Design of Hybrid Systems with Guaranteed Performance[1]

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Abstract

In this paper we show that, for a linear system, any worst-case energy gain greater than the optimal $\mathcal{H}_\infty$ norm is achievable by a logarithmically quantized state feedback. We also show how to derive the coarsest logarithmic quantizer provable via quadratic Lyapunov functions for a given level of performance. The smallest logarithmic base, for a given performance level, is obtained via a bisection algorithm applied to a parametric feasibility LMI problem. The result highlights the tradeoff between performance degradation versus coarseness of quantization. Simulations suggest that the upper bound derived in this paper is a realistic measure of the actual performance under logarithmic quantization. The end result is the systematic design of a discrete event controller that stabilizes a linear system and guarantees a certain level of performance measured in terms of the worst-case close loop energy gain. The resulting hybrid system is implicitly verified.

1 Introduction

We are interested in the following general question: What is the minimal information about the state of a system we need in order to achieve a desired performance level in closed loop? This question is at the basis of many new open problems in control theory, from systematic design of hybrid systems [1], to feedback over communication networks [6, 2].

Particularly, in this paper, we study the problem of designing a discrete-event feedback system that stabilizes a given linear time-invariant system, and guarantees a desired, feasible, value of the worst-case energy gain of the closed loop system.

There are several reasons for considering this performance measure. First, it allows us to study the effect of bounded energy noise acting at the input of the system. Second, its inverse is a well known measure of robustness against model uncertainty. Third, the closed loop system itself could be seen as uncertainty of given norm. This is a useful abstraction in analysis and design of complex systems.

In this paper, we adopt the approach first introduced in [1], where quadratic stability was the only concern, and extend the results to include the worst-case energy gain of the closed loop system as performance measure. We study both continuous-time and the discrete-time systems. We show that any level of performance, γ, strictly greater than the optimal achievable closed loop $\mathcal{H}_\infty$ norm can be achieved by a logarithmic quantizer as defined in [1]. Moreover, we are able to characterize the coarsest quantizer in terms of the solution of a modified $\mathcal{H}_\infty$ Riccati equations or LMI.

We want to point out that we are interested in the systematic (i.e. implicitly verified) design of hybrid systems, rather than concentrating on the verification of a particular scheme, or in the control of a specific hybrid system. Also the central idea in this paper, that hybrid phenomena are effect of information quantization, [3, 4, 7], leads us to consider quantization beneficial rather than undesirable either as noise, or state uncertainty that must be reduced by often complex controllers.

We describe the implicitly verified hybrid automata that emerges from the solution for continuous time systems. We also observe how quantized command signals are generated automatically by the discrete event controller, suggesting a plausible approach for reference command generation for nonlinear systems based on the ideas presented in this paper. We apply the results of the paper to a simple example, and compute a curve of tradeoff between the level of guaranteed performance an the density of the logarithmic quantization. We also notice the occurrence, for continuous time systems, of the phenomenon of optional sampling, also known as Lebesgue sampling, where the quantized state information is used when needed and not necessarily at uniformly spaced sampling times.

[1]This research was supported by ARO-DAAL-03-92-G-0115, and Siemens A.G.

[2]I wish to thank Sanjoy K. Mitter for first getting me interested in this problem and for the many inspiring discussions. The work in this paper extends the approach of our joint paper [1]. I also wish to thank Alexander Megretski and Sekhar Tatikonda for the interesting discussions and useful suggestions.

2 Problem Formulation

In this section we introduce the problem. Specifically, we assume that the system is governed by the following equations:

$$\begin{array}{rcl} x^+ & = & Ax + B_1 w + B_2 u \\ z & = & C_1 x + D_{11} w + D_{12} u \\ y & = & Ix \end{array} \qquad (1)$$

where x^+ is either equal to $\dot{x}$ for continuous time systems, or $x(t+1)$ in the case of discrete time systems. $x \in \mathbb{R}^n$, $z \in \mathbb{R}^m$, $w \in \mathbb{R}^p$, and $u \in \mathbb{R}^q$. y represents the measurements, z the regulated output variables, w the exogenous inputs, and u the control inputs.

From the system equations (1) it can be noted that we are considering the state feedback case. Also, in this paper, we will consider the single control input case, $B_2 \in \mathbb{R}^{n\times 1}$. We make the following extra assumptions:

Assumption 2.1

.1) The systems is stabilizable from u.

