

From PI to \mathcal{H}_∞ Control in a Unified Framework

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Abstract

Based on a recently presented evaluation method, tuning rules are introduced and compared for a series of PID related controllers. For all of them, the integral gain and the high frequency gain (the parameters in a PI controller), are used as the main tuning parameters. As these directly correspond to well known properties of a control system, they are easy to handle for an operator. It is also shown that a well tuned PID controller augmented by a low pass filter, to increase the roll off rate, is quite competitive to a PID weighted \mathcal{H}_∞ controller, and superior to the common PI weighted \mathcal{H}_∞ controller.

Keywords: PID control, PI control, \mathcal{H}_∞ control, tuning rules, robustness

1 Introduction

Within the control community there has been extensive research on comparing new, or modified, controller design strategies with already existing methods, quite often with a PID controller designed by, e.g., Ziegler and Nichols' tuning rules [13]. Such comparisons are not always objective, since only some aspects of performance and robustness issues usually are considered. Typically, step responses and stability margins are compared, neglecting the cost in terms of, e.g., high frequency robustness and control activity. Recently a more fair evaluation method, based on four evaluation criteria was introduced, see [4]. According to this method two control systems can be compared in one respect, while the remaining characters of the two systems are kept constant and equal.

Tuning of a controller always imply that suitable values for some tuning parameters have to be found. The suggested evaluation method has been used to evaluate several controllers, related in one way or the other to

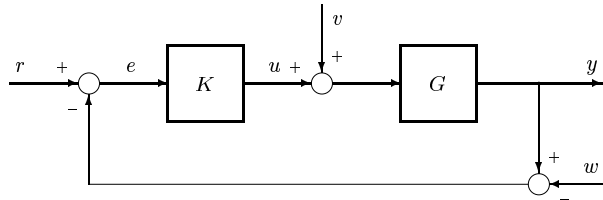


Figure 1: Closed loop system with plant G and controller K .

the PID controller. The goal has been to obtain simple tuning rules for these controllers. Related results can be found in [1, 3, 6, 7, 10, 11, 12]. In this paper it will be shown that tuning rules can often be formulated in a very similar way, starting from the two parameters of the PI controller, the integral gain k_i , and the high frequency gain $k_\infty = K_{PI}(\infty)$, where

$$K_{PI}(s) = \frac{k_i}{s} + k_\infty.$$

Controllers such as PID, lead-lag filter (PIPD), PID with extra roll off, and \mathcal{H}_∞ loop shaping controllers are compared. It is shown that a well tuned PID controller can give just as good performance as a PI, or PID weighted \mathcal{H}_∞ controller, when demands on robustness and the restrictions on control activity are equal.

2 Evaluation

Consider the SISO system in Figure 1, where a plant $G(s)$ is controlled by a controller $K(s)$. It has three inputs, the reference signal $r(t)$, the process disturbance $v(t)$, and the measurement noise $w(t)$. Relevant outputs are the controlled output $y(t)$, the control signal $u(t)$ and the control error $e(t) = r(t) - y(t)$.

The input and output signals can be related to each other by a set of transfer functions denoted G_{outin} .

With the loop $L = GK$, the four sensitivity functions are defined: $S = 1/(1 + L) = G_{er}$, $T = 1 - S = L/(1 + L) = G_{yr} = G_{wr}$, $GS = G/(1 + L) = G_{yv}$, and $KS = K/(1 + L) = G_{ur} = G_{wr}$. Now three evaluation criteria, based on \mathcal{H}_∞ norms and related to performance and robustness of the system in different respects, can be formulated

$$\begin{aligned} J_v &= \left\| \frac{1}{s} G_{yv} \right\|_\infty = \left\| \frac{1}{s} G(s) S(s) \right\|_\infty, \\ J_u &= \|G_{ur}\|_\infty = \|G_{uw}\|_\infty = \|K(s)S(s)\|_\infty, \\ J_{HF} &= \|s^m G_{ur}\|_\infty = \|s^m K(s)S(s)\|_\infty, \end{aligned}$$

where m is the high frequency roll off rate of the controller. This means that a controller with integral action has the following asymptotic properties:

$$K(s) \rightarrow \begin{cases} \frac{k_i}{s}, & s \rightarrow 0, \\ \frac{k_\infty}{s^m}, & s \rightarrow \infty. \end{cases}$$

