Robust Synchronizing Motion Control of Twin–Servo Systems Based on Network Modeling

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Abstract

In this paper, a robust control method for synchronizing motions of a twin–servo system is proposed. Using the correspondence between the mathematical modeling and the circuit representation, a network representation of twin–servo system is proposed. And a stabilizing control input is designed based on robust internal–loop compensator for the separate system in the presence of uncertainty and disturbance. Skew motion compensating control input is also designed to maintain the synchronizing motion during high speed motion. The stability of the whole closed–loop system is proved based on passivity theory. Through experiments using a semiconductor chip mounting device, the performance of the proposed method is evaluated.

1 Introduction

High–accuracy positioning systems emphasizing high performance and high productivity have introduced twin–servo mechanism in many current application areas such as semiconductor chip mounting devices. Twin–servo mechanism is used to increase the payload capacity and speed of high precision system. This consists of two driving motors controlled independently for one reference input. Difficulties are mainly due to the fact that the system of interest requires wide range and high speed motions under significant nonlinear characteristics. Moreover, accurate description of the nonlinear effect is not available because uncertainties always exist and cannot be neglected. Consequently, the control algorithm for twin–servo with high performance must address both synchronizing motion control performance under the dynamic unbalance of twin–servo system and robustness issue under the nonlinearities and uncertainties.

In this paper, we focus on the network modeling of twin–servo system and propose a robust synchronizing motion controller which consists of separate feedback controller and skew motion compensator to meet performance specifications and cancel out the skew motion of two driving systems. Model reference trajectory tracking controller based on robust internal–loop compensator(RIC) which has 2–DOF control structure is proposed as the separate feedback controller. To compensate the skew motion, a symmetric type skew motion compensating controller is designed. The stability of the whole closed–loop system is analyzed based on passivity based approach.

In the next section, the network modeling of twin–servo system is introduced. In Section 3, a separate controller for each driving system is proposed based on RIC, and the disturbance attenuation property and the performance of the separate closed–loop system with RIC are analyzed. In Section 4, the stability analysis of the whole closed–loop system is represented. Experimental results are shown in Section 5, and conclusion follows.

2 Network Modeling of Twin–Servo System

A twin–servo system consists of the primary and secondary servo system with control loop closed separately around them as shown in Fig. 1. The dynamic behaviors of two separate system are functions of each other. The equations of motion for this system are expressed as

$$\begin{aligned} m_p\ddot{y}_p + b_p\dot{y}_p &= \tau_p + f_p \\ m_s\ddot{y}_s + b_s\dot{y}_s &= \tau_s + f_s \end{aligned} \tag{1}$$

where $y_{p,s}$ is the output of interest, $m_{p,s}$ is the mass, and $b_{p,s}$ is the damping coefficient. $f_{p,s}$ is the force that separate feedback controller applies to the primary and secondary motor. Driving forces for synchronizing motion is represented by $\tau_{p,s}$.

It is assumed that the dynamics of the separate feedback controllers can be approximately represented as a simple spring–damper system:

$$\begin{aligned} f_{pf} - f_p &= b_{pc}\dot{y}_p + k_{pc}y_p \\ f_{sf} - f_s &= b_{sc}\dot{y}_s + k_{sc}y_s \end{aligned} \tag{2}$$

