A STOCHASTIC CONFLICT DETECTION METHOD INTEGRATING PLANNED HEADING AND VELOCITY CHANGES

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Abstract

This paper addresses the issue of conflict detection in the domain of air traffic control. In a previous study investigating the position error in conflict estimation, a comprehensive model has been proposed and tuned with real data. Further steps are proposed in this paper consisting firstly of investigating wind effects on aircraft trajectory predictions; secondly by providing a conflict estimation approach that could be applied to aircraft trajectories with heading and velocity changes. Finally, a time-based probabilistic approach instead of position-based is proposed for predicting flight paths to be navigated by existing Flight Management Systems (FMSs).

1 Introduction

Considering air traffic growth, the major challenge facing Air Traffic Control is to enhance air traffic capacity while providing safety improvements. A possible option to address this challenge is to provide some advanced conflict detection capabilities, to be used on the ground to assist the controller, or in the air to support delegation of separation assurance to the pilot. This option raises many questions at different levels. It raises technical questions, typically for airborne applications, the way to transmit the appropriate surveillance information among aircraft. It also raises questions regarding human factor aspects, typically how this facility would impact on working methods, how to present the information to the controller or to the pilot? This paper addresses a theoretical issue: the definition of a reliable conflict detection method.

The field of conflict detection has been widely studied so far both from ground and airborne perspectives: different approaches [KY97b] and evaluation methodologies [Pai98], [Bil98] have been proposed. The probabilistic approach proposed in [PE96], [EPIE97] was validated by simulations on a real traffic, and it also allows the implementation of different assumptions and scenarios in a straightforward manner. For this reason, this approach was used as a basis for our study. A first step has been carried out consisting of an investigation into the position error for conflict estimation [BAB+00]. A comprehensive model based on a Brownian error in the velocity has been proposed and tuned with real data used in [PE96]. Next steps are proposed in this paper. First, an investigation into wind impacts on aircraft trajectory predictions is carried out. Next, an extension of an operational conflict detection method is presented. This operational method [EPIE97] gives an estimate of conflict probability for straight flights at constant velocities and relies on a position-based probabilistic approach. The extention presented here provides a conflict probability estimation for flights that involve heading and velocity changes. Finally, a time-based probabilistic approach is proposed for predicted flight paths to be navigated by existing Flight Management Systems (FMSs).

The paper is organised as follows: section 2 presents a brief state of the art and section 3 provides more insight into the background of the present work. Following sections present the proposed enhancements: section 4 for the wind effects, section 5 for the conflict probability estimation with trajectory heading and velocity changes, and section 6 for the time-based probabilistic approach.

2 State of the art

Trajectory prediction is inherently uncertain mainly due to errors in wind forecast, aircraft modelling, control and navigation. (This will not be consider here, but uncertainty may also result from umpredictable events such as re-planning due to a controller instruction.) Furthermore trajectory prediction becomes more uncertain with time. A reliable trajectory prediction thus relies on an appropriate modelling of each considered source of error. A classification based upon the level of error modelling is proposed. More precisely, three types of trajectory prediction are described here: geometric, worst-case, and probabilistic. The corresponding conflict definition is given and classical conflict detection methods are quoted. Except when it is mentioned, the different conflict detection methods assume that aircraft fly in straight line and at constant velocity.

2.1 Geometric

A first level of modelling is the geometric trajectory prediction based upon nominal trajectories without any uncertainty. A conflict occurs if the distance between the minimum predicted separation (also denoted as distance at Closest Point of Approach) is lower than the separation minimum [KP97].

2.2 Worst-case

A second level of modelling is the worst-case trajectory prediction in which the positions of aircraft are represented by areas of uncertainty that may grow with time. The uncertainty areas can be simple (e.g. rectangular boxes, spheres [FO97]) or more sophisticated (e.g. polygon [DA97] used to model instantaneous aircraft heading changes). A conflict occurs if the minimum separation between two areas of uncertainties is lower than the separation minimum.

