A sequential Monte Carlo filtering approach to fault detection and isolation in nonlinear systems

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Abstract

Much of the development in fault detection schemes have relied on the system being Linear and the noise and disturbances being Gaussian. In such cases, optimal filtering ideas based on Kalman filtering is utilised in estimation followed by a residual analysis for which whiteness tests are typically carried out. Linearised approximations have been used in the nonlinear systems case. However, linearisation techniques, being approximate, tend to suffer from poor detection or high false alarm rates. In this paper, we use the sequential Monte Carlo filtering approach where the complete posterior distribution of the estimates are represented through samples or particles as opposed to the mean and covariance of an approximated Gaussian distribution. We compare the fault detection performance with that using the extended Kalman filtering and investigate the isolation performance on a nonlinear system.

1 Introduction

Fault detection and isolation is an increasingly important issue in designing systems with safety and reliability. One of the central difficulties in the construction of such detection schemes are the trade-off between succesful detections and false alarm rates. These in turn depend on the system dynamics, fault type and the detection/estimation schemes employed.

In the past two decades, a large variety of methods have been proposed for solving the fault detection problem [13], [15], [11], [3], [14], [6], [7], [2]. The problem of fault detection can basically be split into two steps: generation of residuals that reflect the fault on the basis of a system model; then residual evaluation or decision making based on these residuals. For the stochastic systems, much of the development in fault detection schemes has relied on the systm being linear and the noise and disturbances being Gaussian. The optimal state estimator in such cases is the *Kalman filter* [1]. State estimation for nonlinear systems with non-Gaussian noise is a more difficult problem and in general, the optimal solution cannot be expressed analytically. Sub-optimal solutions use some form of approximation such as model linearisation in the extended Kalman filter (EKF) [1]. as a residual generator. Although fault detection has been thoroughly studied and numerous fault detection methods have been developed, very few methods, except EKF based residual generation schemes [16], were concerned with nonlinear non-Gaussian case. The main reason is that the approximate state estimation methods cannot effectively be used in such cases.

The use of Monte Carlo algorithms for nonlinear non-Gaussian state estimation was advocated in [10], but not until powerful computers became available did the methods become popular. Recently, the *particle filter* [8], [9], an extension of the ideas in [10], has attracted much attention [4], [5]. This interest stems from the great advantage of particle filter being able to handle any functional nonlinearity and system or measurement noise of any distribution.

In this paper, the particle filter, a sequential Monte Carlo algorithm, is combined with the innovation-based fault detection techniques to develop a fault detection and isolation scheme. The rest of the paper is organised as follows: in section 2, an on-line fault detection method is outlined followed by a description of the sequential Monte Carlo filter in section 3. This filter based detection and isolation schemes are described in sections 4 and 5 respectively. Experimental results from two simulations are provided in sections 6 and 7 with conclusions and further work in section 8.

2 On-line Fault Detection

A variety of fault detection methods have been devised for dynamic systems depending on the availabile knowledge about the system, fault type and assumptions regarding noise and disturbances [11], [6], [2]. Here we assume that the system dynamics is known and given by,

$$\begin{aligned} x_k &= f_{k-1}(x_{k-1}, w_{k-1}) \\ y_k &= h_k(x_k, v_k) \end{aligned} \quad (1)$$

where x is the system state, y the output measurement, w the system disturbance and v the measurement noise. The functions $f_{k-1}(\cdot)$ and $h_k(\cdot)$ can be both nonlinear or linear and assumed known. The noise and disturbance are assumed to be additive and their characteristics known, generally taken to be zero mean Gaussian white noise.

The type of faults of interest here are the failure type where the system parameter values jump to a new value reflected in a change in the function $f(\cdot)$ and /or $h(\cdot)$. Such faults can be detected using the state observer approach or the filtering approach. The idea is to generate estimates of the states and the predicted outputs from these state estimates. The residuals or innovation from the output prediction are used in a measure which changes significantly under a failure type fault. Such a fault detection scheme facilitates on-line application since the state estimates and the predicted outputs can be generated on-line.