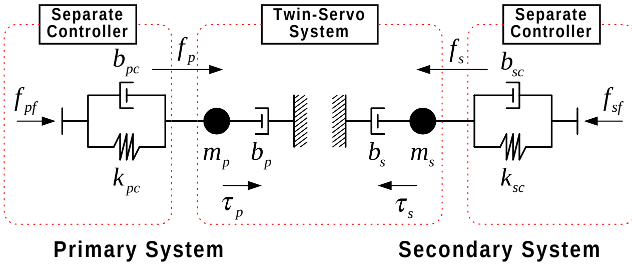


Figure 1: Twin–servo motion control system

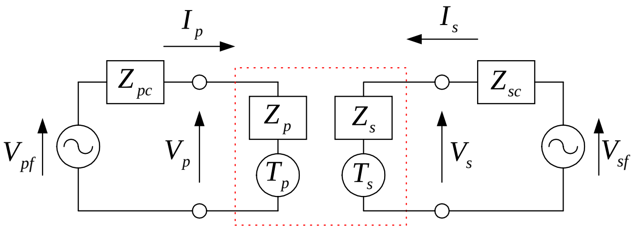


Figure 2: Circuit representation of twin–servo system

where $b_{pc,sc}$ and $k_{pc,sc}$ are the damping coefficient and stiffness of the separate feedback controllers, respectively. $f_{pf,sf}$ denotes primary and secondary feedforward command determined by the desired trajectory.

Synchronizing motion controller is used to synchronize the motion of two motors by cancelling out the skew motion. Hence, this has to recognize skew motion in real time and compensate dynamic difference during high–speed motion. The separate robust feedback controllers compensate different dynamic characteristics of two motors and the skew motion compensating controller is appended to this.

Consider the following control schemes for primary and secondary motor as general expressions which determine compensating forces to synchronize motions:

$$\begin{aligned} \tau_p &= K_{pp}\, y_p - K_{ps}\, y_s \\ \tau_s &= K_{sp}\, y_p - K_{ss}\, y_s \end{aligned} \tag{3}$$

where K_{pp} and K_{ps} are the feedback controllers of the primary motor, whereas K_{sp} and K_{ss} are gains of the secondary motor, respectively. In (3), it is assumed that time delay due to the data transmission between two systems is negligible.

Now, let us consider a two–terminal–pair network which is connected to a power source at each terminal pair as shown in Fig. 2. By regarding the power source as a reference command and two–terminal–pair network as a twin–servo system, the whole system can be replaced by the electrical circuit in Fig. 2. The correspondences between the mathematical modeling and the circuit rep-

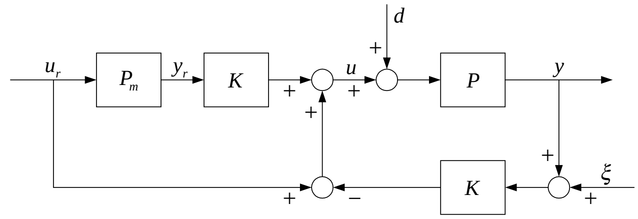


Figure 3: Robust internal–loop compensator structure

resentation in this figure are given by

$$\begin{aligned} \dot{y}_p,\, \dot{y}_s &\longleftrightarrow I_p,\, I_s \\ f_{pf},\, f_{sf} &\longleftrightarrow V_{pf},\, V_{sf} \\ f_p,\, f_s &\longleftrightarrow V_p,\, V_s \\ \tau_p,\, \tau_s &\longleftrightarrow T_p,\, T_s. \end{aligned}$$

(1) and (3) can be transformed from time domain into s domain:

$$\begin{aligned} T_p + V_p &= (m_p s + b_p) I_p \triangleq Z_p I_p \\ T_s + V_s &= (m_s s + b_s) I_s \triangleq Z_s I_s \end{aligned} \tag{4}$$

$$\begin{aligned} T_p &= K_{pp}\frac{1}{s} I_p - K_{ps}\frac{1}{s} I_s \triangleq P_p I_p - R_p I_s \\ T_s &= K_{sp}\frac{1}{s} I_p - K_{ss}\frac{1}{s} I_s \triangleq P_s I_p - R_s I_s. \end{aligned} \tag{5}$$

By eliminating T_p and T_s from (4) and (5), the impedance matrix is obtained as

$$\boldsymbol{Z} = \begin{bmatrix} Z_p - P_p & R_p \\ -P_s & Z_s + R_s \end{bmatrix}. \tag{6}$$

3 Separate Feedback Control

In this section, a robust trajectory tracking controller based on robust internal–loop compensator(RIC) is presented for the separate controller of twin–servo system. Disturbance attenuation characteristics of the proposed controller is shown and the performance is analyzed.

