On the Separation of Estimation and Control in Discrete-Event Systems [1]

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Abstract

The discussion of this paper concerns both centralized and decentralized control of logical discrete-event systems. Of interest are the maximal information sets of the centralized or decentralized supervisors and the potential for control and estimation policy independence. We show that there exists a form of a centralized supervisor's maximal information sets that is independent of the supervisor's control policy. We also show that this method of separation is not generally applicable in decentralized settings. These results are consistent with the literature in stochastic control; however, supervisory control potentially presents a more simple framework in which to explore these concepts.

Keywords: discrete-event systems, decentralized control, separation, policy independence

1 Introduction

Separability in stochastic systems (i.e., independent design of estimation and control policies) results from the policy independence of conditional expectations, and this independence is available for systems with particular types of information structures [5], [6], [7], [8]. The demonstration of policy independence within stochastic control literature generally relies on algebraic manipulation of probability distributions and Bayes rule. During such manipulations, in the words of Witsenhausen [7], the policy dependent quantity "literally factors out".

In this paper, we demonstrate that the "maximal information sets" available in the centralized supervisory control problem have a form which can be determined without knowing the supervisor's control policy. We also demonstrate that this technique does not generally extend to the decentralized supervisory control problem. The concepts of separability and policy independence do not rely on the existence of an underlying probability space; these concepts are connected, instead, to the fundamental structure of a system.

Our discussion here is brief due to space constraints; however, a more thorough discussion can be found in [2]. Furthermore, we assume that the reader is aware of the notation of supervisory control literature, and we will follow that given in [1].

2 Separability and the Concept of State

Centralized Control

The situation where only one agent/supervisor ($|Z| = 1$) observes a plant, G, via a projection operator $\mathbb{P}$ and individually controls the plant is called *centralized* supervisory control and has been well studied. In this case, the dynamics of the closed-loop control system (plant and controller) can be completely determined from the pair

$$(s, \mathbb{P}(s)) \in \mathcal{L}(Z/G) \times \mathbb{P}(\mathcal{L}(Z/G)). \tag{1}$$

The maximal information available to the supervisor to generate a control decision following trace s is denoted here by $\Psi(s)$ and is represented by the set of all traces in the closed-loop behavior that "look" like s to the supervisor:

$$\Psi(s) = \mathbb{P}^{-1}(\mathbb{P}(s)) \cap \mathcal{L}(Z/G). \tag{2}$$

An alternative description of $\Psi(s)$ is the set of all sequences the supervisor "infers" could have occurred given its partial observation of the occurrence of s. Often, difficulties arise for controller synthesis because the maximal information set (2) used by the state estimator depends upon the control policy implicit in $\mathcal{L}(Z/G)$, the control actions depend upon state estimates, and $\mathcal{L}(Z/G)$ may not be known prior to the synthesis of the control policy; control and estimation based on Eqn(2) are dependent. (Note that this difficulty does not arise when it is known *a priori* that the desired language, $\mathcal{L}(H)$, *will* be the language generated by the closed-loop system, i.e., $\mathcal{L}(H) = \mathcal{L}(Z/G)$ is observable and controllable.)

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One method that has been repeatedly used to bypass this difficulty is to assume that all controllable events are observable, $\Sigma_c \subseteq \Sigma_o$ which then produces maximal information sets $\mathbb{P}^{-1}(\mathbb{P}(\cdot)) \cap \mathcal{L}(G)$ which do not depend on the control policy (compare to $\mathbb{P}^{-1}(\mathbb{P}(\cdot)) \cap \mathcal{L}(Z/G)$). The result is that only *normal* languages are considered. Another method [3] prioritizes control actions to provide enough information about the control policy so that state-estimates can be generated: The resulting control policies are guaranteed to be maximal. Both of these methods place some type of assumption on the control policies to enable synthesis. Here, we show that no assumptions are needed to establish a separation.