One of the popular state estimation methods for nonlinear systems is the extended Kalman filter (EKF) [1] where the states are estimated according to the following equations:

$$\begin{array}{lcl} & & \text{Prediction :} \\ \hat{x}_{k|k-1} & = & f(\hat{x}_{k-1}) \\ P_{k|k-1} & = & \Phi_{k-1}^T P_{k-1} \Phi_{k-1} + W \\ & & \text{Correction :} \\ \hat{x}_k & = & \hat{x}_{k|k-1} + g_k(y_k - h(\hat{x}_{k|k-1})) \\ g_k & = & P_{k|k-1}\psi_k[\psi_k^T P_{k|k-1}\psi_k + V]^{-1} \\ P_k & = & P_{k|k-1} - g_k \psi_k^T P_{k|k-1} \end{array} \quad (2)$$

where W and V are the variance of the disturbance and noise respectively and $\Phi_{k-1} = \frac{\partial f(x_{k-1})}{\partial x_{k-1}}$, $\psi_k = \frac{\partial h(x_k)}{\partial x_k}$.

The predicted output based on the EKF state estimate is given by,

$$\hat{y}_k = h(\hat{x}_{k|k-1}) \quad (3)$$

The residual or the innovation is then,

$$r_k = y_k - \hat{y}_k \quad (4)$$

In the stationary linear Gaussian case, the innovations are essentially zero mean Gaussian with covariance

$$Q_k = [\psi_k^T P_{k|k-1}\psi_k + V] \quad (5)$$

Any failures or changes in system dynamics can therefore be detected by a change in the weighted squared residual (WSR) measure

$$l_k = r_k^T Q_k^{-1} r_k \quad (6)$$

This however can lead to false alarms occurring at a particular instant due to disturbances and noise and a more robust change detection measure is the weighted sum squared residual (WSSR) [15], [16],

$$L_k = \sum_{j=k-\kappa+1}^{k} l_k \quad (7)$$

where κ is the length of the window within which the residual measure is summed. Since in the absence of a fault this measure remains almost at a constant value, a threshold can be defined in such a way that the fault alarm is set at time k when the condition

$$L_k > \epsilon \quad (8)$$

is satisfied, ϵ being the threshold. When using EKF to estimate the states and hence r_k, Q_k (using equations (4) and (5)), these represent approximations. The measure L_k will thus consist of fluctuations which can in turn lead to higher false alarm rates and also to faults not being detected.

3 A Sequential Monte Carlo filter

The Bayesian approach to dynamic state estimation problems involves the construction of the probability density function (PDF) of the current state x_k, given the measurements up to time k. If Z_k is denoted to be the set of measurements up to time k, *ie.*, $Z_k = \{y_1, y_2, \cdots, y_k\}$, then the Bayesian solution would be to calculate the PDF $p(x_k|Z_k)$. This PDF will encapsulate all the information about the state x_k which is contained in the measurements Z_k and the prior PDF of x_0. Once $p(x_k|Z_k)$ is known, the estimates of functions of the state x_k conditional on measurements Z_k, can be made. For example, the minimum mean squared error estimate of x_k given Z_k is,

$$\hat{x}_k = \mathbf{E}[x_k|Z_k] = \int x_k p(x_k|Z_k) dx_k \quad (9)$$

For linear Gaussian systems where the PDF can be summarised by means and covariances, the Kalman filter is used to propagate and update the means and covariances of the PDF. For general nonlinear, non-Gaussian systems, there is no simple way to proceed. The reason is that there is no general analytic expression for the required PDF. *Particle filter*, a sequential Monte Carlo filter, was proposed as a new way of representing and recursively generating an approximation to the conditional PDF $p(x_k|Z_k)$ [8]. The key idea is to represent the PDF by a swarm of points called "particles", rather than by a function over the state space. As the number of particles increases, they effectively provide a good approximation to the required PDF.