3.1 Robust Internal–Loop Compensator

Without loss of generality, the proposed control method is presented for systems with a single–input, single–output(SISO), which allow us to develop intuition about the basic aspects of the proposed robust controller design. Fig. 3 shows the control structure in which we will derive the control input based on Lyapunov redesign using RIC [1, 2]. The system we are dealing with is represented by the plant P and its output signal y. The function u_r represents a reference control input signal, u represents a control input signal, d represents a disturbance signal, and ξ represents measurement noise. P_m and K represent dynamic models which are to be designed. We call these as a reference or nominal model and the controller of RIC. Laplace

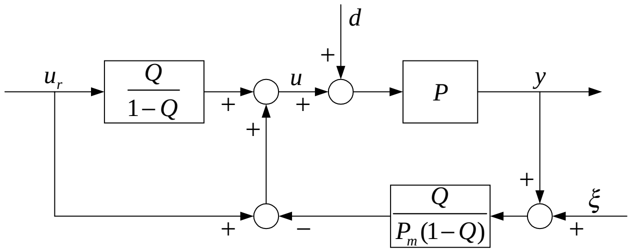


Figure 4: Equivalent structure of RIC using Q function

variable s is dropped for mathematical clarity. The difference between plant output and reference model output is defined as the model following error $e_r = y_r - y$, where y_r is the output of reference model P_m. Then, the RIC based control input has the form of

$$u = u_r + K e_r \tag{7}$$

where the second term on the right–hand side is given by Lyapunov redesign [3]. From the block diagram in Fig. 3, e_r can be expressed as

$$e_r = S\left[(P_m - P)u_r - Pd\right] + T\xi \tag{8}$$

where $S = \frac{1}{1+PK}$ and $T = \frac{PK}{1+PK}$ are given as the sensitivity and complementary sensitivity functions of the typical feedback system, respectively.

3.2 Disturbance Attenuation Characteristics

As shown above, in order to minimize e_r, we need to design the controller K so that S and T have optimal values. However, since we can not know the exact mathematical model of real plant P, it is very difficult to design K directly using the functions of S and T. Therefore, consider an imaginary transfer function Q, which controls P_m using feedback controller K. Hence Q is expressed as

$$Q = \frac{P_m K}{1 + P_m K}. \tag{9}$$

Recalculating this equation for K, it has the form of

$$K = \frac{Q}{P_m(1-Q)}. \tag{10}$$

If we substitute K into Fig. 3, we obtain Fig. 4. This figure can be also transformed equivalently to the well known structure of disturbance observer(DOB) [4]. This means that if we select K as (10), the RIC becomes DOB and the characteristics is the same.

3.3 Performance Tuning

Now, let us analyze the performance of the control system with RIC using input–output relationship. If the tracking error is defined as $e = r - y$, from Fig. 5 and Fig. 6, the relationship among e, r, and d_{eq} can be expressed as

$$e = \frac{1}{1+P_m C}\left(r - \frac{P_m}{1+P_m K}\, d_{eq}\right). \tag{11}$$

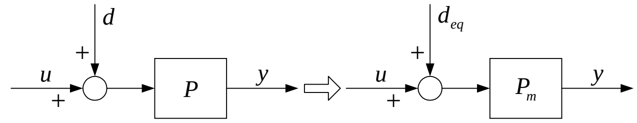


Figure 5: Reconstruction of plant

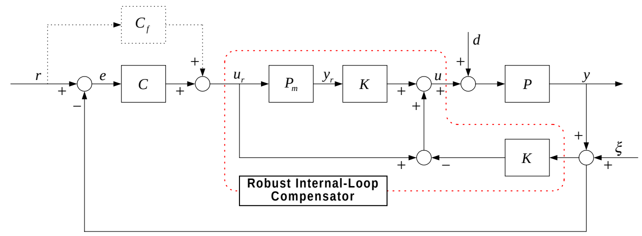


Figure 6: Robust control structure based on RIC

But since e is affected by r, it is difficult to predict its performance. Hence, it is required to eliminate the effect of r on e by using feedforward compensation.