.2) $C_1' D_{12} = 0$, $D_{12}' D_{12} = I$, $D_{11} = 0$ and $D_{22} = 0$.

2.1.1) is standard assumption and clearly necessary, while 2.1.2) is a set of typical assumptions that just simplify the treatment.

The problem we consider is the following one.

Problem 2.1 *We want to find a set*

$$\mathcal{U} = \{u_i \in \mathbb{R} : i = 0, 1, 2 \ldots\}$$

representing possibly a countable number of fixed control values, to be determined, and a function $f : \mathbb{R}^n \to \mathcal{U}$ such that, $u = f(y)$ guarantees that the worst-case energy gain between the input signal $w(t)$ and the output signal $z(t)$, with zero initial conditions, is less than a pre-specified value γ, i.e. $\dfrac{\|z\|_2}{\|w\|_2} \leq \gamma$.

With a slight abuse of terminology f is called the quantizer. Notice that the range of f induces a quantization partition in the state-space of the system, where equivalence classes of states correspond to the same adopted control value, i.e., $\Omega_i = \{x \in \mathbb{R}^n \mid f(x) = u_i\}$. Notice also that by asking for the quantizer to be a function we are implicitly assuming that to each x there corresponds only one element in $\mathcal{U}$.

We want to stress that we want to find the coarsest (in some sense) quantizer that still guarantees the performance with a low complexity controller. Another important difference is that we want to derive the quantizer structure instead of imposing the quantizer structure a priori and then analyze (or verify) the performance of the closed loop system.

3 Approach

We extend the approach used in [1] to incorporate the performance objective. Although in this paper we restrict our attention to quadratic candidate Lyapunov functions (being interested in quadratic stability), the treatment can in principle be extended to consider more general Lyapunov functions. This restriction is reflected in the next definition.

Definition 3.1 *An $\mathcal{H}_\infty$ Control Lyapunov Function with associate performance level γ, in short $\mathcal{H}_\infty(\gamma)$CLF, is a quadratic positive function $V(x) = x'Px$, $P > 0$, such that there exists a control input $u = f(x)$ for which*

$$\dot{V}(x) + z'z < \gamma^2 w'w \qquad (2)$$

for all x and w satisfying (1) with $u = f(x)$, where $\dot{V}(x(t))$ denotes $\frac{dV(x(t))}{dt}$ in the continuous time case, and $V(x(t+1)) - V(x(t))$ in the discrete time case

The existence of a $\mathcal{H}_\infty(\gamma)$CLF implies that there is a feedback law for which System (1) has energy gain less than or equal to γ. We know that the existence of a $\mathcal{H}_\infty(\gamma)$CLF is also a necessary condition in the case of LTI systems with linear controllers.

All the $\mathcal{H}_\infty(\gamma)$CLF's are simply characterized via Linear Matrix Inequality constraints. These LMI's arise from the $\mathcal{H}_\infty$ state-feedback problem.

3.1 Relevant $\mathcal{H}_\infty$ State-Feedback Results

In this section we review some of the results about $\mathcal{H}_\infty$ state-feedback synthesis that are relevant to our treatment. Good reference for the material herein is [9].

3.1.1 Continuous Time: Given system (1) and Assumption 2.1, there is a linear static feedback controller for which the closed loop system has an $\mathcal{H}_\infty$ norm less then or equal to γ if and only if there is a matrix $P > 0$ which satisfies the following constraints

$$A'P + PA + C'C + \frac{PB_1B_1'P}{\gamma^2} - PB_2B_2'P < 0 \qquad (3)$$

A feasible controller is the so called central controller and is given by

$$K_\infty = -B_2'P \qquad (4)$$

The optimal $\mathcal{H}_\infty$ performance γ^o is the infimum over γ for which there is a $P > 0$ satisfying the constraint (3).

It is convenient make the following definition based on (3)

$$-A'P-PA-C'C-\frac{PB_1B_1'P}{\gamma^2}+PB_2B_2'P \triangleq Q > 0 \quad (5)$$

For the given γ the set of all $\mathcal{H}_\infty(\gamma)$CLF, $V(x) = x'Px$ with $P > 0$, is given by all those P that satisfies the constraint (3).

