A strategy for controlling autonomous robot platoons[1]

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Abstract

Development of decentralized control systems for platoons of robotic vehicles often requires direct communication between vehicles. Using basic tools from the field of decentralized control, we develop a framework from which existence of vehicle controllers can be assessed for a given communication structure. This approach to control design allows us to investigate minimum inter-vehicle communication solutions. Simulation results are presented for a platoon of autonomous underwater vehicles.

1 Introduction

Many researchers have conjectured that platoons of cooperating mobile robots or autonomous vehicles provide significant benefits over single-unit approaches for a variety of tasks. Further, cooperating robots or vehicles need not necessarily be sophisticated or expensive to out-perform many advanced independent units for tasks such as material transport, scouting, etc. Unfortunately, large-scale control systems for platoons of cooperating mobile robots or autonomous vehicles are difficult to design for real-world situations. Communication requirements, especially with regard to bandwidth limits, are often challenging obstacles. In this work, we are interested in the development of a design methodology and analysis technique for controlling platoons of autonomous vehicles with a focus on understanding communication requirements for such systems. We present a decentralized control framework applicable to platoons of mobile robots or vehicles and, for illustration, consider a simplified design example for a platoon of autonomous underwater vehicles (AUVs). An underwater example is chosen to highlight the need for control strategies that address reduced communications since communication bandwidth is severely limited underwater.

There are two primary schools of thought on methods for controlling platoons of cooperating mobile robots and vehicles: the system-theoretic and behavior-based approaches. Behavior-based methods (sometimes referred to as reactive control) rely on the use of algorithmic behavior structures without an explicit mathematical model of the subsystems or the environment (see, e.g., [1], [2]). The system-theoretic approach, on the other hand, relies strongly on the use of system dynamics and models of the interactions between the vehicles themselves, as well as between the vehicles and the environment (e.g., [3], [4]).

These two approaches to cooperation have fundamentally differing benefits; neither presents a universal solution to the problem of designing cooperating platoons of autonomous vehicles. The benefits of the system-theoretic approach are that the results are provable and predictable and there are analytic solutions to questions regarding performance. The drawback to system-theoretic techniques is that they are encumbered by the need to approximate complex dynamics, models for which are never fully accurate. A benefit of behavior-based approaches is that they are motivated by biological systems that have shown great survivability and adaptability and can exhibit significant, if hard to quantify, performance and robustness. Behavior-based approaches do not, however, readily admit a closed-form design process and are sometimes as likely to exhibit unexpected and undesirable behavior as they are to perform as desired. Herein, we will focus on systems-theory-based approaches, as we wish to develop rigorous design methodologies while addressing issues of limited communications bandwidth.

Control-theoretic methods for cooperating platoons of robots in use today rely on decentralized control almost exclusively, although not in the formal framework developed in [5] (see, e.g., [3], [4]). These systems typically use local measurements and implicit communications as feedback for local controllers. Herein we propose that the formal framework of decentralized control may offer a more rigorous design procedure for these types of systems and, in addition, offer a strategy for determining what types of explicit communication are required.

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A strong component of our results is that the design technique and analysis scale to arbitrarily large platoons of cooperating vehicles. Indeed, the communication that is required among vehicles in the platoon is independent of the number of vehicles in the platoon.

Control objectives for cooperating platoons of robots have typically consisted of generating specific formations [6] or global behaviors [3] based on local relationships (e.g., the exact location of the nearest neighbor). These types of objectives may require high bandwidth communications under the informal decentralized control schemes used. We choose to control global functions of a platoon, such as the center (average position) of the platoon and the distribution of vehicles about the center. Vehicles are not commanded to be in specific positions. Instead, the vehicles autonomously move to locations that satisfy the center and distribution commands under a decentralized control law with surprisingly little inter-vehicle communication.