Consider the RIC based control system with feedforward compensation shown in Fig. 6. By using Fig. 5, the error can be expressed in term of equivalent disturbance as

$$e = \frac{1}{1+P_m C}\left((1 - P_m C_f)r - \frac{P_m}{1+P_m K}\, d_{eq}\right). \tag{12}$$

If C_f is chosen as the inverse of the reference model P_m, that is $C_f = 1/P_m$, then (12) is arranged as

$$e = \frac{1}{1+P_m C}\left(-\frac{P_m}{1+P_m K}\, d_{eq}\right). \tag{13}$$

Thanks to the feedforward compensation, (13) is not a function of the reference input r differently from (12). Hence, the inequality about the norm of error can be obtained as

$$|e| \le |W| \times \frac{1}{|1 + P_m K|} \tag{14}$$

where $W = \frac{P_m d_{eq}}{1+P_m C}$. Note here that W is the function that is only related to the plant P_m and outer–loop controller C.

3.4 RIC Based Motion Control

Consider the equation of motion for a high–accuracy positioning system:

$$m\ddot{y} + b\dot{y} + F_r(\dot{y}) - d_{ex} = f, \tag{15}$$

where f is the feedback control input to follow the desired trajectory, $F_r(\dot{y})$ is the friction term including stiction and Coulomb friction, and d_{ex} is the uncertain external disturbance whose magnitude is bounded. Tracking error is defined as $e = y_d - y$, where y_d is a desired trajectory.

Now, in order to design a robust motion controller in the RIC framework, consider the following reference model for (15):

$$m_m \ddot{y} + b_m \dot{y} = f \tag{16}$$

where m_m and b_m are the reference values of m and b, respectively. Hence, (15) can be rewritten in terms of m_m and b_m

$$m_m \ddot{y} + b_m \dot{y} = f + d_{eq} \tag{17}$$

where $d_{eq} = (m_m - m)\,\ddot{y} + (b_m - b)\,\dot{y} - F_r(\dot{y}) + d_{ex}$. And the reference control input which can stabilize the reference model given by (16) can be chosen as

$$f_r = m_m \ddot{y}_r + b_m \dot{y}_r. \tag{18}$$

A noteworthy feature of this equation is that the reference input is used to generate internal model state. That is, since m_m and b_m are the parameters which are to be designed, y_r becomes the state of implicit internal model of (18). And then, choose the reference state variable as follows:

$$y_r = y_d + \Lambda \int_0^t e \; d\tau \tag{19}$$

where Λ is an appropriate gain. Hence the model following error is obtained as

$$e_r = e + \Lambda \int_0^t e \; d\tau. \tag{20}$$

Therefore, from (7), the RIC based robust motion controller is formulated as

$$f = m_m \ddot{y}_r + b_m \dot{y}_r + K e_r. \tag{21}$$

From (18), the reference model P_m is obtained as

$$P_m = \frac{1}{m_m s^2 + b_m s}. \tag{22}$$

Hence, various Q can be designed by K using (9). For example, if the controller is chosen as

$$K = (m_m s + b_m) D \tag{23}$$

then Q function has the form of

$$Q = \frac{D}{s + D}. \tag{24}$$

Therefore, it can be roughly said that the disturbances can be attenuated below the cutoff frequency($\omega_c = D\,\mathrm{rad/s}$) of (24). And from (21), the external–loop controller and the feedforward compensator are given by

$$C = (m_m s + b_m)\,\Lambda, \qquad C_f = m_m s^2 + b_m s. \tag{25}$$

Since the feedforward compensator satisfies the condition of $C_f = 1/P_m$ in the above equations, the following inequality is obtained from (14):

$$|e| \le |W| \times \frac{1}{\left|1 + \frac{D}{j\omega}\right|} \tag{26}$$

Therefore, if the gain Λ of C is fixed and the magnitude of D of (23) is increased N times, then it can be roughly said that $|e|$ is reduced by the factor of $\left|\left(1 + \frac{D}{j\omega}\right) / \left(1 + \frac{N \times D}{j\omega}\right)\right|$. Specifically, when D is large enough or the system is operated in low frequency range, we can predict that if D is increased by N times, the error will be reduced to its $1/N$, approximately.