2.3 Probabilistic

A third level of modelling is the probabilistic trajectory prediction in which all possible aircraft positions are weighted. The weighting can be represented by a density function, e.g. Gaussian [PE96]. There are various other uncertainty parameters (such as heading change or avoidance response latency) and associated density functions have been proposed [KY97a]. Random process is another way to model uncertainties [BB93], [BAB$^+$00], the trajectory prediction thus becoming stochastic. In this context of probabilistic prediction, the conflict definition evolves: a conflict occurs when the conflict probability is greater than a given threshold.

2.4 Rationale

The use of nominal trajectories without uncertainty to predict future aircraft positions is a time-limitation for the validity of the conflict detection. Indeed the reliability of the prediction gets lower with time, thus resulting rapidly in an unsafe or inefficient conflict detection. For the worst-case trajectory predictions, the limitation relies on a possible high rate of false alerting or a high rate of missed alerting. False and missed alerting rates depend on the selection of the uncertainty area bounds. Too large area may include low probability aircraft positions thus leading to false alerting whereas too restricted an area may exclude high probability aircraft positions thus leading to missed alerting. The selection of the uncertainty area bounds is a trade-off between expected efficiency and expected safety. Beyond the issue of identifying this trade-off, since all aircraft positions are considered as having the same probability, the worst-case modelling may represent a rough view of the reality. In the probabilistic approach, all uncertainties can be considered and weighted separately. Similarly to the worst-case prediction, a trade-off must be identified between efficiency and safety. This trade-off arises in the selection of the probability threshold to set off the conflict alarm. A reliable conflict detection thus relies on an appropriate model of error along with a judicious choice of the probability threshold.

3 Background

First a brief presentation of the Erzberger and Paielli conflict detection method is proposed. Second the main results of a previous work on the modelling of aircraft position error are presented.

3.1 A widely used probabilistic method

To estimate the probability of conflict in [PE96] [EPIE97], the position error is taken as normally distributed with a constant along-track root mean square (rms) rate that linearly grows with time and a constant cross-track error rms.

The two position errors have to be considered for a pair of aircraft, represented by two co-variance matrices. They are combined into a single equivalent co-variance which can be assigned to one of the aircraft. The other aircraft can thus be regarded as having no position uncertainty. A co-ordinate transformation is proposed that transforms the combined error co-variance into a standard form of a unit circle. Finally, an analytical solution for the conflict probability estimation can be found.

3.2 A stochastic conflict detection model revisited

In a previous study [BAB$^+$00], a dynamic dimension to the position error was introduced as a possible enhancement of the classical probabilistic model. It was suggested that an appropriate modelling of the along-track uncertainty is a trade-off between:

1. a position error normally distributed with a constant rate that linearly grows with time,
2. a position error resulting from a Brownian motion error in the position,
3. a position error resulting from a Brownian motion error in the velocity.

A combination of these three models is a possible solution. This combination provides three degrees of freedom to allow for a more accurate error modelling and beyond a more reliable conflict detection.

4 Wind effects

This section proposes a model of wind effects on trajectory prediction. The wind is decomposed into a mean vector $\vec{w}_{mean}$ plus a noise component $\Delta\vec{w}_{noise}$.

$$\vec{wind} = \vec{w}_{mean} + \Delta\vec{w}_{noise} \tag{1}$$

Although anticipation and compensation of wind noise is inherently not possible, the effects of this noise can be estimated. This will be investigated in this section.

4.1 Assumptions and notation

It is assumed that the aircraft fly with a FMS, that is capable (1) to generate a flyable trajectory from a set of waypoints, altitude or speed constraints, and (2) to guide the aircraft along this trajectory through the autopilot. However existing 3D FMSs control the cross-track position and not the along-track position [1]. Thus while the cross-track error can be reduced and bounded, a significant along-track error may exist specifically in the presence of wind. Introducing some notation:

$\vec{e}^{\,j}$: normalised direction vector of aircraft j.