3.1.2 Discrete Time: Given system (1) and Assumption 2.1, there is a linear static feedback controller for which the closed loop system has an $\mathcal{H}_\infty$ norm less then or equal to γ if and only if there is a matrix $X > 0$ which satisfies the following constraints

$$\begin{array}{l} X > B_1B_1'; \\ \text{and} \\ \begin{pmatrix} X - B_1B_1' + \gamma^2 B_2B_2' & CXA' \\ AXC' & \gamma^2 I - CXC' \end{pmatrix} > 0 \end{array} \quad (6)$$

A feasible controller is the so called central controller and is given by

$$K_\infty = -R^{-1}B_2'YA \quad (7)$$

where

$$Y = (X - B_1B_1')^{-1}, \text{ and } R = \left(\frac{I}{\gamma^2} + B_2'YB_2\right) \quad (8)$$

The optimal $\mathcal{H}_\infty$ performance γ^o is the infimum over γ for which there is a $X > 0$ satisfying the constraint (6). An equivalent and more useful form of (6) is given applying Schur complement to the second inequality. After some rearrangement, we obtain

$$\begin{array}{l} X > B_1B_1'; \\ \text{and} \\ X^{-1} - \frac{C'C}{\gamma^2} - A'YA + A'YB_2R^{-1}B_2YA \triangleq Q > 0 \end{array} \quad (9)$$

For the given γ the set of all $\mathcal{H}_\infty(\gamma)$CLF, $V(x) = x'Px$ with $P > 0$, is given by all those P such that $X = \gamma^2 P^{-1}$ satisfies the constraint (6).

4 Logarithmic Quantizers which Guarantee $\mathcal{H}_\infty$ Closed Loop Performance

The objective of this section is to prove the first main result of the paper. It says that any performance level $\gamma > \gamma^o$, can be achieved by a logarithmic quantizer. In more precise terms,

Theorem 4.1 *Let γ^o denote the optimal $\mathcal{H}_\infty$ norm for System (1) over all stabilizing controllers.*
Assume that $V(x) = x'Px$ is a $\mathcal{H}_\infty(\gamma)$CLF for a performance level $\gamma > \gamma^o$. Then there is a quantizer which delivers a worst-case energy gain less than or equal to γ in closed loop. The fixed control values used by the quantizer follow a logarithmic law

$$\mathcal{U} = \{\pm u_i, : u_{i+1} = \rho(\gamma)u_i, -\infty \le i \le +\infty\} \cup \{0\}.$$

The quantizer induces a logarithmic partition of the state-space into stripes orthogonal to K_∞' defined as

$$\Omega_i^\pm = \\ = \{x \in I\!R^n \mid \alpha_{i+1} < K_\infty x \le \alpha_i, \text{ with } \alpha_{i+1} = \rho(\gamma)\alpha_i\}$$

where $\Omega_i^\pm$ is a compact notation to describe both the symmetric sets Ω_i^+ and Ω_i^- associated with $+u_i$ and $-u_i$, respectively, while $\Omega_{zero} = \{x \in I\!R^n \mid K_\infty x = 0\}$ is associated to the 0 control value.

The constant $0 \le \rho(\gamma) < 1$ is given by the following expression for continuous time systems

$$\rho(\gamma) = \frac{\sqrt{B_2'PQ^{-1}PB_2} - 1}{\sqrt{B_2'PQ^{-1}PB_2} + 1} \quad (10)$$

where K_∞ and Q are defined in equations (4) and (5) respectively. While for discrete time systems

$$\rho(\gamma) = \frac{\sqrt{\frac{B_2'YAQ^{-1}A'YB_2}{R}} - 1}{\sqrt{\frac{B_2'YAQ^{-1}A'YB_2}{R}} + 1} \quad (11)$$

where K_∞, Y, R, and Q are defined in equations (7), (8), and (9) respectively.

Moreover, this quantization is minimal in the sense that, for any $\rho < \rho(\gamma)$, Inequality (2) will not be satisfied for a non-zero Lebesgue measurable set of states.

Remark 4.1 *This theorem is providing a logarithmic partition of the system state-space in stripes orthogonal to the optimal direction of quantization K_∞, which still guarantees that the energy gain between w and z is bounded by γ.*

In the continuous time case, we are obtaining through the quantized feedback a pure reactive system. As soon as the state crosses the boundaries between partition, the appropriated control value is generated at the input of the plant. This is along the spirit of Lebesgue sampling introduced by Astrom in [11].