4 Stability Analysis

In this section, we discuss the stability of proposed synchronizing controller including separate primary and secondary feedback controller. Synchronizing motion controller makes the system reciprocal so that the necessary and sufficient condition can be calculated analytically.

4.1 Passivity Based Approach

The motion of primary motor is affected by two control inputs: f_p, the separate control command of the primary and τ_p, the skew motion compensating command of twin–servo mechanism. Control inputs of secondary motor are similar to those of the primary. Since the primary and secondary motor are interconnected in a feedback loop, the dynamics of the whole closed–loop system should be considered.

From electric circuit representation of Section 2, the twin–servo system can be expressed as

$$\boldsymbol{b} = \boldsymbol{S}\boldsymbol{a} \tag{27}$$

where the matrix $\boldsymbol{S}$ is called scattering matrix, $\boldsymbol{a} = \frac{\boldsymbol{V}+\boldsymbol{I}}{2}$ and $\boldsymbol{b} = \frac{\boldsymbol{V}-\boldsymbol{I}}{2}$ are input and output wave, where $\boldsymbol{V} = \begin{bmatrix} V_p & V_s \end{bmatrix}^T$ and $\boldsymbol{I} = \begin{bmatrix} I_p & I_s \end{bmatrix}^T$. The scattering matrix $\boldsymbol{S}$ of the system is given by

$$\begin{aligned} \boldsymbol{S} = & \frac{1}{D + z_{11} + z_{22} + 1} \\ & \times \begin{bmatrix} D + z_{11} - z_{22} - 1 & 2z_{12} \\ 2z_{21} & D - z_{11} + z_{22} - 1 \end{bmatrix} \end{aligned} \tag{28}$$

where $D = |\boldsymbol{Z}|$. The system is passive if the following inequality is satisfied,

$$\|\boldsymbol{S}\|_\infty \le 1. \tag{29}$$

Therefore, if the system is reciprocal, that is, $\boldsymbol{S}$ is symmetric, we can analyze the stability of the system using (29) [5].

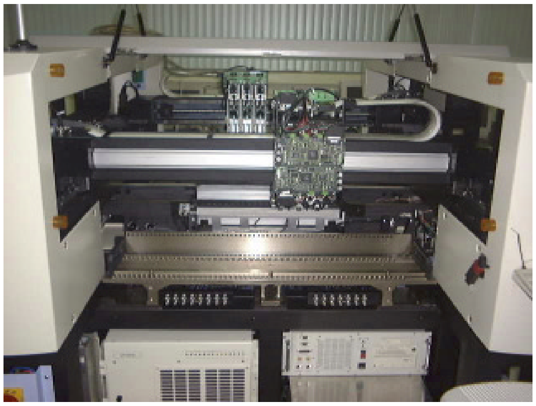


Figure 7: Twin–servo high–accuracy positioning system

4.2 Stability Analysis

In this paper, skew motion compensating scheme is chosen as a symmetric type PD control by which one motor follows the position of the other. Therefore the control algorithm is expressed as

$$\begin{aligned} \tau_p &= K_p(y_s - y_p) + K_d(\dot{y}_s - \dot{y}_p) \\ \tau_s &= K_p(y_p - y_s) + K_d(\dot{y}_p - \dot{y}_s) \end{aligned} \tag{30}$$

where K_p and K_d are the proportional and derivative gains, respectively. From (24), the dynamic characteristic can be assigned so that the primary and the secondary system have equivalent parameters in the low frequency range. Hence, we assume that the dynamic equations of the two systems are represented as (22). The scattering matrix is symmetric when the system is reciprocal, so that it is easier to analyze stability. Substituting the parameter of (30) into (28), we get