$\vec{e}^{\,j} = \begin{pmatrix} x^j \\ y^j \end{pmatrix}$

$\Delta\vec{w}_{noise} = \begin{pmatrix} \Delta\vec{w}_1 \\ \Delta\vec{w}_2 \end{pmatrix}$

$\Delta\vec{p}^{\,j}_{wind}$: position difference resulting from wind on aircraft j.

$\Delta\vec{p}^{\,j}_{error}$: predicted position error of aircraft j.

ξ^j: coefficient representing aircraft j characteristics such as mass, size.

Q^j: individual error position co-variance matrix for aircraft j.

Q^{12}: cross-correlation term.

Q^{12}_{wind}: wind cross-correlation term.

M: combined error co-variance.

4.2 Effects on trajectory predictions

The error of position resulting from wind can be written as follows:

$$\Delta\vec{p}^{\,j}_{wind} = \xi^j \Delta\vec{w}_{noise}.\vec{e}^{\,j}\vec{e}^{\,j} \tag{2}$$

Following the widely used method previously presented, the combined error co-variance M can be computed.

$$M = cov(\Delta\vec{p}^{\,1}_{error} - \Delta\vec{p}^{\,2}_{error}) = Q^1 + Q^2 - Q^{12} \tag{3}$$

For simplicity, only the cross-correlation term resulting from wind effects is considered here. Therefore the cross-correlation term represents the wind effects correlation.

$$Q^{12} = Q^{12}_{wind} = E(\Delta\vec{p}^{\,1}_{wind}\Delta\vec{p}^{\,2T}_{wind} + \Delta\vec{p}^{\,2}_{wind}\Delta\vec{p}^{\,1T}_{wind}) \tag{4}$$

At a fixed small time interval of dt it gives:

$$Q^{12} = \xi^1\xi^2(\vec{e}^{\,1}\vec{e}^{\,2T} + \vec{e}^{\,2}\vec{e}^{\,1T})(x^1x^2(\Delta w_1)^2 + (x^1y^2 + x^2y^1)\Delta w_1\Delta w_2 + y^1y^2(\Delta w_2)^2) \tag{5}$$

By integration, it produces:

$$Q^{12} = \xi^1\xi^2(\vec{e}^{\,1}\vec{e}^{\,2T} + \vec{e}^{\,2}\vec{e}^{\,1T})(x^1x^2var^2(\Delta w_1) + (x^1y^2 + x^2y^1)cov(\Delta w_1, \Delta w_2) + y^1y^2var^2(\Delta w_2)) \tag{6}$$

Therefore the cross-correlation between the two position errors resulting from wind can be decomposed in three parts, where two are independent from each other. This result is independent of the noise model chosen and can be directly adapt for a 3D study.

5 Changes of heading and velocity at given times

In this section a direct extension of the Erzberger and Paielli conflict detection method is proposed for aircraft trajectories involving changes of heading and velocity. Firstly the context and the main assumptions are presented. Secondly some notation is introduced. Finally the conflict detection method is described and discussed.

5.1 Context and assumptions

The Erzberger and Paielli method can be applied to 2D and 3D aircraft trajectories. The 3D extension however leads to a more complex formulation and for the sake of simplicity, only 2D trajectories will be considered here. It is assumed that the two aircraft fly in the same horizontal plane, and their trajectories are predicted for 20 or 30 minutes ahead. A route point where an aircraft will change one of its flight parameters (e.g. heading, speed) is called a Trajectory Change Points (TCP). For each aircraft new trajectory points are calculated to have the same time division. In other words, the predicted states of an aircraft when the other fly over a TCP are computed. It is assumed that for all aircraft, the errors resulting from winds can be taken into account by dividing the airspace into independent areas of weather uncertainty. In each area, the mean wind direction is known and the variance of position errors resulting from winds is given. The trajectory prediction error in the along track without wind, is modelled as resulting from a Brownian error in the velocity (a mixed model could be used as shown in [BAB+00]). Concerning heading and velocity, it has been decided to assume instantaneous changes as in [DA97]. (It should be noted that apparently no more sophisticated modelling

