

Observability Properties of Petri Nets

Alessandro Giua, Carla Seatzu

Department of Electrical and Electronic Engineering,

University of Cagliari, Piazza d'Armi — 09123 Cagliari, Italy

Tel: +39 (70) 675-5892. Fax: +39 (70) 675-5900. Email: {giua,seatzu}@diee.unica.it.

Abstract

In this paper we discuss the problem of estimating the marking of a Place/Transition net based on event observation. We assume that the net structure is known while the initial marking is unknown.

We define several observability properties and show how they can be proved. In particular we set up a hierarchy considering the possibility that the above properties are satisfied by a net N starting from an initial marking M_0 , by a net N starting from any initial marking M reachable from an initial marking M_0 , or by a net N starting from any marking in $\mathbb{N}^m$, where m is the number of places of the net.

1 Introduction

This paper presents a set of analytical tools to determine the observability properties of Petri nets, i.e., algorithms to determine under which conditions it is possible to reconstruct the marking of a Place/Transition net based on event observation.

Observability is widely studied by automatic control researchers. It is a fundamental property because it allows one to estimate states that cannot be measured. The idea of constructing estimates of the unknown plant state for discrete-event systems has been already investigated in the literature [1, 5, 6], even if there exists very few work dealing with observability in Petri nets [3, 12].

In this paper we consider the marking estimation problem presented in [3] where an algorithm was given to estimate the actual marking of the net based on the observation of a word of events (i.e., transition firings), under the assumption that the net structure is known while the initial marking is not known. The estimate is always a lower bound of the actual marking. The system that compute the estimate is called an observer.

The error function between the actual marking and the estimate was shown in [3] to be a monotonically non-increasing function of the observed word length. Observed words that lead to a null error are said to be “complete”. Complete observers are the discrete-event counterpart of asymptotic observers for time-driven systems.

This framework provides a useful paradigm that can be applied to different settings, from discrete event control, to failure diagnosis and error recovery. The assumption that only event occurrences may be observed, while the plant state cannot, is common in discrete event control. The assumption that the state of the plant is not known (or is only partially known) is natural during error recovery. Consider for instance the case of a plant remotely controlled: if the communication fails the state may evolve and when the communication is re-established the state will be at best partially known.

In a manufacturing environment, one may consider the case in which resources (i.e., tokens) enter unobserved, or in which we know how many resources have entered the system but not their exact location.

In this paper we define several observability properties and show that they are decidable. In particular we consider two main properties. *Marking observability* (MO) means that there exists at least one word that is complete, while *strongly marking observability* (SMO) means that all words can be completed in a finite number of steps into a complete word.

We set up a hierarchy considering the possibility that the two properties are satisfied by a net N starting from an initial marking M_0 , by a net N starting from any marking M reachable from an initial marking M_0 (uniform observability) or by a net N starting from any marking in $\mathbb{N}^m$ (structural observability) where m is the number of places of the net.

All the considered properties can be proved by reducing them to other decision problems (e.g., home-space properties, marking reachability, existence of repetitive sequences) that can be checked using algorithms well known from the literature.

2 Background

In this section we provide some basic definitions that will be used in the following of the paper. We first recall some basic terminology on Petri nets, then we provide the definition of both linear and semi-linear sets and we recall the main results on decidability of home-space property. Finally, we recall some preliminary concepts already presented by Giua in [3] that are the basis for the new results of this paper.

2.1 Petri nets

In this subsection we recall the Petri net formalism used in this paper. For a more comprehensive introduction to Petri nets see [9]. A *Place/Transition net* (P/T net) is a structure $N = (P, T, Pre, Post)$, where P is a set of m places; T is a set of n transitions; $Pre : P \times T \rightarrow \mathbb{N}$ and $Post : P \times T \rightarrow \mathbb{N}$ are the *pre-* and *post-incidence functions* that specify the arcs. The *incidence matrix* of the net is defined as $C(p, t) = Post(p, t) - Pre(p, t)$.