$$\begin{aligned} \boldsymbol{S} = &\frac{1}{(1+\alpha)(1+\alpha+2\beta)} \\ &\times \begin{bmatrix} \alpha(\alpha+2\beta)-1 & -2\beta \\ -2\beta & \alpha(\alpha+2\beta)-1 \end{bmatrix} \end{aligned} \tag{31}$$

where $\alpha = m_m s + b_m$ and $\beta = \frac{k_p}{s} + k_d$. Therefore, the singular values of $\boldsymbol{S}$ are given by

$$\sigma_1 = \frac{|\alpha - 1|}{|\alpha + 1|} \le 1, \qquad \sigma_2 = \frac{|\alpha + 2\beta - 1|}{|\alpha + 2\beta + 1|} \le 1. \tag{32}$$

Both of them never violate the inequality (29). Therefore, the stability of twin–servo system when the proposed control algorithm is applied has been guaranteed.

5 Experimental Results

The system we are dealing with in this paper is the high–accuracy XY positioning system used as the semiconductor chip mounting devices, where y–axis is twin–servo system. Fig. 7 shows the experimental setup. The proposed separate feedback controller in (21) is used to stabilize the whole system and track the desired position accurately:

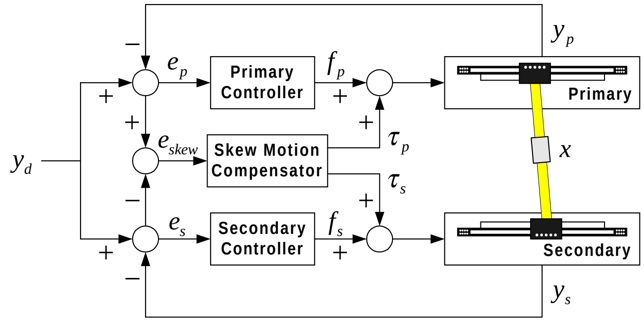


Figure 8: Robust synchronizing motion control structure

$$\begin{aligned} f^x &= m_m^x \ddot{x}_r + b_m^x \dot{x}_r + K^x e_r^x \\ f^y &= m_m^y \ddot{y}_r + b_m^y \dot{y}_r + K^y e_r^y. \end{aligned} \tag{33}$$

For x–axis, m_m^x is 0.075, b_m^x is 0.4, and Λ^x is 250. For y–axis, m_m^y is 0.04, b_m^y is 0.3, and Λ^y is 200. And RIC controller K is chosen as (23):

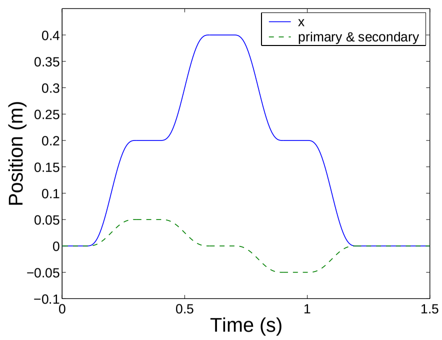
$$\begin{aligned} K^x &= (m_m^x s + b_m^x) D^x \\ K^y &= (m_m^y s + b_m^y) D^y \end{aligned} \tag{34}$$

where D^x is 400 and D^y is 250. The skew motion compensator for y–axis is selected as a symmetric type PD controller in (30):