Following [8], the sequential Monte Carlo filter algorithm can be described as follows:

- Assume that there is a set of random samples (particles) $\{x_{k-1}(i) : i = 1, 2, \cdots, N\}$ from the PDF $p(x_{k-1}|Z_{k-1})$.
- **Prediction:** Sample N values $\{w_{k-1}(i) : i = 1, 2, \cdots, N\}$ from the PDF of system noise w_{k-1}. Use these to generate new swarm of points

$\{x_k^*(i) : i = 1, 2, \cdots, N\}$, where,

$$x_k^*(i) = f_{k-1}(x_{k-1}(i), w_{k-1}(i)) \quad (10)$$

based on equation (1).

- **Update:** Assign each $x_k^*(i)$ a weight $q_k(i)$ for $i = 1, 2, \cdots, N$, after measurement y_k is received. The weights are given by,

$$q_k(i) = \frac{\tilde{q}_k(i)}{\sum_{j=1}^{N} \tilde{q}_k(j)} \quad (11)$$

where $\tilde{q}_k(i)$ are the un-normalised weights

$$\tilde{q}_k(i) = p(y_k | x_k^*(i)) \quad (12)$$

This defines a discrete distribution over $\{x_k^*(i) : i = 1, 2, \cdots, N\}$, which assigns probability mass $q_k(i)$ to the element $x_k^*(i)$.

- **Resample:** Resample independently N times from the above discrete distribution. The resulting particles $\{x_k(i) : i = 1, 2, \cdots, N\}$ satisfies

$$P\{x_k(i) = x_k^*(j)\} = q_k(j) \quad \text{for all } i \quad (13)$$

and forms an appropriate sample from the posterior PDF $p(x_k | Z_k)$.

- The prediction, update and resample steps form a single iteration and is recursively applied for each k.

4 Sequential Monte Carlo Filter for Fault Detection

The advantages of using the complete PDF of the system state in a fault detection scheme is bound to be superior than one which uses approximations, such as in the extended Kalman filter (EKF). Our approach is precisely to replace the EKF based estimation scheme by the sequential Monte Carlo filter, and the weighted sum squared residual (WSSR) measure by an appropriate innovations likelihood measure as the fault detection criteria.

In fact, the innovations likelihood is none other than the model or hypothesis likelihood, useful in model comparison [12]. Spurious false alarms may occur with this criteria if it is based on a single output measurement. A robust criteria is obtained by determining the likelihood over a window of length κ as before, which leads to a detection criteria which makes use of the complete PDF state information given by the sequential Monte Carlo filter.

The following algorithm describes the complete sequential Monte Carlo filter based fault detection scheme:

- **State prediction:** Samples $\{x_k^*(i) : i = 1, 2, \cdots, N\}$ are generated as in sequential Monte Carlo filter prediction step.

- **Output prediction:** The output prediction samples $\{y_k^*(i) : i = 1, 2, \cdots, N\}$ are generated using the measurement equation in (1), where,

$$y_k^*(i) = h_k(x_k^*(i)) \quad (14)$$

- **Residual generation:** Sample mean of the predicted measurements is computed as,

$$r_k = y_k - y_k^*(i) \quad (15)$$

- **Fault detection:** The innovations likelihood is given by,

$$p(r_k | Z_k) = \frac{1}{N} \sum_{i=1}^{N} \tilde{q}_k(i) \quad (16)$$

The windowed likelihood is

$$D(k) = \prod_{j=k-\kappa+1}^{k} p(r_k | Z_k) \quad (17)$$

or equivalently the negative log likelihood is

$$\mathcal{L}(k) = \sum_{j=k-\kappa+1}^{k} - \ln\left(p(r_k | Z_k)\right) \quad (18)$$

is computed and the condition $\mathcal{L}(k) > \epsilon$ is tested for the presence of a fault.

- **State update:** Weights $\tilde{q}_k(i)$ for the samples $\{x_k^*(i) : i = 1, 2, \cdots, N\}$ are generated as in sequential Monte Carlo filter update step.

- **Resample:** Samples $\{x_k(i) : i = 1, 2, \cdots, N\}$ are obtained from resampling as in sequential Monte Carlo filter resample step.

- The steps are repeated recursively for each k.

Of course, different weighting can also be given instead of the windowed function in (18).

The accuracy of the sequential Monte Carlo filter state estimation to that of EKF suggests that the above fault detection scheme should have a superior performance over the EKF based one suggested in [16].