For a proper, but not strictly proper, controller as, e.g., the PID controller with a first order filter, $m = 0$ and hence, $J_{HF} = J_u$. It is obvious that J_v is a measure of the ability of the loop to handle process disturbances, related to the low frequency range. J_u is a measure of the control activity, related to the mid and high frequency region around, or somewhat above, the closed loop bandwidth. The last criterion J_{HF} is a measure of the robustness to unmodeled dynamics in the high frequency region. Recall the small gain theorem ($\|T\Delta\|_\infty < 1$) and the fact that KS is related to the complementary sensitivity function as $KS = G^{-1}T$.

The stability margin can be expressed by the fourth criterion, the *Generalized Maximum Sensitivity*

$$\begin{aligned} GM_S &= \max(\|S\|_\infty, \alpha \|T\|_\infty, \\ &\quad \gamma \max_\omega W_L(j\omega) |L(j\omega) - \frac{1}{m A_m}|). \end{aligned} \quad (1)$$

The two constants $\alpha = M_S/M_T$ and $\gamma = 0.5m_{A_m}M_S$, and ω_{π_L} is the frequency where the loop has a phase lag of 180° . The weight function $W_L(j\omega) = 0$ for $\omega < \omega_{\pi_L}$ and $W_L(j\omega) = 1$ for $\omega \geq \omega_{\pi_L}$. The stability margins M_S and M_T are guaranteed upper limits for the sensitivity functions S and T respectively, and m_{A_m} is a guaranteed lower limit of the amplitude margin. Default limit values throughout this paper are $M_S = 1.7$, $M_T = 1.3$, and $m_{A_m} = 3.0$. By fixing the value of GM_S , an upper bound on $\|S\|_\infty$, $\|T\|_\infty$, and a lower bound on the amplitude margin, is obtained. Decreasing GM_S will imply an increase of the robustness of the system. To find the optimal tuning of a controller, the following problem has to be solved:

$$\min_\rho J_v(\rho), \quad GM_S \leq C_1, \quad J_u \leq C_2, \quad (J_{HF} \leq C_3). \quad (2)$$

The default value of C_1 , is in this paper 1.7, while C_2 may be given different values. The last restriction is relevant only for strictly proper controllers. To solve the optimization problem (2), the Matlab optimization toolbox [8], was used.

3 PID Controllers

In this section differently designed PID controllers are treated, tuning formulas are introduced, and different properties of the closed loop system are compared for the various controllers.

3.1 The PID controller with a first order low pass filter

Traditionally, the well known PID controller with a first order filter on the derivative part is formulated with the parameters controller gain K , integral time T_i , derivative time T_d , and filter constant T_f

$$K_{PID}(s) = K \left(1 + \frac{1}{sT_i} + \frac{sT_d}{1 + sT_f} \right).$$

As has been shown in several papers, see e.g. [4], the optimal PID controller of this type, at least for stable plants, has a complex pair of zeros. This corresponds to the ratio T_i/T_d being well below 4, and often in the vicinity of 2.5. This fact makes it natural to formulate the transfer function of the controller as

$$K_{PID}(s) = k_i \frac{1 + 2\zeta\tau s + (\tau s)^2}{s(1 + s\frac{\tau}{\beta})}, \quad (3)$$

where the damping ratio ζ and the inverse of the natural frequency τ for the zeros are among the design parameters. The ratio of the breaking point of the denominator to that of the numerator is expressed by β . The fourth parameter is the integral gain k_i . The high frequency gain becomes $K_{PID}(\infty) = k_\infty = k_i\tau\beta$. From now on, focus is set on control of stable, non oscillating plants (plants with real poles in the left half plane). The difficulty to control such plants can be expressed by the character κ , a number in the interval $[0, 1]$, introduced in [2] as

$$\kappa = \frac{|G(j\omega_{\pi_G})|}{|G(0)|}.$$

ω_{π_G} is the frequency where the plant has a phase lag of 180° , sometimes called the *phase cross over frequency*, and $|G(0)|$ is the low frequency gain. Note that a higher value of κ implies that the plant is more difficult to control. Stable, non oscillating plants may be divided into two classes; those with $\kappa \geq 0.1$ and those with $\kappa < 0.1$. The recommended tuning rules will be slightly different