5 Optimal Quantization over all $\mathcal{H}_\infty$CLF

In the previous section we have computed the optimal quantizer for a given $\mathcal{H}_\infty(\gamma)$CLF. In this section, we search over all $\mathcal{H}_\infty(\gamma)$CLF to find the coarsest quantizer, i.e., the smallest $\rho(\gamma)$, for a given $\gamma > \gamma^o$.

The main result is a feasibility condition, given γ and a candidate $\rho(\gamma)$, we show that a quantizer with logarithmic base $\rho(\gamma)$ is feasible in achieving performance γ, if a modified $\mathcal{H}_\infty$ problem has a solution. A bisection algorithm can then implemented to find the optimal $\rho^*(\gamma)$ within the desired accuracy.

Define

$$\rho^*(\gamma) = \inf_{\substack{V = x'Px \\ V \in \mathcal{H}_\infty(\gamma)CLF}} \rho(\gamma)$$

Also, for a given $\gamma > \gamma^o$ let α denote $\sqrt{B_2' P Q^{-1} P B_2}$ in the continuous time case, and $\sqrt{\frac{B_2' Y A Q^{-1} A' Y B_2}{R}}$ in the discrete time case. Then $\rho(\gamma) = \frac{\alpha-1}{\alpha+1}$ which is monotonically increasing for $\alpha > 0$. Thus minimizing $\rho(\gamma)$ is equivalent to minimizing α (or α^2) over $P > 0$, $V = x'Px$, $\mathcal{H}_\infty(\gamma)$CLF for system (1).

Thus we focus on the equivalent problem:

$$\alpha^* = \inf_{\substack{V = x'Px \\ P > 0 \\ V\, \mathcal{H}_\infty(\gamma)CLF}} \alpha$$

Theorem 5.1 *The performance level γ is achievable by a logarithmic quantizer with base $\rho(\gamma)$ if*
*(***Continuous Time***) There exists a symmetric matrix X satisfying the following Linear Matrix Inequality constraints*

$$\begin{bmatrix} -XA' - AX - \frac{B_1 B_1'}{\gamma^2} + \frac{B_2(\alpha^2-1)B_2'}{\alpha^2} & XC' \\ C'X & I \end{bmatrix} > 0;$$
$$X > 0$$

or equivalently, there exists a symmetric positive definite matrix P satisfying the following Riccati equation.

$$A'P + PA + C'C - PB_2 \frac{\alpha^2 - 1}{\alpha^2} B_2' P + \frac{P B_1 B_1' P}{\gamma^2} = 0$$

where 0 *denotes vectors of zeros element of appropriate dimensions.*

*(***Discrete Time***) There exists a symmetric matrix X satisfying the following Linear Matrix Inequality constraints*

$$\Sigma(X) > \Sigma_0,\ X > 0,\ X > B_1 B_1'$$

where $\Sigma(X) =$

$$= \begin{bmatrix} \alpha^2 X - AXA' & -AXA' & CXA' \\ -AXA' & \frac{\alpha^2 X}{\alpha^2 - 1} - AXA' & CXA' \\ AXC' & AXC' & -C'XC \end{bmatrix}$$

$$\Sigma_0 = \begin{bmatrix} \alpha^2 B_1 B_1' & 0 & 0 \\ 0 & \frac{B_1 \alpha^2 B_1'}{\alpha^2 - 1} - \frac{B_2 \gamma^2 \alpha^2 B_2'}{\alpha^2 - 1} & 0' \\ 0 & 0 & -\gamma^2 I \end{bmatrix}$$

Remark 5.1 *We want to point out that both the LMI and the Riccati equation that solve the continuous time case, are the solution to an $\mathcal{H}_\infty$ problem for a modified system which has $B_2 \frac{\sqrt{\alpha^2-1}}{\alpha}$ instead of B_2 and for the rest is equal to system (1)*

6 Implicitly Verified Hybrid Automata

We want to highlight that, the quantized feedback makes the closed loop system completely reactive or event driven, and that the result of Theorem 5.1 allows us to compute the tightest upper bound on the performance provable via quadratic Lyapunov functions.

For a continuous-time system, the resulting hybrid automata that represents the closed loop behavior, shown in Figure 1, is implicitly verified. This is the first known example of a hybrid automata systematically designed that emerges directly from the performance specifications and the concept of minimal information. This is of significance in light of the results in [8]. Although the automata requires an infinite countable number of states, a finite state automata could be obtained from it for practical implementations, as shown in [1].