2 Background and Problem Statement

We consider a platoon composed of r heterogeneous vehicle subsystems, each described by the dynamics

$$\dot{x_i}(t) = f_i(x_i(t), u_i(t)) \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, and $i = 1, \ldots, r$. The dynamics of the platoon are completely uncoupled and all interaction between subsystems must be in the form of either implicit or explicit communication. Each subsystem has a local controller that generates the local control signal $u_i(t)$ based on measured signals produced by the subsystem or on signals communicated to the subsystem from elsewhere. This is a *decentralized* control structure, and we borrow heavily from the decentralized control literature in the analysis and design of the platoon controller. Early work on the existence of decentralized controllers, as in [5], [7], and [8], develops essential tools that we use in determining what communication is required between subsystems for a decentralized controller to exist. Since our primary interest is in examining communications structures, we take a local viewpoint and design linear decentralized controllers. Through simulation of a nonlinear platoon model, we find that the linear controllers perform well.

2.1 Regulation of Platoon-Level Functions

Our objective is to regulate *platoon-level* functions, such as the average position of the vehicles in a platoon or the distribution of the vehicles about the average position. The platoon-level function is denoted $h_c(x_1, \ldots, x_r) \in \mathbb{R}^{p_d}$, a function of the entire platoon state.

We adopt the working assumption that only a single vehicle has the capability to measure the platoon-level function or, equivalently, that the platoon-level function is measured by an exogenous system and transmitted to one vehicle in the platoon. Certainly there are practical considerations involved with this assumption, including issues related to single-point failure. Yet this assumption allows for the use of active sensors for the measurement since no crosstalk is present to degrade performance. For platoon-level measurements such as average vehicle position and distribution, fine-grained measurements (i.e., exact position of each vehicle) are unnecessary–only measurements that indicate vehicle position density are required.

Without loss of generality, the platoon-level functions are assumed to be measured by subsystem 1. In other words, we assume that $h_c(x_1, \ldots, x_r)$ is an output of subsystem 1. To ensure zero steady-state tracking error, integrators are connected in series to the output representing the global functions, yielding a new state variable

$$\dot{q}(t) = h_c(x_1(t), \ldots, x_r(t))$$

where $q(t) \in \mathbb{R}^{p_d}$ is the integrator state.

2.2 Platoon

Again, the platoon is the parallel connection of the r subsystems. Since our control system is designed from a local decentralized viewpoint, we linearize the platoon dynamics (1) at an equilibrium value of the subsystem states and inputs and write, somewhat loosely,

$$\dot{\bar{x}}(t) = \begin{bmatrix} 0 & H_c \\ 0 & F \end{bmatrix} \bar{x}(t) + \begin{bmatrix} \hat{G}_1 & \cdots & \hat{G}_r \end{bmatrix} \bar{u}(t) \tag{2}$$

where

$$F = \begin{bmatrix} F_1 & 0 & \cdots & 0 \\ 0 & F_2 & & \\ \vdots & & \ddots & \\ 0 & & & F_r \end{bmatrix}, \quad \hat{G}_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ G_i \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$H_c = \begin{bmatrix} H_{c1} & \cdots & H_{cr} \end{bmatrix}, \quad F_i = \frac{\partial}{\partial x_i} f_i(x_i^o, u_i^o)$$

$$G_i = \frac{\partial}{\partial u_i} f_i(x_i^o, u_i^o), \quad H_{ci} = \frac{\partial}{\partial x_i} h_c(x_1^o, \ldots, x_r^o)$$

where $F \in \mathbb{R}^{n \times n}$, $\hat{G}_i \in \mathbb{R}^{(n+p_d) \times m_i}$, $F_i \in \mathbb{R}^{n_i \times n_i}$, $G_i \in \mathbb{R}^{n_i \times m_i}$, $H_{ci} \in \mathbb{R}^{p_d \times n_i}$. The partial derivatives are evaluated at equilibrium values of the state and input, x_i^o and u_i^o, and the deviation variables are defined