for the two classes. For all illustrations throughout the paper, the following plant model (with $\kappa = 0.136$, and $\omega_\pi = 2.53$) has been used:

$$G(s) = e^{-0.3s}/(1+s)^2,$$

where the time delay has been modeled by a fourth order padé approximation. The value of ζ in the controller has shown to be not very crucial for the properties of a PID controlled system. For an optimized PID controller it is always around 0.8, corresponding to $T_i/T_d \approx 2.5$. When ζ is fixed to this value, the properties of the resulting system are very close to those of a corresponding system with an optimized PID controller. The optimal natural frequency depends on the plant, and to less extent on the demanded value of J_u . Based on investigations of a lot of plants, it has been found that, for moderate J_u values, it can well be fixed to $1/\tau = \omega_{\pi_G}(\kappa + 0.37)$. With τ chosen by this relation during the optimization, very little is in fact lost compared to when τ is a free parameter, see Figure 2. The PID controller, optimized with fixed $\zeta = 0.8$ and τ as above will from now on, be denoted $PID_{\zeta\tau}$.

Then two parameters, k_i and β or the PI parameters k_i and k_∞ remain. These can be used by the operator to adjust the controller to his specific situation. The high frequency gain k_∞ is closely related to J_u , and is an expression of the permitted control activity, while the integral gain k_i is related to J_v and thus, is a measure of the low frequency performance (compare with Section 2). Note that $|GS| \rightarrow \omega/k_i$ when $\omega \rightarrow 0$, and $|KS| \rightarrow k_\infty$ when $\omega \rightarrow \infty$.

Figure 2 shows the relations between J_u and J_v , when five differently tuned controllers are compared for the plant. The curve at the top corresponds to the very common situation, when the relations between the time parameters of the traditional PID controller (PID_{tr}) are fixed to $T_i/T_d = 4$ and $T_d/T_f = 10$. It is obvious that this curve is far from the curve at the bottom, corresponding to the PID controller PID_{opt} , optimized with all parameters free. On the contrary the curve for $PID_{\zeta\tau}$ is close to the optimal one. It can also be noticed that the optimal curve tends to be almost horizontal for higher values of J_u . This tendency is the same for all plants of this type. Therefore, it may be asserted that there is an "economic" limit above which it is no use to increase J_u because the benefits in terms of decreased J_v are too small. For the actual plant this limit goes in the vicinity of $10/|G(0)|$.

The following set of tuning rules for stable, non oscillating plants with $\kappa \geq 0.1$, leads to J_u values close to the "economic" limit and to $GM_S \approx 1.7$, and was first suggested in [5]:

$$k_\infty = \frac{1}{|G(0)|}(13 - 20\kappa), \quad \kappa \leq 0.5, \quad (4)$$

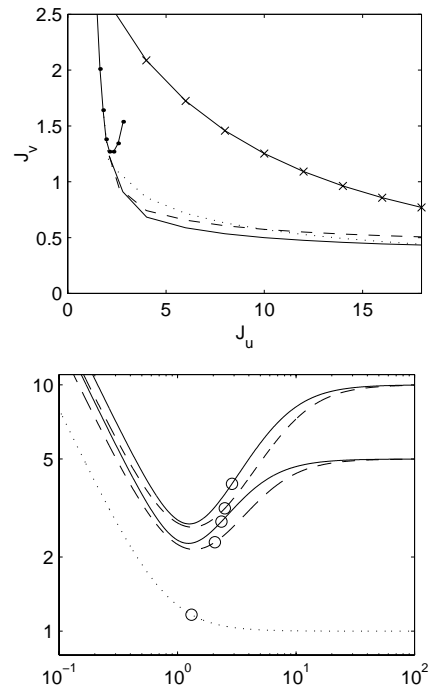


Figure 2: Top: $J_v = f(J_u)$ for PID_{opt} (solid), $PID_{\zeta\tau}$ (dotted), PIPD (dashed), PI (solid with dots) and PID_{tr} (solid with x), Bottom: Gains of PID_{opt} (solid) and PIPD (dashed) for $J_u = 5$ and $J_u = 10$ and for PI (dotted) $J_u = 2.1$. The circles show the closed loop bandwidth for the five cases.