5 Augmented States Model for Fault Isolation

The scheme outlined in the previous section is a fault detection scheme which cannot readily be extended to fault isolation. One approach to fault isolation is to estimate the parameters of the model and track the changes in their values. Such a procedure will require the simultaneous estimation of the state and the parameters, which can be achieved using the augmented states model [1]. The use of a sequential Monte Carlo filter for estimating the states and the parameters has been recently proposed [9].

The idea is to use an augmented state $\xi^T = [x^T \quad \theta^T]$ and re-write the state-space model in terms of ξ. The following set of equations result:

$$\begin{aligned} \begin{bmatrix} x_k \\ \theta_k \end{bmatrix} &= \begin{bmatrix} f_{k-1}(x_{k-1}, \theta_{k-1}, w_{k-1}) \\ \theta_{k-1} + w'_{k-1} \end{bmatrix} \\ y_k &= h_k(x_k, \theta_k, v_k) \end{aligned} \quad (19)$$

where the dependency of the functions $f_{k-1}(\cdot), h_k(\cdot)$ on the parameters θ are made explicit. The disturbance term w'_{k-1} is introduced by the use of a random walk model for parameter evolution to allow the exploration of the parameter space, as is typically done.

Given the above state space representation, the sequential Monte Carlo filter, outlined in section 3, can be used to estimate the states and the parameters. Such an estimate based on equation (9) is given by,

$$\hat{\xi}_k = \sum_{i=1}^{N} q_k(i) \xi_k^*(i) \quad (20)$$

The estimate is essentially a weighted average of the particles or samples representing the underlying distribution. The parameter estimates $\hat{\theta}_k$ can be compared to the nominal values θ_0 as a means for fault detection and its deviation $\tilde{\theta}_k = \theta_0 - \hat{\theta}_k$ be used for fault isolation.

The augmented state space model is attractive in principle. However, this increases the dimensionality of the model and thereby demands the increase in the number of particles for suffciently accurate results. This computational burden is unnecessary during the normal operation since fault isolation is not an issue. This suggests that we can use the following scheme:

- *Under normal operation*: Apply sequential Monte Carlo filter based fault detection.
- *After detection of on-set of fault*: Initiate augmented states model scheme for fault isolation.

The combined fault detection and isolation (FDI) scheme above undergoes a dimensionality change at the point of transfer between the detection and isolation schemes. This requies initialisation of the prior distribution for the augmented states. The prior for the parameters θ are chosen on the basis of the fault types and levels that are to be isolated. The prior for the states are re-initialised to represent a larger uncertainty that reflects the change in the system.

6 Example – Weakly Nonlinear System

Two examples are presented to illustrate the operation of the proposed sequential Monte Carlo filter based algorithm. The fault detection performance is compared with that using the EKF-based approach. The first example is a ship propelling system fault detection problem taken from [16]. The system is first order nonlinear, given by the dynamical equations,

$$\begin{aligned} x_k &= 0.1ax_{k-1}^2 + x_{k-1} + 0.1bu_k + w_{k-1} \\ y_k &= x_k + v_k \end{aligned} \quad (21)$$

where x is the speed of the ship, a is the hull resistance and b is the efficiency of the propeller. The normal values for a and b are: $a = -0.58$, $b = 0.2$. The disturbance w_k and noise v_k are uncorrellated zero mean Gaussian white noise with variance $W = 0.0001$ and 0.01 respectively. The system input u_k is a square wave.

In the simulation, the length of the data window for WSSR calculation is $N_1 = 30$, the threshold for WSSR fault detector is selected as $\epsilon = 2$ and the number of particles or sample size in the particle filter is $N = 500$. Both the EKF and the sequential Monte Carlo filter are initiated with the prior probability density function $p(x_0) = \mathbf{N}(0, 0.1)$ being Gaussian. The fault is simulated to occur at time $k = 701$ at which time the parameter a jumps from a value of -0.58 to -0.29 (50% fault) while b remains unchanged.

The weighted sum squared residual (WSSR) results for the EKF is shown in Figure 1 and for the sequential Monte Carlo filter in Figure 2.