[1] The future 4D FMS will handle this point through a time-based control of position.

for conflict estimation have been proposed so far.) The aircraft positions can be represented by ellipses with principal axes in the along-track and cross-track directions. The wind noise on the position is modelled as white noise on the acceleration.

5.2 Notation

p_i^j: position of aircraft j at i-th TCP be in the Earth reference co-ordinate system.
Δp_i: relative position at time t_i.
$\vec{v}_i^j$: velocity.
Δv_i: relative velocity during the period $[t_i; t_{i+1}[$.
t_i estimated arrival time at TCP_i.
For the period $[t_i; t_{i+1}[$:
σ_{ip}^h: cross track error rms. of aircraft j.
σ_{hvi}^j: along track rms. resulting from a Brownian motion error W_{hvi}^j in the velocity.
σ_{hwi}^j: along track error rms. resulting from a Brownian motion error W_{hwi}^j in the velocity, due to the wind.
α_i^j: changing of heading for aircraft j at TCP_i, with the first heading estimated in a fixed Earth co-ordinate system.
$Q^j(t)$: individual co-variance matrix for aircraft j, in a Earth-fixed reference system.
R_i^j: corresponding rotation matrix that transforms the heading-aligned co-ordinates to the Earth-fixed reference system in a period $[t_i; t_{i+1}[$.

5.3 Combined co-variance

For each period between two successive TCPs, the probability of conflict is estimated and the method of Erzberger and Paielli is adapted. As described in the background section, the two errors can be combined and assigned to one of the aircraft.
If $t \in [t_i; t_{i+1}[$ then:

$$Q^j(t) = (\sigma_{hp}^j)^2 \sum_{k=0}^{i} R_k^j \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} R_k^{jT} + \sum_{k=0}^{i-1} (t_{k+1} - t_k)^3 A_k^j + (t - t_i)^3 A_i^j \quad (7)$$

where:

$$A_k^j = R_k^j((\sigma_{hwk}^j)^2 + (\sigma_{hvk}^j)^2) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} R_k^{jT} \quad (8)$$

$$R_i^j = \begin{bmatrix} \cos\alpha_i^j & -\sin\alpha_i^j \\ \sin\alpha_i^j & \cos\alpha_i^j \end{bmatrix} \quad (9)$$

Therefore the combined co-variance matrix is:

$$M(t) \equiv Q^1(t) + Q^2(t) - Q^{12}(t) \quad (10)$$

Where the cross-correlation term Q^{12} represents the correlated effects of wind. Other types of correlation may exist (e.g. due to instrument error), but are ignored here to focus on wind effects. Moreover, if the two aircraft are far from each other, Q^{12} can be considered as negligible. Otherwise, it is necessary to calculate it with the method described in the wind effects section. That leads to the following total encounter matrix correlation:

$$Q^{12}(t) = \sum_{k=0}^{i-1} \pm((\vec{e}_k^{\,2} \vec{e}_k^{\,1T} + \vec{e}_k^{\,1} \vec{e}_k^{\,2T}) \sigma_{hwk}^1 \sigma_{hwk}^2 \frac{(t_{k+1} - t_k)^3}{3}) \pm((\vec{e}_i^{\,2} \vec{e}_i^{\,1T} + \vec{e}_i^{\,1} \vec{e}_i^{\,2T}) \sigma_{hwi}^1 \sigma_{hwi}^2 \frac{(t - t_i)^3}{3}) \quad (11)$$