6.1 Automatic Quantized Trajectory Generation

Seen at a different level of abstraction, there is an automatic generation of quantized trajectories induced by the discrete event controller. The set of primitive commands, is given by the fixed control values logarithmically spaced with each value corresponding to a state of the hybrid automata. In response to a given input disturbance, the controller produces a command by selecting a sequence of primitive commands and of switching times.

This architecture has striking similarities to that proposed in [10] for the more complex problem of maneuvering of an autonomous helicopter. However, while in [10] the states of the hybrid automata are chosen a priori, based on engineering and physical considerations, the switching times are the result of online mixed integer programming optimization, and the whole architecture is verified to work correctly only at the end of the design, in here the structure is emerging from our solution to the problem of performance guaranteed with quantized state information and is implicitly verified.

The ideas and results in this paper when appropriately extended to nonlinear systems could provide a new criterion and a systematic method for generating quantized reference commands for nonlinear systems. Such extensions are in principle quite straightforward and will be he subject of a future research effort.

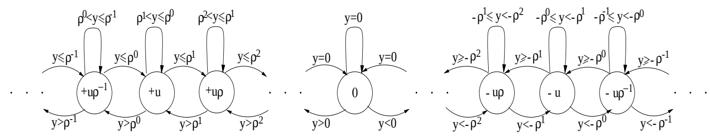

Figure 1: Hybrid automata for the closed loop system with guaranteed worst-case energy gain.

7 Tradeoff Curve

From the result in Theorem 5.1 it follows that we can either fix γ and perform a bisection algorithm on α, or we can fix ρ and apply the bisection algorithm on γ to find the smallest guaranteed performance level for such quantizer.

In this section we apply the above algorithm to a second order continuous time system with the following model.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}; \; B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \; B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$
$$C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}; \; D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The system is unstable. The optimal $\mathcal{H}_\infty$ norm is γ^o=4.0212. Figure 2 shows ρ versus $\frac{\gamma}{\gamma^o}$, and indicates the tradeoff between performance and quantization accuracy. For example, if we choose $\rho = 0.5$ we will have

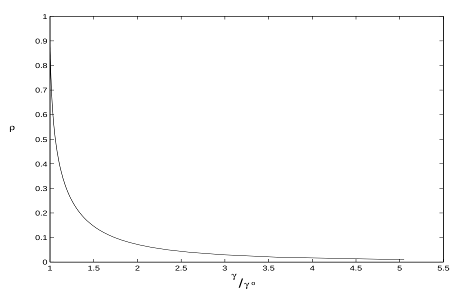

Figure 2: Performance vs. Accuracy Tradeoff

at most a 6% degradation in the performance level with respect to the optimal $\mathcal{H}_\infty$ norm achievable with infinite accuracy. Although γ^o is certainly a lower bound on the achievable performance it is left to future investigation to find a tighter lower bound on the achievable $\mathcal{H}_\infty$ norm for a given logarithmic quantizer.

Clearly, a simple way to obtain a lower bound is through simulation. For $\rho = 0.5$, the lower bound on the achievable $\mathcal{H}_\infty$ norm is obtained by computing the power gain for a unit step input. The resulting value is $\underline{\gamma} = 4.2307$, or equivalently $\frac{\underline{\gamma}}{\gamma^o} = 5.23\%$ which is rather close to $\frac{\gamma}{\gamma^o} = 6\%$.

For $\rho = 0.1$, we expect at most 73.3% degradation of the optimal $\mathcal{H}_\infty$ performance with infinite precision from

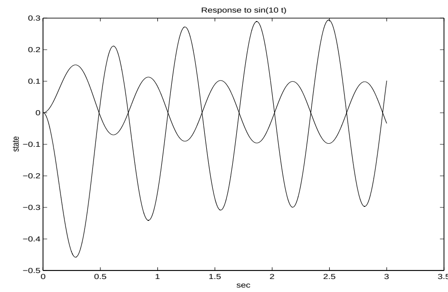

Figure 3: State Evolution to Sinusoidal Input

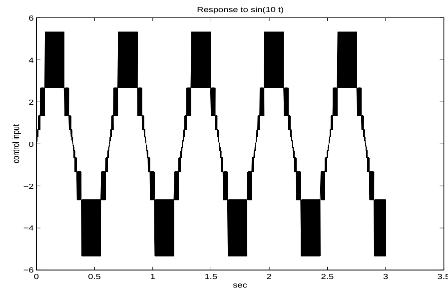

Figure 4: Quantized Control Input to Sinusoidal Input

Figure 2. By simulating the response to a step input, we obtain a lower bound on the achievable $\mathcal{H}_\infty$ norm given by $\underline{\gamma} = 6.8606$, which results in a $\frac{\gamma}{\gamma^o} = 70.61\%$ once again, it is rather close to the upper bound of 73.31%. This may suggest that the performance upper bound developed in this paper is a realistic measure of the actual performance achievable by logarithmically quantized state feedback.