$$k_\infty = \frac{3.0}{|G(0)|}, \quad \kappa > 0.5, \quad (5)$$

$$\zeta = 0.8, \quad (6)$$

$$\frac{1}{\tau} = \omega_{\pi_G}(0.37 + \kappa), \quad (7)$$

$$\beta = \frac{k_\infty}{\tau k_i}, \quad (8)$$

$$k_i = \frac{\omega_{\pi_G}}{|G(0)|}(1.6\kappa^2 - 2.3\kappa + 1.1) \quad (9)$$

An interesting property that is not involved in the optimization process is the closed loop bandwidth. Independent of J_u and κ , it has been found that the ratio ω_b to the gain cross over frequency ω_c , $\omega_b/\omega_c = 2.0 - 2.3$. It has also been found that $\omega_b/\omega_{\pi_G} = 0.5 - 1.2$, with increased ω_b for larger J_u and smaller κ .

For the PI controller, the J_v/J_u curve has a clear minimum, also noted in [10]. This means that higher values of J_u , corresponding to poorer high frequency robustness and higher control costs, will not be rewarded by better low frequency performance. It is also interesting to notice that the PI curve in Figure 2 seems to be a continuation towards low frequencies of the optimal PID curve. The figure also shows that, when $J_u \approx 2$, there is very little to be won by the derivative part of the PID controller. On the other hand, when J_u is al-

lowed to increase a little, that is higher control activity can be accepted, the PID, as could be expected, can offer enhanced low frequency performance with equal stability margins. For the actual plant, the ideal J_u value for a PI controller is $J_u \approx 2$, corresponding to $k_\infty \approx 1$. Note that for this controller, $|KS(\omega)|$ has a peak close to ω_b , resulting in a maximum larger than k_∞ . When $GM_S \approx 1.7$ is demanded, the following tuning rules for PI controllers will lead to J_v values close to the minimum:

$$\begin{aligned} k_\infty &= \frac{1}{|G(0)|}(1.18 - 1.72\kappa), \quad \kappa \leq 0.5, \\ k_\infty &= \frac{1}{|G(0)|}(0.50 - 0.36\kappa), \quad \kappa > 0.5, \\ k_i &= \frac{\omega_\pi}{|G(0)|}(0.33 - 0.17\kappa). \end{aligned}$$

3.2 The PIPD Controller

Sometimes the PID controller has the form of a lead/lag filter with infinite lag-ratio, i.e., a PI controller in series with a PD controller, that is a PIPD controller. In this case, the controller can not have a pair of complex zeros. However, it has been shown that the optimal PIPD controller has a double zero. Therefore, it can be formulated as in (3) with $\zeta = 1.0$, and only three tuning parameters are left to compute. Figure 2 shows that the J_v/J_u relation of the optimal PIPD controller is almost equal to that of $PID_{\zeta\tau}$ in the vicinity of $J_u = 10$, while $PID_{\zeta\tau}$ is the looser at lower frequencies and the winner at higher, though the differences are small.

In Figure 2, the gains $|K(j\omega)|$ of five controllers are compared. In the PID_{opt} , and the PIPD cases, the proposed value $J_u = 10$ is chosen, but also the gains when $J_u = 5$, corresponding to somewhat slower controllers, are shown. It is obvious that a higher value of the integral gain k_i has to be paid by an increased value of k_∞ , corresponding to an increased value of J_u , to keep the demand on $GM_S = 1.7$. The differences between PID_{opt} and PIPD, in terms of k_i are moderate for both J_u values. However, the minima have slightly different characters.

In the same figure, the gain of the optimal PI controller is also shown for $J_u = 2.1$, the value of the minimum of the J_v/J_u relation. The closed loop bandwidth in each case is marked by a circle. It is marginally greater in the PID_{opt} case than in the PIPD case for both J_u values. In all these cases ω_b is, however, not far from $\omega_{\pi_G} = 2.5$. For the PI controller ω_b is of course significantly lower, compared to the other controllers.