$$\bar{x}(t) = \begin{bmatrix} q(t) - q^o \\ x_1(t) - x_1^o \\ \vdots \\ x_r(t) - x_r^o \end{bmatrix} \quad \bar{u}(t) = \begin{bmatrix} u_1(t) - u_1^o \\ \vdots \\ u_t(t) - u_r^o \end{bmatrix}$$

where $\bar{x}(t) \in \mathbb{R}^{p_d+n}$ and $\bar{u}(t) \in \mathbb{R}^m$. The matrix $\hat{G}_i$ is partitioned into $r+1$ blocks, with dimensions $p_d \times m_i$, $n_1 \times m_i$, $\ldots$, $n_n \times m_i$. The first $p_d \times m_i$ submatrix is always zero. The remaining partitioned blocks are all zeros except for entries corresponding to G_i (where G_1 is the second block, etc.). We also define $n = n_1 + \cdots + n_r$ and $m = m_1 + \cdots + m_r$.

2.3 Output Functions

In the decentralized control framework, a separate controller is designed for each subsystem. The signal measured by the controller for subsystem i is denoted $y_i(t)$ and partitioned to distinguish those components that are generated locally by subsystem i from those that are exogenous, such as implicit or explicit communication from another subsystem. The local signal might be the subsystem's measured position and velocity, whereas the exogenous signal might be the position of another subsystem that must be explicitly communicated. The partitions of $y_i(t)$ are denoted

$$y_i(t) = h_i(x(t)) = \begin{bmatrix} h_{ia}(x_i(t)) \\ h_{ib}(x(t)) \end{bmatrix} \in \mathbb{R}^{p_i}$$

where $h_{ia}(x_i) \in \mathbb{R}^{p_{ia}}$ is locally generated and $h_{ib}(x) \in \mathbb{R}^{p_{bi}}$ is exogenous to subsystem i. For local analysis, we use the notation

$$H_i = \frac{\partial h_i(x^o)}{\partial x} \in \mathbb{R}^{p_i \times n}, \quad H_{ia} = \frac{\partial h_{ia}(x_i^o)}{\partial x_i} \in \mathbb{R}^{p_{ia} \times n_i}$$
$$H_{ib} = \frac{\partial h_{ib}(x^o)}{\partial x} \in \mathbb{R}^{p_{ib} \times n_i} \quad \text{for } i = 1, \ldots r.$$

3 Decentralized Control

In the decentralized control structure, each subsystem is regulated by a separate dynamic output feedback controller

$$\begin{aligned} \dot{z}_i(t) &= A_i z_i(t) + B_i y_i(t) \\ u_i(t) &= C_i z_i(t) \end{aligned} \qquad i = 1, \ldots, r. \tag{3}$$

where $A_i \in \mathbb{R}^{n_{ki} \times n_{ki}}$ and n_{ki} is the dimension of the controller state. The output signal $y_i(t)$ of subsystem i is the signal measured by the controller. The controller output $u_i(t)$ is the forcing function for subsystem i. There are r such controllers in the platoon–one for each subsystem. Note that there is no interaction between the controllers. All interaction is due to the subsystem output signal $y_i(t)$ that can be a function of the states of other subsystems in the platoon.

The existence of decentralized controllers was studied extensively in works including [7] and [8]. In [5], the idea of decentralized fixed modes was introduced. Essentially, these are modes of the system that cannot be moved by decentralized controllers, as in (3). If a system has no decentralized fixed modes, then a stabilizing decentralized controller such as (3) can be found. An existence test, suitable for our purposes, was presented in [9] based on combining results in [5] and [7]. We adapt that test for the platoon dynamics at hand.