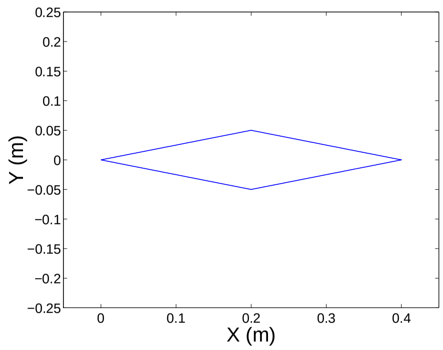
$$\begin{aligned} \tau_p &= K_p\, e_{skew} + K_d\, \dot{e}_{skew} \\ \tau_s &= -K_p\, e_{skew} - K_d\, \dot{e}_{skew} \end{aligned} \tag{35}$$

where the skew error $e_{skew} = y_s - y_p$ and the gains are selected as $K_p = 1500$ and $K_d = 10$. Fig. 8 shows the robust synchronizing motion control structure for y–axis. The 5th order polynomial function is used to specify the position, velocity, and acceleration at the beginning and end of path. Fig. 9 shows the desired trajectory graph. The control frequency is set to 1000 Hz, and the positions are measured by rotary encoder attached in the motors, whose resolutions are 0.5 μm and 0.125 μ at the rectilinear motion for x–axis and y–axis, respectively. The velocity is simply obtained through the backward differentiation of position signal.

From (33), the reference model of RIC is given by (22) and Q is obtained as (24). Fig. 10 shows the experimental results. Fig. 10 (a), (b), and (c), respectively show the tracking errors without skew compensation, the tracking errors with skew compensation, and the skew errors. As can be seen here, the tracking errors show good performance and the skew error is halved if the proposed skew compensation is applied. From the experimental results, we can conclude that our proposed control schemes successfully maintain the stability of overall system and can meet the performance specifications for synchronized motions.



(a) Desired trajectory



(b) Trajectory in Cartesian plane

Figure 9: Desired trajectory

6 Conclusions

We proposed a modeling and network representation of twin–servo system including robust synchronizing motion control algorithm which consists of separate feedback controller and skew motion compensator. Model reference tracking controller based on RIC was proposed as the separate feedback controller and symmetric type PD controller which makes the system reciprocal was designed as the skew motion compensator. The stability analysis of the proposed method was shown based on passivity based approach. The effectiveness of the proposed algorithm was verified through trajectory tracking experiment using a semiconductor chip mounting device and the results showed excellent performance under various uncertainties and disturbances for twin–servo XY system.

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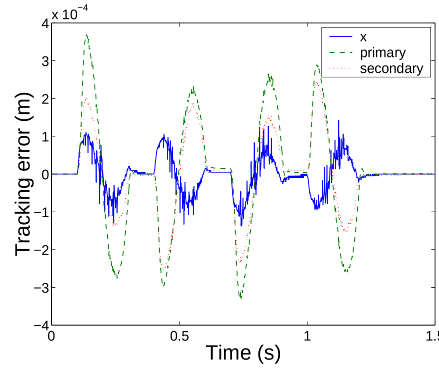
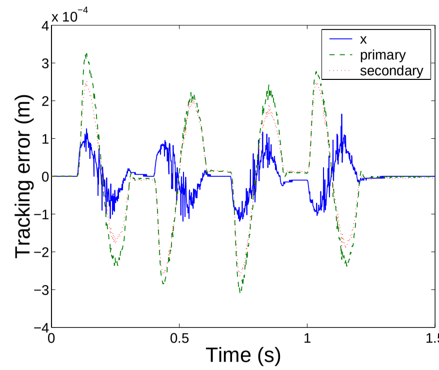
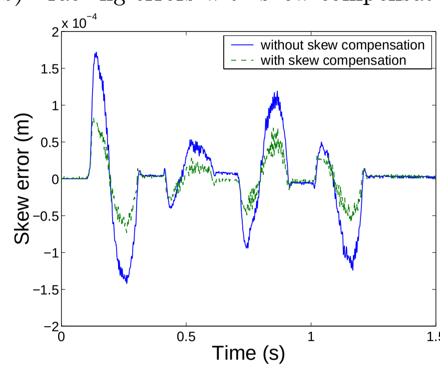


(a) Tracking errors without skew compensation



(b) Tracking errors with skew compensation



(c) Skew error

Figure 10: Experimental results of twin–servo XY system

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