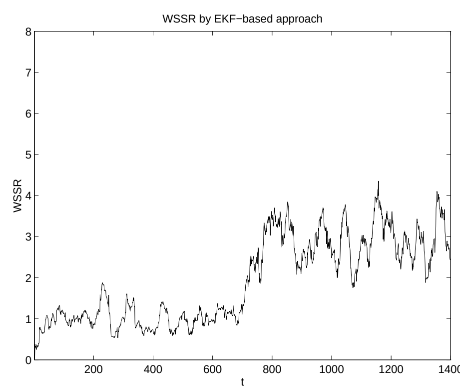

Figure 1: Weighted sum squared residual – EKF

For the fault detection threshold chosen, the EKF based scheme detected the fault at $k = 722$ and the sequential Monte Carlo filter based scheme at $k = 723$. However, the change in detection criteria is more pronounced in the latter and hence the robustness of the sequential Monte Carlo filter based scheme is likely to be superior. Neverthless, the detection performance for the

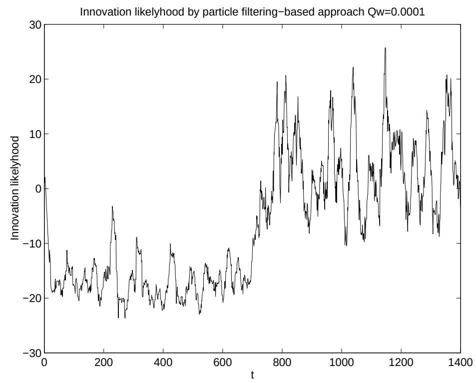

Figure 2: Negative log likelihood – Particle Filter

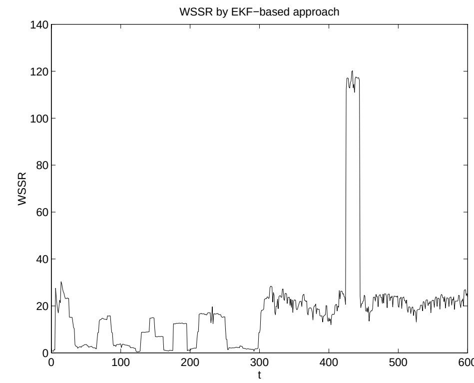

Figure 3: Weighted sum squared residual – EKF

two schemes are comparable which may be due in part to the system being weakly nonlinear.

7 Example – Strongly Nonlinear System

The second example is a univariate growth model taken from [8], where the system is is described by,

$$\begin{aligned} x_k &= 0.5x_{k-1} + a_1 \tfrac{x_{k-1}}{(1+x_{k-1}^2)} + a_2 \cos(1.2(k-1)) \\ &\quad + w_{k-1} \\ y_k &= a_3 x_k^2 + v_k \end{aligned} \tag{22}$$

where w_k and v_k are uncorrelated zero mean Gaussian white noise with variance $W = 0.1$ and $V = 1$ respectively. The nominal values of the parameters are $\bar{a}_1 = 25$, $\bar{a}_2 = 8$ and $\bar{a}_3 = 0.05$.

In the simulation, the length of the data window for WSSR calculation is $N_1 = 20$ and the number of particles or sample size in the sequential Monte Carlo filter is $N = 500$. Both the EKF and sequential Monte Carlo filter are initiated with the prior probability density function $p(x_0) = \mathbf{N}(0, 2)$. The fault is simulated to occur at time $k = 301$ at which time the parameter a_1 jumps from a value of $\bar{a}_1$ to $0.5\bar{a}_1$ while a_2, a_3 remain unchanged.

The weighted sum squared residual (WSSR) results for the EKF is shown in Figure 3 and the negative log likelihood for the sequential Monte Carlo filter in Figure 4.

The EKF based approach fails to detect the occurrence of the fault around $k = 301$ as evidenced by Figure 3 where there is no significant change in WSSR. The significant change takes place much after the onset of the fault and hence the detection performance is unacceptable. On the otherhand, the sequential Monte Carlo filter based scheme detects the fault at $k = 307$ for a threshold value of $\epsilon = 5$. The change in log likelihood is quite pronounced following the onset of the fault.