Lemma 3.1 *There exists a decentralized output feedback control system as in* (3) *that stabilizes the plant* (2) *with linearized output functions given by $y_i = H_i x$ if*

$$\operatorname{rank}\left[\begin{array}{cc|ccc} \lambda I & -H_c & \hat{G}_{i_1} & \cdots & \hat{G}_{i_\mu} \\ 0 & \lambda I - F & & & \\ \hline \multicolumn{2}{c|}{H_{j_1}} & & & \\ \multicolumn{2}{c|}{\vdots} & & 0 & \\ \multicolumn{2}{c|}{H_{j_\nu}} & & & \end{array}\right] \geq n + p_d \tag{4}$$

is satisfied for all $\lambda \in \mathbb{C}$ and indexes $i_1, \cdots i_\mu$, and $j_1, \cdots, j_\nu$, $(0 \leq \mu \leq r,\ 0 \leq \nu \leq r)$ with

$$\{i_1, \cdots i_\mu\} = \{1, \cdots, r\} \setminus \{j_1, \cdots, j_\nu\}$$

The hypothesis of Lemma 3.1 need only be checked for values λ that are eigenvalues of F_i and for $\lambda = 0$. An important benefit of our systems-theory approach is that Lemma 3.1 can be used to study the communication structures, both implicit and explicit, that are required to stabilize the platoon. In general, one would first choose H_i to represent only the implicit communication that is available between subsystems and test the hypothesis of Lemma 3.1. If the rank test succeeds, then no explicit communication is required between subsystems. If the rank test fails, then H_i is altered to represent additional explicit communication channels (e.g., states corresponding to the position of subsystem 2 are explicitly transmitted to subsystem 3). The hypothesis of Lemma 3.1 is checked again, and this iterative processes continues, increasing the explicit communications burden, until the rank test is satisfied.

Due to the structure of the platoon dynamics at hand, more specific conclusions about inter-vehicle communications can be generated. To state these conclusions precisely, we first present some underlying assumptions.

Assumption 3.2 *Each subsystem is locally controllable and locally observable. That is,*

$$\operatorname{rank}\begin{bmatrix} \lambda I - F_i \\ H_{ia} \end{bmatrix} = n_i \quad \textit{and} \quad \operatorname{rank}\begin{bmatrix} \lambda I - F_i & G_i \end{bmatrix} = n_i$$

for all $\lambda \in \mathbb{C}$ and $i = 1, \ldots, r$.

Assumption 3.3 *The platoon is free of transmission zeros at zero relative to the integrator output. That is, the matrix*

$$\left[\begin{array}{c|ccc} -H_c & \hat{G}_1 & \cdots & \hat{G}_r \\ -F & & & \end{array}\right]$$

has full rank.

Assumption 3.3 states that an integrator can be placed at the output of the integrated function without causing a pole/zero cancelation.

Given that the platoon dynamics satisfy Assumptions 3.2 and 3.3, it is possible to examine specific communication strategies that satisfy Lemma 3.1. One such communication strategy requires that the integrator states $q(t)$ are broadcast directly to each vehicle in the platoon. In our notation, this is equivalent to choosing $h_{ib}(x(t)) = q(t)$ for all $i = 1, \ldots, r$. This choice of explicit communication is desirable in that the bandwidth of the communication is dependant only on the number of integrator states, not on the number of vehicles in the platoon.

Corollary 3.4 *There exists a decentralized output feedback control system as in* (3) *that stabilizes the platoon* (2) *if Assumptions 3.2 and 3.3 are satisfied and if $h_{ib}(x) = q(t)$ for $i = 1, \ldots, r$.*

The proof of Corollary 3.4 appears in [10], and we present a brief outline here. Row and column exchanges of (4) are used to display the matrices in Assumption 3.2, from which the rank of (4) is determined to be at least n. Depending on the selection of $\{i_1, \cdots i_\mu\}$ and $\{j_1, \cdots, j_\nu\}$, an additional p_d independent rows or columns are obtained from either Assumption 3.3 or from the hypothesis of the corollary, $h_{ib}(x) = q$.

