t = \Delta_t^T P \Delta_t + tr\left\{\widetilde{W}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right\} + tr\left\{\widetilde{W}_{2,t}^T K_2^{-1} \widetilde{W}_{2,t}\right\} \tag{13}$$

where $P \in \Re^{n\times n}$ is positive define matrix. According to (8), the derivative is

$$\begin{array}{l} \dot{S}_t = \Delta_t^T \left(PA + A^T P\right)\Delta_t \\ +2\Delta_t^T P \widetilde{W}_{1,t}\sigma(\widehat{x}_t) + 2\Delta_t^T P \widetilde{W}_{2,t}\phi(\widehat{x}_t)\gamma(u_t) \\ +2\Delta_t^T P \widetilde{f}_t + 2\Delta_t^T P\left[W_1^* \widetilde{\sigma} + W_1^* \widetilde{\phi}\gamma(u_t)\right] \\ +2tr\left\{\dot{\widetilde{W}}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right\} + 2tr\left\{\dot{\widetilde{W}}_{2,t}^T K_2^{-1} \widetilde{W}_{2,t}\right\} \end{array}$$

Since $\Delta_t^T P W_1^* \widetilde{\sigma}_t$ is scalar, using **A1** and matrix inequality

$$X^T Y + \left(X^T Y\right)^T \leq X^T \Lambda^{-1} X + Y^T \Lambda Y \tag{14}$$

where $X, Y, \Lambda \in \Re^{n\times k}$ and for any positive defined matrix $\Lambda = \Lambda^T > 0$, we obtain

$$\begin{array}{r} 2\Delta_t^T P W_1^* \widetilde{\sigma}_t \leq \Delta_t^T P W_1^* \Lambda_1^{-1} W_1^{*T} P \Delta_t + \widetilde{\sigma}_t^T \Lambda_1 \widetilde{\sigma}_t \\ \leq \Delta_t^T \left(P\overline{W}_1 P + D_\sigma\right)\Delta_t \\ 2\Delta_t^T P W_2^* \widetilde{\phi}_t \gamma(u_t) \leq \Delta_t^T \left(P\overline{W}_2 P + \overline{u} D_\phi\right)\Delta_t \end{array} \tag{15}$$

So we obtain

$$\begin{array}{l} \dot{S}_t \leq \Delta_t^T \left[\begin{array}{c} PA + A^T P + P\left(\overline{W}_1 + \overline{W}_2\right)P \\ +\left(D_\sigma + \overline{u} D_\phi + Q_0\right) \end{array}\right]\Delta_t \\ +2tr\left\{\dot{\widetilde{W}}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right\} + 2\Delta_t^T P \widetilde{W}_{1,t}\sigma(\widehat{x}_t) + 2\Delta_t^T P \widetilde{f}_t \\ +2tr\left\{\dot{\widetilde{W}}_{2,t}^T K_2^{-1} \widetilde{W}_{2,t}\right\} + 2\Delta_t^T P \widetilde{W}_{2,t}\phi(\widehat{x}_t)\gamma(u_t) \\ -\Delta_t^T Q_0 \Delta_t \end{array}$$

Since $\dot{\widetilde{W}}_{1,t} = \dot{W}_{1,t}$, if we use the updating law as in (12) and **A2**, we have

$$\dot{S}_t \leq -\Delta_t^T Q_0 \Delta_t + 2\Delta_t^T P \widetilde{f}_t \tag{16}$$

From Definition 1, if we define the input as $\widetilde{f}_t$ and the output as $2P\Delta_t$, then the system is strictly passive with

$$0 \leq V_t \leq \Delta_t^T Q_0 \Delta_t$$

■

Remark 2 *Since the updating rate is $K_i P$ $(i = 1, 2)$, and K_i can be any positive matrix, the learning process of dynamic neural network (12) is free of P, the solution of Riccati equation (11).*

Corollary 1 *If only parameters uncertainty present ($\widetilde{f}_t = 0$), then the updating law as (12) can make the identification error asymptotic stable,*

$$\begin{array}{c} \lim_{t\to\infty} \Delta_t = 0 \quad \lim_{t\to\infty} \dot{W}_{1,t} = 0, \quad \lim_{t\to\infty} \dot{W}_{2,t} = 0 \\ W_{1,t} \in L_\infty, \quad W_{2,t} \in L_\infty, \end{array} \tag{17}$$

Proof: Since the dynamic of identification error (8) is passive, from the *Property 1* the storage function $S(x_t)$ satisfies

$$\dot{S}(x_t) \leq \widetilde{f}_t^T 2P\Delta_t = 0$$

The positive defined $S(x_t)$ implies Δ_t, $W_{1,t}$ and $W_{2,t}$ are bounded.