5.4 Conflict probability estimation

The Closest Point of Approach (CPA) could be defined as follows: it is the relative position of the two aircraft when the range between them is a minimum. For constant velocities on the path segments and for a relative velocity different from zero, the time at which the minimum separation occurs during the period $[t_i; t_{i+1}[$ is:

$$t_{CPA} = \begin{cases} t_{i+1} & \text{if} \quad t_C \geq t_{i+1} - t_i \\ t_i + t_C & \text{if} \quad t_{i+1} - t_i \geq t_C \geq 0 \\ t_i & \text{if} \quad t_C \leq 0 \end{cases} \quad (12)$$

Where: $t_C = -\frac{\Delta p_i^T \Delta v_i}{\Delta v_i^T \Delta v_i}$.
For each period $[t_i; t_{i+1}[$, the elliptical conflict zone can be projected along a line parallel to the relative velocity. Then, it can be bounded with the maximum and minimum values of two points denoted x_C and y_C to obtain a rectangle R, where x_C is the abscissa and y_C is the ordinate of a point located on the ellipsoid conflict zone at a fixed time.
The conflict probability on the segment is:

$$P_C(t \in [t_i; t_{i+1}[) = \int\int_R p(x,y) dx dy = \int_Y p(y) dy \int_X p(x) dx \quad (13)$$

with $R = X \times Y$.
Y can be defined as explained in [PE96] by: $Y = \left[\min_{t \in [t_i; t_{i+1}[} y_C(t); \max_{t \in [t_i; t_{i+1}[} y_C(t)\right]$.
The search of X is different from [PE96]. Indeed, here X is equal to:

$$X = \left[\min_{t \in [t_i; t_{i+1}[} x_C(t); \max_{t \in [t_i; t_{i+1}[} x_C(t)\right] \quad (14)$$

Therefore the total conflict probability can be bounded as follows:

$$\max_i P_C(t \in [t_i; t_{i+1}[) \leq P_C \leq \sum_i P_C(t \in [t_i; t_{i+1}[) \quad (15)$$

Similarly the worst case conflict probability on a trajectory is given by:

$$P_{worst} = \max_i P_C(t \in [t_i; t_{i+1}[) \quad (16)$$

The main advantage of this method is to integrate the changes of velocity or heading. However, that implies a strong assumption on the way aircraft are guided along their trajectories. It is assumed that the aircraft change their flight parameters at given times instead of given points. Nevertheless, this method can be easily adapted for 3D trajectories.

6 Arrival time at a TCP as a random variable

Conflict detection methods usually consider the actual position at TCP as non determinist (i.e. probabilist) and the arrival time at TCP as determinist. However as discussed in section 4.1, existing FMSs control the cross-track position and not the along-track position. To ensure maximum simplicity of the formulation and reliability of the conflict detection, it is assumed that the prediction model should follow the same principles as the guidance model. For that purpose, the actual position at TCP will be considered as determinist while the arrival time will be considered as a random variable.

6.1 Notation and assumptions

χ^j: trajectory of aircraft j, it is a continuous path with affine geometry lines.
C: potential conflict zone.
$C = S^1 \times S^2$: where S^1 and S^2, trajectory segments of aircraft 1 and 2.
d_{min}: separation minimum.
$X^j(\tau) \in \chi^j$: position of aircraft j at time τ.
$\underline{x}^j$: entry point in the segment S^j for aircraft j.
$\overline{x}^j$: exit point of the segment S^j for aircraft j.
$\underline{\tau}^j$: first entry time in the segment S^j for aircraft j.
$\underline{\tau}^j = \inf\{\tau, X^j(\tau) = \underline{x}^j\}$.
$\overline{\tau}^j$: first exit time of the segment S^j for aircraft j.
$\overline{\tau}^j = \inf\{\tau, X^j(\tau) = \overline{x}^j\}$.
$\vec{v}^j$: nominal velocity of aircraft j.
W^j: Brownian motion error in the velocity for aircraft j, independent from $\underline{\tau}^j$.
σ^j: associated variance.
$u = \underline{\tau}^2 - \underline{\tau}^1$. Two assumptions are needed to model the conflict detection problem.