4 Example

Our decentralized control design method is demonstrated with an example based on controlling a platoon composed of four AUVs. For clarity of presentation, we limit the example to the 2 dimensional plane and choose a simplistic model for the AUV dynamics. Our goal is to control the average and variance of the AUV positions with limited inter-vehicle communications. The dynamics of each vehicle are written

$$\begin{bmatrix} \dot{x}_{1_i} \\ \dot{x}_{2_i} \\ \dot{y}_{1_i} \\ \dot{y}_{2_i} \end{bmatrix} = \begin{bmatrix} x_{2_i} \\ f_i \cos(\theta_i) - 2x_{2_i} \\ y_{2_i} \\ f_i \sin(\theta_i) - 2y_{2_i} \end{bmatrix} \tag{5}$$

where f_i represents the force created by the vehicle's thruster, θ_i is the angle of the AUV in an inertial frame, and the $-2x_{2_i}$ and $-2y_{2_i}$ terms represent viscous damping. The states x_{1_i} and y_{1_i} represent the position of the vehicle within an inertial coordinate frame and x_{2_i} and y_{2_i} are their respective velocities.

It is well known that dynamics such as (5) are nonholonomic. Linearized at a constant operating point, they are not controllable, and thus the vehicles violate the hypothesis of Corollary 3.4. However, the linearized AUV dynamics are controllable when they are linearized about a trajectory. This agrees well with our AUV example, as underwater vehicles are often designed to be in motion.

The AUV dynamics are linearized about a constant velocity given by $\dot{x}_{1_i} = x_{2_i} = \frac{1}{2}$ and $\dot{y}_{1_i} = y_{2_i} = 0$. Inputs corresponding to this trajectory are $f_i = 1$ and $\theta_i = 0$. Selecting a change of variables,

$$\begin{bmatrix} \bar{x}_{1_i} \\ \bar{x}_{2_i} \\ \bar{y}_{1_i} \\ \bar{y}_{2_i} \end{bmatrix} = \begin{bmatrix} x_{1_i} - \frac{1}{2}t \\ x_{2_i} - \frac{1}{2} \\ y_{1_i} \\ y_{2_i} \end{bmatrix}, \qquad \begin{bmatrix} \bar{f}_i \\ \bar{\theta}_i \end{bmatrix} = \begin{bmatrix} f_i - 1 \\ \theta_i \end{bmatrix},$$

where t is time, and then linearizing yields the linear time-invariant system

$$\begin{bmatrix} \dot{\bar{x}}_{1_i} \\ \dot{\bar{x}}_{2_i} \\ \dot{\bar{y}}_{1_i} \\ \dot{\bar{y}}_{2_i} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} \bar{x}_{1_i} \\ \bar{x}_{2_i} \\ \bar{y}_{1_i} \\ \bar{y}_{2_i} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{f}_i \\ \bar{\theta}_i \end{bmatrix} \tag{6}$$

Our goal is to regulate the average position of the vehicles and the distribution of the vehicles about the average position in these new coordinates. The average is computed in two dimensions

$$v_x = \left(\bar{x}_{1_1} + \bar{x}_{1_2} + \bar{x}_{1_3} + \bar{x}_{1_4}\right)/4 \tag{7}$$
$$v_y = \left(\bar{y}_{1_1} + \bar{y}_{1_2} + \bar{y}_{1_3} + \bar{y}_{1_4}\right)/4 \tag{8}$$

and the sample variance, which gives the distribution of the vehicles, is also computed in two dimensions

$$w_x = \sum_{i=1}^{4} \left(\bar{x}_{1_i} - v_x\right)^2/3 \tag{9}$$
$$w_y = \sum_{i=1}^{4} \left(\bar{y}_{1_i} - v_y\right)^2/3. \tag{10}$$

The global (platoon–level) functions (7)–(10) are entries of the vector function $h_c(\bar{x}) = [v_x, v_y, w_x, w_y]^T$

and are integrated to ensure zero steady-state error. Measurement and integration of the global variables is accomplished by subsystem 1, which we consider the *mothership* AUV. Subsystem 1 may have the capability to directly measure the global variables, or to indirectly measure the global variables by measuring the position of every other vehicle in the platoon and computing (7) – (10). Importantly, subsystem 1 is the only subsystem with this capability.