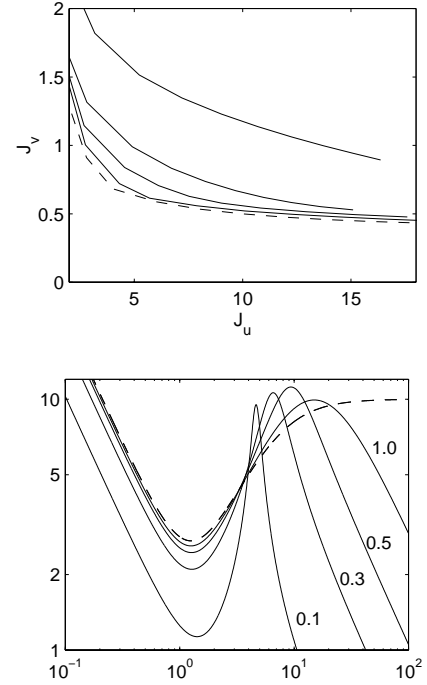


Figure 3: Top: $J_v = f(J_u)$ for PID_{ro} (solid) when ζ_f is (from the top to the bottom) 0.1, 0.3, 0.5 and 1.0 and for PID_{opt} (dashed), Bottom: Gains of PID_{ro} with different values of ζ_f (solid) and of PID_{opt} (dashed), $J_u = 10$.

4 PID Controllers with roll off

When there is a demand on more high frequency roll off in the loop than can be offered by the plant, the PID controller can be augmented by a lowpass filter of arbitrary order. If the first order filter is exchanged by a filter of second order, the controller can be formulated as

$$K_{PID_{ro}}(s) = k_i \frac{1 + 2\zeta\tau s + (\tau s)^2}{s(1 + 2\zeta_f \frac{\tau}{\beta} s + (\frac{\tau}{\beta} s)^2)}.$$

The J_v/J_u relation of PID_{ro} is shown in Figure 3, for different values of the damping ratio of the complex poles ζ_f . It is obvious that if ζ_f is too small, this strictly proper PID controller will show a considerably less favorable relation than PID_{opt} . However, the increase of the filter order does not influence very much on the low frequency performance when $\zeta_f \geq 0.5$. Note that the maximum of the control sensitivity function $KS(J_u)$, is equal for the five systems in the figure, even if the maximum of the controller gains are somewhat different. Note that $|KS| \approx |K|$ for higher frequencies since $|S| \approx 1$ in this region. The high frequency criterion J_{HF} goes from 298 to 44 when ζ_f decreases from 1.0 to 0.1, with $J_{HF} = 122$ for $\zeta_f = 0.5$. Hence, even here there is a trade off between reduced performance (J_v) and improved robustness in terms of roll off (J_{HF}).

The tuning rules for PID_{ro} may then be the same as for $PID_{\zeta\tau}$, i.e. (4)-(9), with the addition

$$\zeta_f = 0.5.$$

This means that also for this controller, at least three of the five tuning parameters can be computed by very simple rules, and only the two PI parameters k_i , and k_∞ are left for the operator.

5 The \mathcal{H}_∞ Controller

An interesting challenge is to investigate how a PID controller can assert itself against a controller designed with the \mathcal{H}_∞ loop shaping procedure introduced by McFarlane and Glover [9]. The main idea in this method is to augment the plant with a weight function W , and modify this until a desired open loop shape is obtained for the augmented plant $\overline{G} = WG$. The controller \overline{K} thus found, is then combined with the weight function to give the final controller $K_{\mathcal{H}_\infty} = W\overline{K}$. It is quite common to use a PI filter as the weight function. However, when greater control activity can be accepted, a PID filter (3), may be an interesting alternative.

In the design of the \mathcal{H}_∞ controllers, the constrained optimization (2) was performed in two steps. An outer

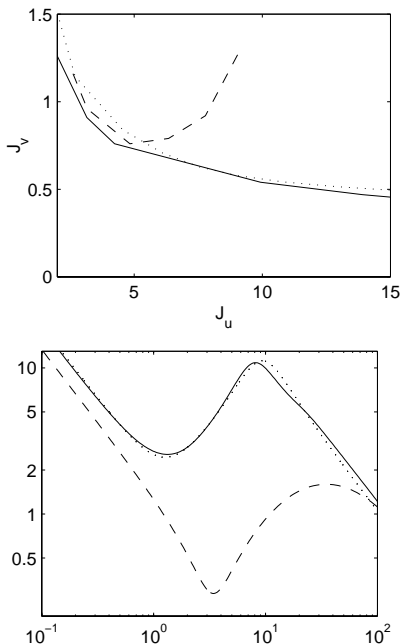


Figure 4: Top: $J_v = f(J_u)$ for a PID_{ro} controller with $\zeta_f = 0.5$ (dotted) together with two \mathcal{H}_∞ controllers with a PI weight (dashed) and a PID weight (3) (solid). Bottom: Gains of PID_{ro} , $J_u = 10$ (dotted), $\mathcal{H}_\infty PID$, $J_u = 10$ (solid) and $\mathcal{H}_\infty PI$, $J_u = 5$ (dashed).