From the error equation (8) $\dot{\Delta}_t \in L_\infty$

$$\frac{d}{dt} V_t \leq -\Delta_t^T Q_0 \Delta_t \leq 0$$

Integrate (16) both sides

$$\int_0^\infty \|\Delta_t\|_{Q_0} \leq S_0 - S_\infty < \infty$$

So $\Delta_t \in L_2 \cap L_\infty$,, using Barlalat's Lemma we have (17). As u_t, $\sigma(\widehat{x}_t)$, $\phi(\widehat{x}_t)$ and P are bounded, $\lim_{t\to\infty} \dot{W}_{1,t} = 0$ and $\lim_{t\to\infty} \dot{W}_{1,t} = 0$. ■

Corollary 2 *If the modeling error $\widetilde{f}_t$ has a special relationship with identification error Δ_t*

$$\widetilde{f}_t = -K_f \Delta_t, \quad K_f = K_f^T > 0$$

where K_f is a positive defined constant matrix, then the identification error is asymptotic stable.

Proof: Because $\Delta_t = h(x_t, \widehat{x}_t)$ is independent of $\widetilde{f}_t$, the feedback loop with $\widetilde{f}_t = -K_f \Delta_t$ is well posed. The time derivative of S satisfies

$$\dot{S}(x_t) \leq -\Delta_t^T K_f \Delta_t \leq 0$$

So the equilibrium $\Delta_t = 0$ of $\dot{\Delta}_t = f(\Delta_t, \widetilde{f}_t)$ is stable. Base on the Invariance Principle, the bounded solutions of $\dot{\Delta}_t = f(\Delta_t, \widetilde{f}_t)$ converge to the largest invariant set of $\dot{\Delta}_t = f(\Delta_t, 0)$ contained in $E = \{\Delta_t \mid h(x_t, \widehat{x}_t) = 0\}$, this set is $\Delta_t = 0$, so the asymptotic stable is proved. ■

Remark 3 *For model matching case, Lyapunov-like analysis can reach the same result as Corollary 1 and Corollary 2 [18]. But in the case of modeling error, we will give a new conclusion on neuro identification: the gradient algorithm (12) is also robust respect to unmodeled dynamic, bounded disturbance and stochastic noise.*

Theorem 2 *Using the updating law as (12), the dynamic of neuro identifier (8) is input-to-state stable (ISS).*

Proof: In view of the matrix inequality (14),

$$2\Delta_t^T P \widetilde{f}_t \leq \Delta_t^T P \Lambda_f P \Delta_t + \widetilde{f}_t^T \Lambda_f^{-1} \widetilde{f}_t$$

(16) can be represented as

$$\begin{aligned} \dot{S}_t &= -\Delta_t^T Q_0 \Delta_t + 2\Delta_t^T P \widetilde{f}_t \\ &\leq -\lambda_{\min}(Q_0) \|\Delta_t\|^2 + \Delta_t^T P \Lambda_f P \Delta_t + \widetilde{f}_t^T \Lambda_f^{-1} \widetilde{f}_t \\ &\leq -\alpha_{\|\Delta_t\|} \|\Delta_t\| + \beta_{\|\widetilde{f}_t\|} \left\|\widetilde{f}_t\right\| \end{aligned}$$

where $\alpha_{\|\Delta_t\|} := [\lambda_{\min}(Q_0) - \lambda_{\max}(P \Lambda_f P)] \|\Delta_t\|$, $\beta_{\|\widetilde{f}_t\|} := \lambda_{\max}\left(\Lambda_f^{-1}\right) \left\|\widetilde{f}_t\right\|$. We can select a positive defined matrix Λ_f such that

$$\lambda_{\max}(P \Lambda_f P) \leq \lambda_{\min}(Q_0) \tag{18}$$

So α and β are $\mathcal{K}_\infty$ functions, S_t is an ISS-Lyapunov function. Using Theorem 1 of [15], the dynamic of identification error (8) is input to state stable. ■

Corollary 3 *If the modelling error $\widetilde{f}_t$ is bounded, then the updating law as (12) can make the identification procedure stable*

$$\Delta_t \in L_\infty, \quad W_{1,t} \in L_\infty, \quad W_{2,t} \in L_\infty$$

Proof: From Property 2 we know input-to-state stable means that the behavior of the dynamic neural networks should remain bounded when its input is bounded. ■

Remark 4 *Since the state and output variables are physically bounded, the modelling error $\widetilde{f}_t$ can be assumed to be bounded too (see, for example [6][11][13]). The condition (18) can be established if Λ_f is a small enough constant matrix. Unlike robust adaptive laws, such as dead-zone [11] and $\sigma-$modification [9], we do not need to know the upper bound of uncertainties.*