We define $p^\bullet = \{t \in T \mid Pre(p, t) > 0\}$ as the set of output transitions of place p .

A *marking* is a vector $M : P \rightarrow \mathbb{N}$ that assigns to each place of a P/T net a non-negative number of tokens, represented by black dots. A *P/T system* or *net system* $\langle N, M_0 \rangle$ is a net N with an initial marking M_0 .

A transition t is enabled at M if $M \geq Pre(\cdot, t)$ and may fire yielding the marking $M' = M + C(\cdot, t)$. We write $M \xrightarrow{w} M'$ to denote that the enabled sequence of transitions w may fire at M yielding M' ; we use the

notation $M' = w(M)$ and $M = w^{-1}(M')$. Moreover, we denote $w(M_0) = M_w$. Finally, we denote as w_0 the sequence of null length. The set of all sequences firable in $\langle N, M_0 \rangle$ is denoted $L(N, M_0)$ (this is also called the prefix-closed free language of the net). If the firing sequence w is enabled at M_0 , we also say that w is a word in $L(N, M_0)$.

Let $w = t_{\alpha_1}, t_{\alpha_2}, \dots, t_{\alpha_k}$ be a sequence in $L(N, M_0)$. The sequence $w_i = t_{\alpha_1}, \dots, t_{\alpha_i}$ with $i \in \mathbb{N}$ and $i < k$ is a prefix of w of length i and we write $w_i \preceq w$.

A marking M is *reachable* in $\langle N, M_0 \rangle$ iff there exists a firing sequence w such that $M_0 [w] M$. The set of all markings reachable from M_0 defines the reachability set of $\langle N, M_0 \rangle$ and is denoted $R(N, M_0)$.

A *repetitive* sequence w is such that $M[w]M'$ with $M' \geq M$. Then $\forall i \geq 1$, w^i is enabled at M . A repetitive sequence w is said to be *non-stationary* if $M[w]M'$ with $M' \geq M$: such a sequence strictly increases the token count of one or more places.

Three useful elementary facts about Petri nets that will be used in the paper are the following.

Fact 1. *If $M \leq M'$ then $L(N, M) \subseteq L(N, M')$.*

Fact 2. *If w is enabled at M and M' then: $M - M' = w(M) - w(M')$.*

Fact 3. *The reachability set $R(N, M_0)$ is infinite iff there exists a non-stationary repetitive sequence in $L(N, M_0)$.*

Finally, we denote $\vec{0}_m$ ($\vec{1}_m$) a $m \times 1$ vector of zeros (ones).

2.2 Home space property

Linear and semi-linear sets were firstly introduced in [10] in order to study some problems from formal language theory.

Definition 4. *We say that $\mathcal{E} \subseteq \mathbb{N}^m$ is a linear set if there exists some $V \in \mathbb{N}^m$ and a finite set $\{V_1, \dots, V_n\} \subseteq \mathbb{N}^m$ such that*

$$\mathcal{E} = \{V' \in \mathbb{N}^m \mid V' = V + \sum_{i=1}^n k_i V_i \text{ with } k_i \in \mathbb{N}\},$$

V is called the base of $\mathcal{E}$, and $V_1, \dots, V_n$ are called its periods.

A semi-linear set is the finite union of a family of linear sets.

A first result regarding decidability is the following.

Theorem 5 ([2]). *Given a net system $\langle N, M_0 \rangle$ and a semi-linear set E it is decidable if $R(N, M_0) \cap E = \emptyset$.*

Finally, we introduce the definition of home space [8] and an important theorem that will be used when proving some properties of estimates.

Definition 6 ([8]). *Let $\mathcal{HS}$ be a set of markings. We say that $\mathcal{HS}$ is a home space of a P/T net $\langle N, M_0 \rangle$ iff $\forall M \in R(N, M_0)$, $\exists M' \in \mathcal{HS}$ such that $M' \in R(N, M)$. If $\mathcal{HS}$ is a singleton, we call its unique element a home state.*

Theorem 7 ([2]). *The property of being a home space for finite unions of linear sets having the same periods, is decidable.*

2.3 Estimate and error

The aim of this subsection is that of recalling some preliminary concepts already presented in [3]. The proofs of all propositions are omitted and can be found in [3, 4].