loop for tuning the weight function subject to the constraints, and an inner loop for the standard \mathcal{H}_∞ optimization. Figure 4 shows the J_v/J_u relation for three controllers, all of them optimized with constrained J_u and $GM_S = 1.7$. The PI weighted \mathcal{H}_∞ controller $\mathcal{H}_\infty PI$, tuned by k_i and k_∞ , shows the same characteristic minimum as the PI controller. Here the optimal J_u value is around 5. The result for the PID weighted \mathcal{H}_∞ controller $\mathcal{H}_\infty PID$, is similar to that of the PID_{ro} , except for very low values of J_u . Here, the \mathcal{H}_∞ controllers are optimized with all weighting filter parameters free, while PID_{ro} is optimized with fixed $\zeta = 0.8$ and $\zeta_f = 0.5$. However, the two sets of controllers have the same GM_S , and J_{HF} values. In fact, the value of J_{HF} obtained for PID_{ro} has been introduced as a constraint in the optimization of the weighting filter in the \mathcal{H}_∞ design.

It is important to notice that the \mathcal{H}_∞ loop shaping procedure is only a tool to minimize J_v and to obtain a fair comparison between different controllers. Included in the description by McFarlane and Glover, is a scaling factor α . The theoretically optimal robustness corresponds to $\alpha = 1$, but for practical reasons α slightly larger than one, e.g. 1.05, is generally chosen. When the \mathcal{H}_∞ optimizations were carried through, it was found that GM_S did not reach the limit 1.7, when the restriction on J_{HF} was added (to achieve a fair comparison to PID_{ro}). Though, by allowing somewhat larger α , this could be taken care of, which not at all is obvious. In fact, small variations in α resulted in dramatically different optimal solutions with respect to low frequency performance. This implies that α is an important and sensitive tuning parameter for \mathcal{H}_∞ controller design.

The controller gains of $\mathcal{H}_\infty PID$, $\mathcal{H}_\infty PI$ and PID_{ro} are compared in Figure 4. For all of them $GM_S = 1.7$. $J_u = 10$ and $J_{HF} = 122$ for both $\mathcal{H}_\infty PID$ and PID_{ro} , whereas for $\mathcal{H}_\infty PI$, the optimal value $J_u = 5$ has been chosen. As can be seen, the curves for PID_{ro} and $\mathcal{H}_\infty PID$ almost coincide while $\mathcal{H}_\infty PI$ has a quite different behavior. Note that the order of the PID_{ro} controller is 3, whereas the \mathcal{H}_∞ controller is of order 10. Finally, Figure 5 show some simulations. The reference, and process disturbance step responses, and the control signals after reference steps are shown for five systems, with the same plant and different controllers.

Just as could be expected, both disturbance and reference step responses are very similar for the two PID controllers, PID_{ro} and PID_{opt} . However, it is interesting to see how well even the responses from $\mathcal{H}_\infty PID$ agree with those two. The $\mathcal{H}_\infty PI$ on the contrary, has a somewhat slower reference response but less overshoot and a little greater maximum error regarding disturbance rejection. The PI controller is the looser in both respects. The control activity, on the other hand, is