Remark 5 *It is well known that structure uncertainties will cause parameters drift for adaptive control, so one has to use robust modification to make identification stable [5]. Robust adaptive methods may be extended to neuro identification directly [6][11] [13]. But neuro identification is a kind of "black-box" method, nobody needs structure information and all of uncertainties are inside the black box. Although robust adaptive algorithms are suitable for neuro identification, they are not the simplest. By means of passivity technique, we success to prove our conclusion: pure gradient algorithm is robust with respect to all kinds of bounded uncertainties for neuro identification.*

4 Simulation

The engine operation at idle is a nonlinear process that is far from its optimal range. Because it does not require any large degree of instrumentation or external sensing capabilities, the idle speed control is also accessible and can be formatted as a benchmark problem for control society. The process of engine at idle has time delays that vary inversely with engine speed and is time-varying due to aging of components and environmental changes such as engine warm-up after a cold start. The measurement of system outputs occurs asynchronously with the calculation of control signals. We assume that the occurrence of plant disturbances, such as engagement of air conditioner compressor, shift from neutral to drive in automatic transmissions, application and release of electric loads, and power steering lock-up, are not directly measured. The dynamic engine model a two inputs and two outputs system [12]:

$$\begin{aligned}\dot{P} &= k_P\left(\dot{m}_{ai}-\dot{m}_{ao}\right)\\ \dot{N} &= k_N\left(T_i-T_L\right)\\ \dot{m}_{ai} &= \left(1+k_{m1}\theta+k_{m2}\theta^2\right)g\left(P\right)\\ \dot{m}_{ao} &= -k_{m3}N-k_{m4}P+k_{m5}NP+k_{m6}NP^2\end{aligned}$$

The engine model parameters are for a 1.6 liter, 4-cylinder fuel injected engine

$$g\left(P\right)=\begin{cases}1 & P<50.6625\\ 0.0197\sqrt{101.325P-P^2} & P\geq 50.6625\end{cases}$$
$$T_i=-39.22+325024m_{ao}-0.0112\delta^2+0.635\delta$$
$$+\tfrac{2\pi}{60}\left(0.0216+0.000675\delta\right)N-\left(\tfrac{2\pi}{60}\right)^2 0.000102N^2$$
$$T_L=\left(\tfrac{N}{263.17}\right)^2+T_d$$
$$m_{ao}=\dot{m}_{ao}\left(t-\tau\right)/\left(120N\right)$$

$k_P=42.40$, $k_N=54.26$, $k_{m1}=0.907$, $k_{m2}=0.0998$, $k_{m3}=0.0005968$, $k_{m4}=0.0005341$, $k_{m5}=0.000001757$, $\tau=45/N$. The system outputs are manifold press P (kPa) and engine speed N (rpm). The control inputs are thottle angle θ (degree) and the spark advance δ (degree). Disturbances act to the engine in the form of unmeasured accessory torque T_d (N-m). The variable $\dot{m}_{ai}$ and $\dot{m}_{ao}$ refer to the mass air flow into and out of the manifold. m_{ao} is the air mass in the cylinder. The parameter τ is a dynamic transport time delay. The function $g\left(P\right)$ is a manifold pressure influence function. T_i is the engine's internally developed torque, T_L is the load torque. If we define $x=\left(P,\ N\right)^T$, $u=\left(\theta,\ \delta\right)^T$, then the dynamic of vehicle idle speed are

$$\dot{x}_t=\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix}=\begin{pmatrix}f_1(x,u)\\ f_2(x,u)\end{pmatrix}. \tag{19}$$

f_1 and f_2 are assumed to be unknown and only x and u are measurable. In order to do the simulation, we select input as $\delta=30\sin\frac{t}{2}$, θ is sawtooth wave with amplitude 10, frequency $\frac{1}{2}$, T_d is square wave with amplitude 20, frequency $\frac{1}{4}$, $x_0=[10,500]^T$. Let us select dynamic neural network as

$$\dot{\widehat{x}}_t=A\widehat{x}_t+W_{1,t}\sigma(\widehat{x}_t)+W_{2,t}\phi(\widehat{x}_t)u_t,\quad W_{1,t},W_{2,t}\in R^{2\times 2}$$

The sigmoid functions are

$$\sigma\left(x_i\right)=\frac{2}{1+e^{-2x_i}}-0.5,\quad \phi\left(x_i\right)=\frac{0.2}{1+e^{-0.2x_i}}-0.05$$