1. The aircraft does not "return during the flight". That is to say, $X^j(\tau)$ enters in S^j by $\underline{x}^j$, exits by $\overline{x}^j$, and then never returns in the segment S^j.

2. C does not meet the bound of the segments $S^1 \times S^2$, in other words, if $(X^1(\tau), X^2(\tau)) \in C$ then the aircraft 1 and 2 are not yet exiting the segment, but they are already entering.

6.2 Conflict definition

The potential conflict zone C is constructed geometrically. The distance between the two aircraft paths is compared to the separation minimum. When the distance between two points is smaller than this separation minimum, the points are selected and added to the potential conflict zone. A conflict occurs when the two aircraft are in the potential conflict zone at the same time, that is to say when the conditions C_1 or C_2 hold true. Where:

$$C_1 = \{\tau^1 \leq \tau^2 \text{ and } \exists \tau \geq 0 / (X^1(\underline{\tau}^2 + \tau), X^2(\underline{\tau}^2 + \tau)) \in C\} \tag{17}$$

under the assumption 1 that gives: $\tau \leq \min(\overline{\tau}^1, \overline{\tau}^2) - \underline{\tau}^2$

$$C_2 = \{\tau^2 \prec \tau^1 \text{ and } \exists \tau \geq 0 / (X^1(\underline{\tau}^1 + \tau), X^2(\underline{\tau}^1 + \tau)) \in C\} \tag{18}$$

under the assumption 1 that gives: $\tau \leq \min(\overline{\tau}^1, \overline{\tau}^2) - \underline{\tau}^1$
The conflict probability is equal to the probability that condition C_1 or C_2 holds. Due to the symmetry of the problem only one case need be studied here.

6.3 Conflict probability estimation

Once the segments $S^1 \times S^2$ are reached by the aircraft, the past is forgotten. Let the new position of aircraft j be $Y^j(T)$.

$$Y^j(T) = \underline{x}^j + \frac{\vec{v}^j}{\|\vec{v}^j\|} \left[\|\vec{v}^j\| T + \sigma^j \int_0^T W^j(s) ds \right] \tag{19}$$

Let $\overline{T}^j$ be the first entry time of Y^j at $\overline{x}^j$. Therefore, the rule of $(X^1(\underline{\tau}^2 + \tau), X^2(\underline{\tau}^2 + \tau))$ under the condition $\{\tau^1 \leq \tau^2\}$ is equal to the law of $\overline{T}^j$. Thus the probability of conflict when $\{\tau^1 \leq \tau^2\}$, is equal to:

$$P(C_1) = P((\underline{\tau}^2 - \underline{\tau}^1) \geq 0 \text{ and } \exists \tau \leq \min(\overline{\tau}^1, \overline{\tau}^2) - \underline{\tau}^2, \|X^1(\tau + \underline{\tau}^2 - \underline{\tau}^1) - X^2(\tau)\| \leq d_{min}) \tag{20}$$

$$P(C_1) = \int_0^{+\infty} dP_{\underline{\tau}^2 - \underline{\tau}^1} \quad P(\exists T \succ 0, T \leq \min(\overline{T}^2, \overline{T}^1 - u), \|Y^1(T + u) - Y^2(T)\| \leq d) \tag{21}$$

Let T_u be the random time at which the CPA is reached. It is a linear function of u.

$$T_u = \sup\{-\frac{\Delta \underline{x}^T \Delta \vec{v}}{\Delta \vec{v}^T \Delta \vec{v}} - \frac{\Delta \vec{v}^{1T} \Delta \vec{v}}{\Delta \vec{v}^T \Delta \vec{v}} u, 0\} \tag{22}$$