To partially satisfy the hypothesis of Corollary 3.4, the integrated global variables are broadcast to the platoon. This is equivalent to setting $h_{ib}(x(t)) = q(t)$, $i = 2, 3, 4$. In addition, we assume that each vehicle can measure its own position so that each subsystem is locally observable. The platoon is linearized at $(\bar{x}_{1_1}, \bar{y}_{1_1}) = (1.1, 2)$, $(\bar{x}_{1_2}, \bar{y}_{1_2}) = (2, 1.5)$, $(\bar{x}_{1_3}, \bar{y}_{1_3}) = (0.7, 1.1)$, and $(\bar{x}_{1_4}, \bar{y}_{1_4}) = (1.5, 0.7)$. Inspection of the linearized subsystem dynamics (6) shows that each subsystem is locally controllable and that the platoon is free of zeros at zero. Thus Assumptions 3.2 and 3.3 are satisfied, which implies that the hypothesis of Corollary 3.4 is satisfied and a controller (3) for each subsystem exists such that the platoon as a whole is stable and is able to track global function commands. The requirements for such a decentralized control system are that one vehicle can measure and integrate the global variables and that the integrated global variables are broadcast to the other vehicles in the platoon. The only communication that is required is the broadcast of the integrated global variables.

4.1 Synthesis of Decentralized Controllers

From Corollary 3.4, a decentralized controller exists for the platoon operating near the stated equilibrium point. In general, however, designing such a controller is a difficult task. For the system at hand, we note that the output of each system, $y_i(t) = H_i x(t)$ is not observable. Thus a number of design methods reported in the literature are not applicable. Instead, we adapt a synthesis technique recently reported in [11] to design a decentralized controller for the platoon at hand. Our focus here is on existence of decentralized controllers with attention to communication requirements. Thus we refer the reader to [11], and references therein, for a detailed discussion of decentralized controller synthesis. A summary of this particular control design problem is found in [10].

We found the four subsystem controllers with controller state dimension $n_{k1} = \cdots = n_{k4} = 15$. These controllers are able to stabilize the closed-loop system in the neighborhood of where the system was linearized with adequate performance. The algorithm is numerically intensive which causes the controller design process to be prohibitively time consuming. Though we have proved existence of decentralized controllers for the vehicle platoon, efficient methods of control synthesis remain an open issue.

4.2 Simulation

The AUV platoon was simulated using nonlinear global (platoon–level) variable functions, nonlinear dynamics, and linear decentralized controllers. Commanded and actual trajectories for the average and variance along the x- and y-axis are shown in Fig. 1 and 2, respectively. Along the x-axis, the data is shown with respect to the transformed state variables $\bar{x}_{1_i}$, for convenience. The trajectories of the vehicles are shown in Fig. 3 and close-up snapshots of the vehicles over time are shown in Fig. 4.

Note that the vehicle spacing about the mean position is not regular or specified. There are an infinite number of possible full system states that satisfy the required regulation of the global variables. This allows for much more flexible system behavior than that of a platoon under rigid formation control while still guaranteeing that the desired objectives are met.

5 Concluding Remarks

Methods presented here allow the designer to determine what explicit communication strategies are sufficient for a stabilizing decentralized control to exist. Using a simplified model, we have shown that it is indeed possible to regulate global variables of a platoon of autonomous underwater vehicles: in particular, the center of the platoon and the distribution of the vehicles about the center. The only explicit communication is a broadcast–only communication structure (single-source, unidirectional). Further, the approach presented is scalable to any number of cooperating vehicles without the need for additional communication, although there is a practical limit on the size of the platoon. A number of open research areas remain, including analysis of the performance of the system under disturbances, failure of a subsystem, and efficient methods for synthesizing decentralized controllers.