Firstly, we recall an algorithm for estimating the state of a net system $\langle N, M_0 \rangle$ whose marking cannot be directly observed under the following assumptions.

- A1) The structure of the net $N = (P, T, Pre, Post)$ is known, while the initial marking M_0 is not.
- A2) The event occurrences (i.e., the transition firings) can be observed.

After the word w has been observed we define the set $\mathcal{M}(w)$ of w consistent markings as the set of all markings in which the system may be given the observed behaviour.

Definition 8. *Given an observed word w , the set of w consistent markings is $\mathcal{M}(w) = \{M' \mid \exists M \in \mathbb{N}^m, M'[w]M\}$.*

Given an evolution of the net $M_{w_0}[t_{\alpha_1}]M_{w_1}[t_{\alpha_2}]\dots$, we use the following algorithm to compute the estimate μ_{w_i} of each actual marking M_{w_i} based on the observation of the word of events $w_i = t_{\alpha_1}, t_{\alpha_2}, \dots, t_{\alpha_i}$.

Algorithm 9 ([3] Mark. Est. with Event Obs.).

1. Let the initial estimate be $\mu_{w_0} = \vec{0}_m$.
2. Let $i = 1$.
3. Wait until t_{α_i} fires.
4. Update the estimate $\mu_{w_{i-1}}$ to μ'_{w_i} with

$$\mu'_{w_i}(p) = \max\{\mu_{w_{i-1}}(p), Pre(p, t_{\alpha_i})\}.$$

5. Let $\mu_{w_i} = \mu'_{w_i} + C(\cdot, t_{\alpha_i})$.
6. Let $i = i + 1$.
7. Goto 3. ■

Note that in step 4. of the algorithm we update the previously computed estimate $\mu_{w_{i-1}}$, since the firing of t_{α_i} implies that $M_{w_{i-1}} \geq Pre(\cdot, t_{\alpha_i})$. In the following we will always denote the estimate computed by this algorithm after having observed the word w as μ_w .

The estimate computed by Algorithm 9 is a lower bound on the actual marking of the net.

Proposition 10 ([3]). *Let $w = t_{\alpha_1}t_{\alpha_2}\dots \in L(N, M_0)$ be an observed string and w_i its prefix of length i . Then*

$$\forall i, \quad \mu_{w_i} \leq \mu'_{w_{i+1}} \leq M_{w_i}.$$

In [3] it has been given an easy characterization of the set of consistent markings in terms of estimate.

Theorem 11 ([3]). *Given an observed word $w \in L(N, M_0)$ and the corresponding estimated marking μ_w computed by Algorithm 2, the set of w consistent markings is*

$$\mathcal{M}(w) = \{M \in \mathbb{N}^m \mid M \geq \mu_w\}.$$

In this paper we also define a meaningful measure of the place estimation error, as the token difference between a marking and its estimate in a given place.

Definition 12. Let us consider a place $p \in P$ and an observed word $w \in L(N, M_0)$. Let M_w and μ_w be the corresponding marking and its estimate. The place estimation error in p is $e_p(M_w, \mu_w) = M_w(p) - \mu_w(p)$ and its update after the firing of t is $e_p(M_w, \mu'_{wt}) = M_w(p) - \mu'_{wt}(p)$.

Analogously, it is possible [3] to define a measure of the estimation error, as the token difference between a marking and its estimate.

Definition 13 ([3]). Given a marking M_w and its estimate μ_w , the estimation error is $e(M_w, \mu_w) = \sum_{p \in P} e_p(M_w, \mu_w) = \vec{1}_n^T \cdot (M_w - \mu_w)$ and its update after the firing of t is $e(M_w, \mu'_{wt}) = \vec{1}_n^T \cdot (M_w - \mu'_{wt})$.