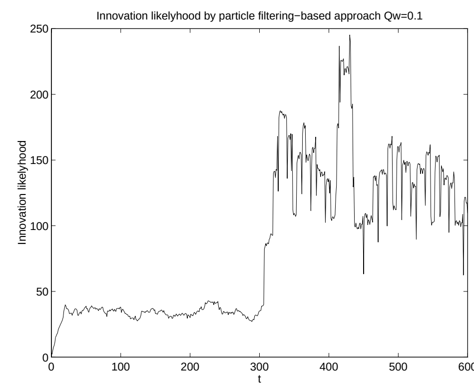

Figure 4: Negative log likelihood – Particle Filter

This example clearly demonstrates the inadequacy of EKF and perhaps other approximation schemes to be used in fault detection of highly nonlinear systems and that the sequential Monte Carlo filtering approach offers much promise. Experiments have been conducted to test the sensitivity of the threshold parameter to the performance of the system. Although not reported here, sequential Monte Carlo filter exhibited better robustness properties in comparison to the EKF based scheme.

The combined FDI scheme proposed in section 5 was also applied to the above example. Here, the fault is simulated to occur at time $k = 201$ and the number of particles used were $N = 1000$. The same type and level of fault was introduced as before.

Figure 5 shows that the change in the parameter a_1 is tracked following an initial transient and the estimates for the other parameters hover around the nominal parameters. Thus we can safely conclude that the fault in the system is due to the change in the parameter a_1 and an estimate of its level could also be obtained.

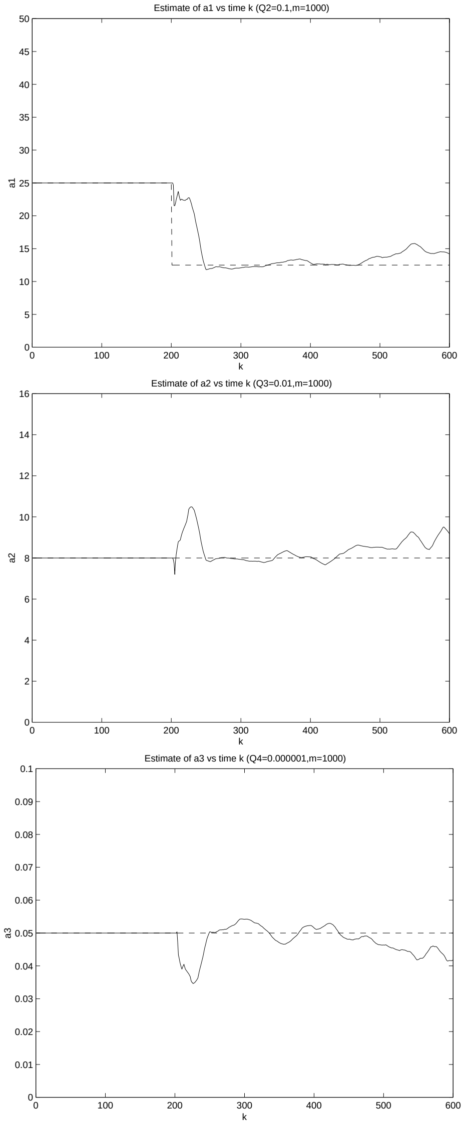

Figure 5: Parameter a_1, a_2, a_3 estimates (–) from Sequential Monte Carlo Filter and true values (- -).

8 Conclusions

In this paper, a sequential Monte Carlo filter based approach to fault detection and isolation scheme is developed. This new algorithm is applicable to general nonlinear system with non-Gaussian noise and disturbances in contrast to the extended Kalman filter (EKF) based approach which rely on approximations arising from linearisation and Gaussianisation. The fault detection performance compared with that using the EKF. The results from simulation show that the detectability of the sequential Monte Carlo filtering approach is superior to the EKF based scheme, especially in the case where the system model is highly nonlinear. The fault isolation scheme is also shown to identify the parameter associated with the fault and the level of the fault. Further work is being carried out on the fault isolation scheme and robustify the performance of the developed schemes.

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