That gives the following relation:

$$\begin{aligned} Y^1(T+u) - Y^2(T) &= \Delta \underline{x}_u + \Delta \vec{v}(T - T_u) \\ &+ \sigma^1 \frac{\vec{v}^1}{\|\vec{v}^1\|} \int_0^{T_u + u} W^1(s) ds + \sigma^1 \frac{\vec{v}^1}{\|\vec{v}^1\|} \int_{T_u + u}^{T+u} W^1(s) ds \\ &- \sigma^2 \frac{\vec{v}^2}{\|\vec{v}^2\|} \int_0^{T_u} W^2(s) ds - \sigma^2 \frac{\vec{v}^2}{\|\vec{v}^2\|} \int_{T_u}^{T} W^2(s) ds \end{aligned} \tag{23}$$

Therefore,

$$Y^1(T+u) - Y^2(T) = Y^1(T_u + u) - Y^2(T_u) + \Delta \vec{v}(T - T_u) + R(T) \tag{24}$$

As in [PE96], the period of encounter is assumed to be short. As a consequence, $R(T)$ is negligible for $T \approx T_u$ and:

$$E(R(T)) = 0 \tag{25}$$

$$var(R(T)) = ((\sigma^1)^2 + (\sigma^2)^2)\frac{(T - T_u)^3}{3} \tag{26}$$

Under the two following assumptions:

1. $\tau^1 \prec \tau^2$
2. $T \approx T_u$

the following approximation can be made:

$$\begin{aligned} P(\exists T \succ 0, T \leq \min(\overline{T}^2, \overline{T}^1 - u), \|Y^1(T+u) - Y^2(T)\| \leq d) \\ \approx P(\exists T near T_u, \|Y^1(T+u) - Y^2(T)\| \leq d) \\ = P(\exists T near 0, \|Y^1(T_u+u) - Y^2(T_u)\| \leq d) \end{aligned} \tag{27}$$

where:

$$Y^1(T+u) - Y^2(T) = \Delta\underline{x}_u + \Delta\vec{v}T_u$$
$$+\sigma^1\frac{\vec{v}^1}{\|\vec{v}^1\|}\int_0^{T_u+u} W^1(s)ds - \sigma^2\frac{\vec{v}^2}{\|\vec{v}^2\|}\int_0^{T_u} W^2(s)ds \tag{28}$$

with: $\Delta\underline{x}_u + \Delta\vec{v}T_u = \Delta\overline{x}(T_u)$.
Therefore the method for combining co-variance matrixes presented in the previous section can be applied to calculate the probability of the event C_1, through the combined co-variance matrix of position error equal to:

$$cov[Y^1(T_u+u) - Y^2(T_u)] = (\sigma^1)^2\frac{(T_u+u)^3}{3} + (\sigma^2)^2\frac{T_u^3}{3} \tag{29}$$

$$\int_0^{+\infty} dP_{\underline{\tau}^2 - \underline{\tau}^1} \tag{30}$$

Nevertheless, even if approximations are possible, the exact computation of the previous integral is still missing. Future work will involve working out an explicit expression for the first arrival times $\underline{\tau}^1$ and $\underline{\tau}^2$.

7 Conclusion

This paper has proposed three enhancements of a model previously developed for conflict estimation. Firstly, an investigation of wind correlated effects on conflict estimation was performed thus providing more insight into the understanding on how wind may "significantly" affect trajectory predictions. The proposed model can be applied to any type of wind. Secondly, an extension of the method to integrate heading and velocity changes was presented. Finally, while the existing approaches rely on a position-based probabilistic principle, a time-based probabilistic principle was proposed that follows the guidance model of existing FMSs. In addition to provide capabilities for conflict estimation, the proposed method can also be used for monitoring potential or solved conflicts typically by considering the influence of variations of arrival time over waypoints. Future work will involve the investigation into an analytical solution of the conflict probability estimation. An evaluation of the proposed method will then be carried out to estimate its performances compared to the existing methods.

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