Finally Figure 3 shows the state evolution of the system in response to a sinusoidal input $w(t) = \sin 10t$, with zero initial conditions, and $\rho = 0.5$. Figure 4 shows the associated quantized control action.

It can be noted form the figure that the control input is highly oscillating, this is due to the high gain of the controller. Form one end, this could be justifiable by the fact that γ is very close to the optimal $\mathcal{H}_\infty$ norm to which correspond an high gain controller. On the other hand, the fast switching also happens in the other case with $\rho = 0.1$ and $\gamma = 6.97$, where we are far form $\mathcal{H}_\infty$ optimality, but still get high gain K_∞. In fact, the high gain is due to being close to the optimality of ρ for the given γ. This hypothesis is confirmed by the next two figures, which show the state and control response to the same sinusoid, $w(t) = \sin 10t$, where $\rho = 0.5$ (same as before) but gamma has been relaxed form $\gamma = 4.2307$ to $\gamma = 4.3589$.

Figure 5 shows the state response of the hybrid system (solid line), and the state response of the linear closed loop system without quantization where only K_∞ was

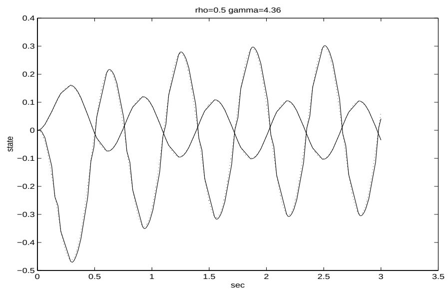

Figure 5: State response with (solid) and without (dotted) quantizer in feedback

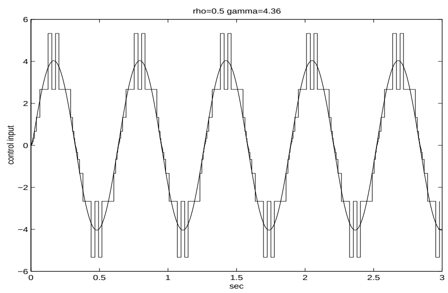

Figure 6: Control input with (stairs) and without (smooth) quantizer in feedback

used to close the loop (dotted line). They are rather close to each other. Figure 6 shows the quantized control input (staircase), and the control input of the linear system in loop with K_∞ but without the quantization (smooth sinusoid). The switching in the control is much reduced in comparison with Figure 4.

Note also the phenomenon of optional sampling, or Lebesgue sampling [11]. New quantized state information is used by the controller only when it is necessary and not at predefined fixed sampling times. As shown in Figure 6, the quantized control values only change when the continuous-time system state crosses the guards conditions of the hybrid automata of Figure 1. The amount of time one control value is used depends implicitly on the state of the system and the input.

8 Conclusions and Future Work

In this paper we have shown that, for a linear system, any energy gain greater than the optimal $\mathcal{H}_\infty$ norm is achievable by a logarithmically quantized state feedback. We have also shown how to derive the logarithmic quantizer associated with the smallest upper bound on the achievable energy gain provable via quadratic Lyapunov functions. In this sense the logarithmic quantization is the coarsest provable via quadratic Lyapunov functions. We have shown, through an example, how to derive a tradeoff curve between performance degradation versus coarseness of quantization. We have noticed, through simulation, that the actual achievable worst case energy gain delivered by the quantizer is close to the computed upper bound; this suggests that the upper bound derived in this paper in a realistic measure of the actual performance. The end result is the systematic design of a discrete event controller that stabilizes a linear system and guarantees a certain level of performance measured in terms of the worst case close loop energy gain. The resulting hybrid system is implicitly verified. There are many possible extensions of the results and the approach presented in this paper. In the short term, more work is needed to derive a lower bound which is computable without simulation. In the longer term, we plan to extend the results to multi-input systems, to output feedback, and to more general Lyapunov functions. It should also be possible to extend the approach based on robust control Lyapunov functions to some special class of nonlinear systems.

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