We also select $A=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}$, $K_1P=K_2P=2I$, $\widehat{x}_0=[-5,-5]^T$. We use the learning law

$$\dot{W}_{1,t}=-K_1P\Delta_t\sigma(\widehat{x}_t)^T,\quad W_{1,0}=\begin{bmatrix}1 & 10\\ 10 & 1\end{bmatrix}$$
$$\dot{W}_{2,t}=-K_2P\phi(\widehat{x}_t)^Tu_t\Delta_t^T,\quad W_{2,0}=\begin{bmatrix}.1 & 0\\ 0 & .1\end{bmatrix}$$

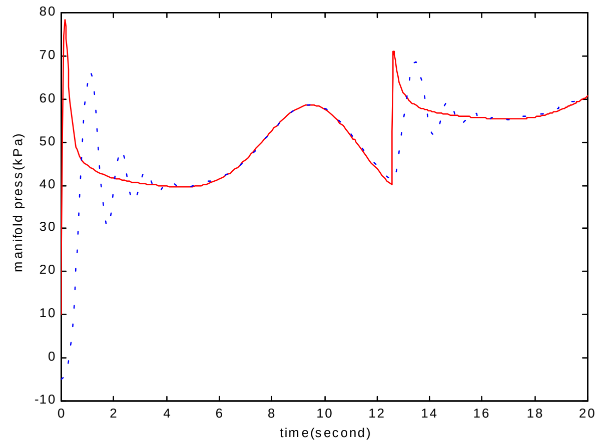

Figure 1

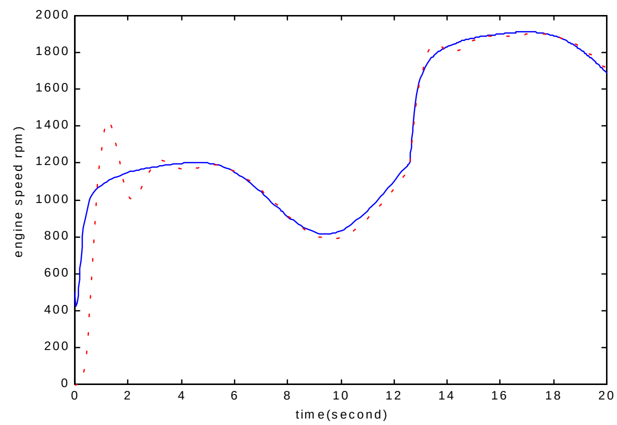

Figure 2

The following simulation results show the identification ability of dynamic neural network. Figure 1 and Figure 2 are with internal uncertainties (unmodeled dynamics and accessory torque). Figure 3 and Figure 4 show identification results with white noise (variances are 1 and 100).

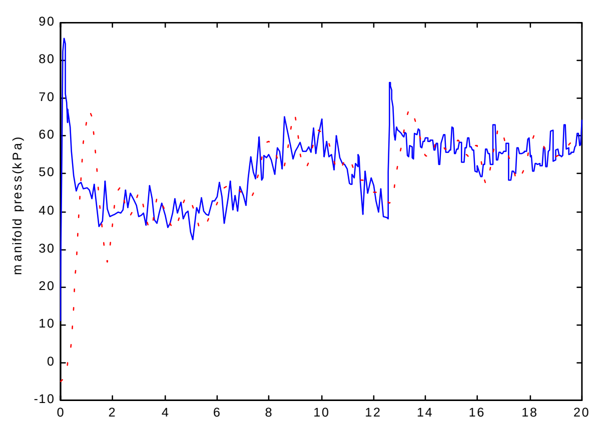

Figure 3

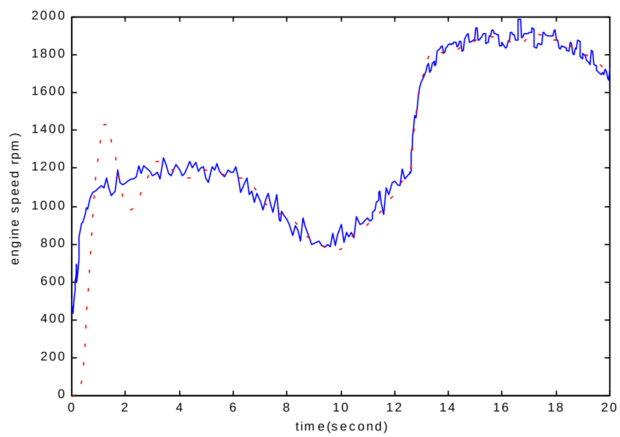

Figure 4

One can see that the gradient algorithm of dynamic neural networks is robust with respect to bounded uncertainties.

5 Conclusion

By means of passivity technique, we give new results on neuro identification. Compared with other robustness analysis of neuro identifications, we propose a new result: gradient learning algorithm may guarantee the identification error robust stable with respect to bounded uncertainties.

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