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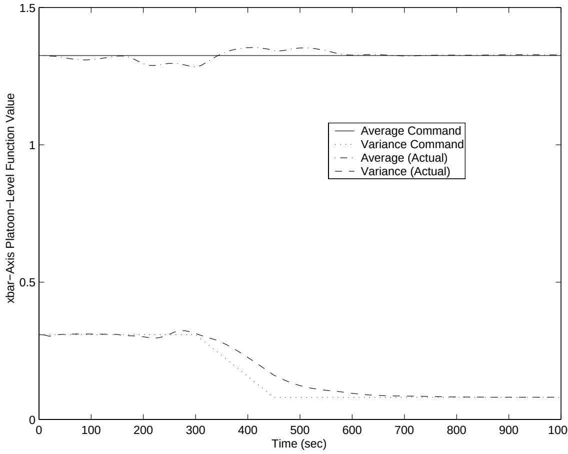

Figure 1: Actual and commanded average position and variance for $\bar{x}$.

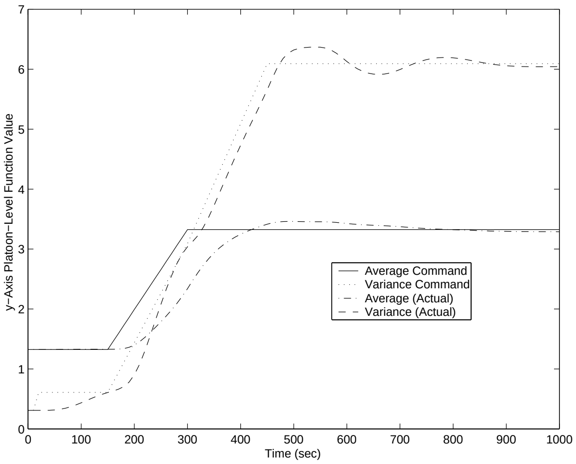

Figure 2: Actual and commanded average position and variance for y.

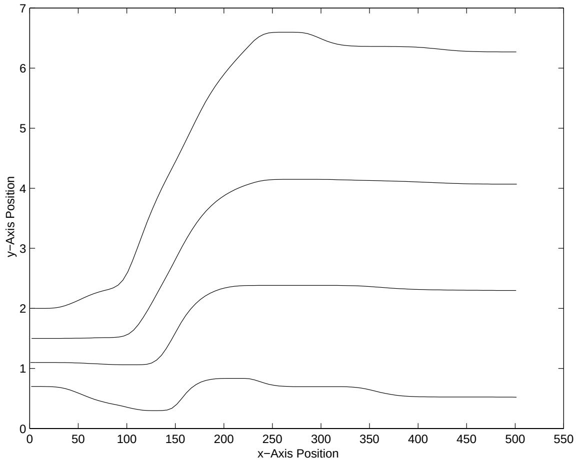

Figure 3: Trajectories of vehicles.

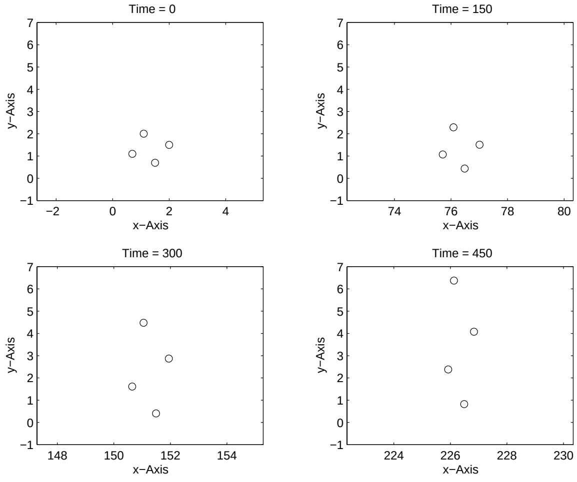

Figure 4: Snapshots of vehicle positions over time.