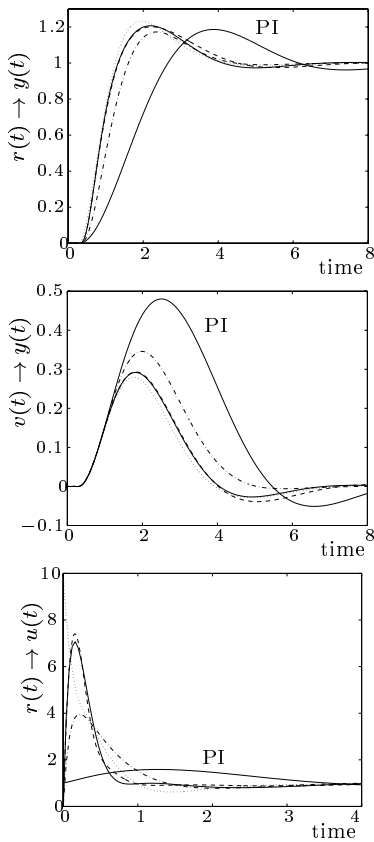


Figure 5: Step responses from five systems with the following controllers $\mathcal{H}_{\infty PID}$ (solid), $\mathcal{H}_{\infty PI}$ (dashdotted), PID_{ro} (dashed), PID_{opt} (dotted) and PI (solid). $J_u = 10$ except for $\mathcal{H}_{\infty PI}$ with $J_u = 5$ and PI with $J_u = 2$.

greatest for PID_{opt} , equal for PID_{ro} and $\mathcal{H}_{\infty PID}$ and a little smaller for $\mathcal{H}_{\infty PI}$. For PI it is very small but also very sluggish. With no restrictions on J_{HF} , $\mathcal{H}_{\infty PID}$ can do marginally better with respect to J_v ($\approx 2\%$), but with much larger J_{HF} .

6 Conclusions

A set of PID related controllers have been compared in this paper. It has been shown that all of them, from the simple PI controller to the modern \mathcal{H}_{∞} controller, can be tuned by the same parameters. The integral gain and the high frequency gain of the controller have been chosen as the main tuning parameters, since both of them have a clear physical meaning to an operator. Furthermore, it has been pointed out that a PID controller with a first order filter, with two parameters fixed to suitable values and tuned only by the two parameters named above, can give almost the same properties as an optimally tuned controller of the same type. It is also shown that a PID controller with a second order filter (order 3), resulting in controller roll off, is well

competitive to a PID weighted \mathcal{H}_{∞} controller (order 10) and superior to the quite common PI weighted \mathcal{H}_{∞} controller.

References

- [1] K. J. Åström and T. Häggglund. *PID Controllers: Theory, Design and Tuning*. Instrument Society of America, 1995.
- [2] C. C. Hang, K. J. Åström, and W. K. Ho. Refinements of the Ziegler Nichols tuning formula. 138(2), 1991.
- [3] W.K. Ho, C.C. Hang, and L. S. Cao. Tuning of pi controllers based on gain and phase margin specifications. In *Preprints IEEE Int Symposium on Industrial Electronics*, P.R. China, 1992.
- [4] B. Kristiansson and B. Lennartson. Optimal PID controllers including roll off and Smith predictor structure. *IFAC World Congress*, F:297–302, 1999.
- [5] B. Kristiansson and B. Lennartson. *Near optimal tuning rules for PI and PID controllers*. IFAC workshop on digital control: PID'00, Terrassa, Spain, April 2000.
- [6] J. Langer and I. D. Landau. Combined pole placement/sensitivity function shaping method using convex optimization criteria. *Automatica*, 35(6):1111–1120, 1999.
- [7] R.J. Mantz and E.J. Tacconi. Complementary rules to ziegler and nichols rules for regulating and tracking controller. *Int. J. Control*, Vol 49:1465–1471, 1989.
- [8] MathWorks. *Matlab Optimization Toolbox, User's Guide*. Natick, MA, USA, 1996.
- [9] D. McFarlane and K. Glover. A loop shaping design procedure using \mathcal{H}_{∞} synthesis. *IEEE Trans. Autom. Contr.*, 37(6):759–769, 1992.
- [10] H. Panagopoulos. PID control, design, extension, application. Ph.d. thesis, Dept of Automatic Control, Lund Institute of Technology, Lund Sweden, 2000.
- [11] T. S. Schei. Automatic tuning of PID controllers based on transfer function estimation. *Automatica*, 30(12), 1994.
- [12] P. van Overschee and B. D. Moor. Optimal PID control of chemical batch reactor. In *Proc. Eur. Control Conference*, Karlsruhe, Germany, 1999.
- [13] J. G. Ziegler and N. B. Nichols. Optimum settings for automatic controllers. *Trans. ASME*, 64:759–768, 1942.