Theorem 2.1 *The centralized supervisor's maximal information sets, $\Psi(\cdot)$, have a form which do not depend on the supervisor's control policy.*

The idea behind the proof of Theorem 2.1 (see [2]) is to show there is a map, $\varphi : (2^{\Sigma_c} \times \Sigma_o)^* 2^{\Sigma_c} \to 2^{\Sigma}$ such that if $s \in \mathcal{L}(G)$ (with $\mathbb{P}(s) = \sigma^1\sigma^2 \ldots \sigma^k$) is the trace that has actually occurred in the closed-loop system, then $\varphi(\gamma^0\sigma^1\gamma^1\sigma^2\gamma^2 \ldots \sigma^k\gamma^k) = \Psi(s) = \mathbb{P}^{-1}(\mathbb{P}(s)) \cap \mathcal{L}(Z/G)$, where σ^i is the i-th observed event and γ^i is the i-th control action. The proof begins with the base case of the form

$$\varphi(\gamma^0) = (\Sigma_{uo} \setminus \gamma^0)^* \cap \mathcal{L}(G), \tag{3}$$

and proceeds by induction. The result is that the information set $\mathbb{P}^{-1}(\mathbb{P}(s)) \cap \mathcal{L}(Z/G)$ can be "recovered" using only observations and control *actions* and without knowledge of the control *policy*. Furthermore, if the plant and desired behaviors are regular, then the sufficient statistic $\varphi(\gamma^0\sigma^1\gamma^1 \ldots \sigma^k\gamma^k)$ has a finite-state representation. Hence, policy separation is possible in the centralized case using a finite statistic when the behaviors are regular.

Decentralized Control

Now, the discussion will shift focus to the situation where two controllers ($|Z| = 2$) are to act in concert on a plant. Here, the dynamics of the closed-loop control system can be completely determined from the three-tuple $(s, \mathbb{P}_1(s), \mathbb{P}_2(s)) \in \mathcal{L}(Z/G) \times \mathbb{P}_1(\mathcal{L}(Z/G)) \times \mathbb{P}_2(\mathcal{L}(Z/G))$, where, we have basically the same form as in (1) above.

If it is known a priori (via a polynomial test for coobservability [4] and controllability) that the desired language $\mathcal{L}(H)$ is achievable, then the maximal information sets available to each controller following some trace s are $\mathbb{P}_1^{-1}(\mathbb{P}_1(s)) \cap \mathcal{L}(H)$, and $\mathbb{P}_2^{-1}(\mathbb{P}_2(s)) \cap \mathcal{L}(H)$.

As in the centralized case, synthesis difficulties can occur when $\mathcal{L}(Z/G)$ is not known *a priori*; however, given that decentralized controllers have analogous forms of the centralized equations (1) and (2), it is natural to ask whether Theorem 2.1 generalizes to the multiple controller case. The answer is generally negative. Attempting to write an analogous Eqn(3) for the decentralized controllers reveals the problem immediately:

$$\begin{aligned}\varphi_i(\gamma_i^0) = {} & (\Sigma_{uo,i} \setminus \gamma_i^0)^* \cap \mathcal{L}(G) \cap \\ & \Bigg[\varphi_j(\gamma_j^0) \cup \\ & \quad \bigcup_{\sigma^1 \in \Sigma_{o,j} \setminus \Sigma_{o,i}} \varphi_j(\gamma_j^0\sigma^1\gamma_j^1) \\ & \quad \vdots \\ & \quad \bigcup_{\sigma^1, \sigma^2, \ldots, \sigma^k \in \Sigma_{o,j} \setminus \Sigma_{o,i}} \varphi_j(\gamma_j^0\sigma^1\gamma_j^1 \ldots \sigma^k\gamma_j^k) \\ & \quad \vdots \quad \Bigg]\end{aligned} \tag{4}$$

The "additional" components of Eqn(4) that Eqn(3) does not have represent an apparently inescapable dependence of controller i's information sets on controller j's control policy. Basically, controller i needs to know all of controller j's contingency plans for all event sequences that controller i does not disable and/or does not observe.

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