Note that the place estimation error is a monotonically non-increasing function of the observed word length.

Proposition 14 ([4]). Let $w = t_{\alpha_1} t_{\alpha_2} \dots \in L(N, M_0)$ be an observed word and w_i its prefix of length i . Then $\forall i$ and $\forall p$:

$$e_p(M_{w_i}, \mu_{w_i}) \geq e_p(M_{w_i}, \mu'_{w_{i+1}}) = e_p(M_{w_{i+1}}, \mu_{w_{i+1}}), \quad (1)$$

and

$$e_p(M_{w_i}, \mu'_{w_{i+1}}) = \min \{e_p(M_{w_i}, \mu_{w_i}), M_{w_i} - \text{Pre}(p, t_{\alpha_{i+1}})\}. \quad (2)$$

Thus, it follows that also the estimation error is a monotonically non-increasing function of the observed word length.

Proposition 15 ([3]). Let $w = t_{\alpha_1} t_{\alpha_2} \dots \in L(N, M_0)$ be an observed word, w_i the prefix of w of length i , and μ_{w_i} and μ'_{w_i} the estimate and the updated estimate of M_{w_i} . Then $\forall i$:

$$e(M_{w_i}, \mu_{w_i}) \geq e(M_{w_i}, \mu'_{w_{i+1}}) = e(M_{w_{i+1}}, \mu_{w_{i+1}}).$$

3 Properties of estimates

It is natural to ask under which conditions the estimated marking computed by algorithm 9 converges to the actual marking. This motivated us to define the following properties.

Definition 16. A word $w \in L(N, M_0)$ is marking complete with respect to (wrt) $\langle N, M_0 \rangle$ if $\mu_w = M_w$, i.e., $e(M_w, \mu_w) = 0$.

Thus a marking complete word allows one to reconstruct the actual marking of the net. Sometimes, however, only the marking of a subset of places can be reconstructed.

Definition 17. A place $p \in P$ is observable in $\langle N, M_0 \rangle$ if there exists a word $w \in L(N, M_0)$ such that $\mu_w(p) = M_w(p)$, i.e., $e_p(M_w, \mu_w) = 0$.

Finally we can define these properties of a net system.

Definition 18. A net system $\langle N, M_0 \rangle$ is:

- marking observable (MO) if there exists a marking complete $w \in L(N, M_0)$;
- strongly marking observable (SMO) in k steps if:
 1. $\forall w \in L(N, M_0)$ such that $|w| \geq k$, w is marking complete,
 2. $\forall w \in L(N, M_0)$ such that $|w| < k$, either w is marking complete or $\exists t \in T$ such that $M_0[w t]$.

In this definition we note that the observability properties depend not only on the net structure N , but also on the initial marking M_0 , that we assume is unknown. Thus, it may seem that those properties have little significance per se. In effect, we will use the characterization of MO and SMO to prove two more general properties that have greater significance.

Definition 19. A net system $\langle N, M_0 \rangle$ is:

- uniformly marking observable (uMO) if $\forall M \in R(N, M_0)$, $\langle N, M \rangle$ is MO;
- uniformly strongly marking observable (uSMO) in k steps if $\forall M \in R(N, M_0)$, $\langle N, M \rangle$ is SMO in k steps.

The property of uMO and uSMO are important if we consider the following problem: we consider a system whose initial marking M_0 is known. Due to a communication failure the system evolves unobserved. When the communication is re-established, we can only be sure that the actual marking belongs to the set $R(N, M_0)$. We want to know if the marking can be reconstructed starting from any of these reachable markings.

Definition 20. A net N is:

- structurally marking observable (sMO) if it is MO for any initial marking $M_0 \in \mathbb{N}^m$;
- structurally strongly marking observable (sSMO) if $\langle N, M_0 \rangle$ is SMO (in a number of steps k that depends on M_0) for any initial marking $M_0 \in \mathbb{N}^m$.

The properties of sMO and sSMO are even more general and only depend on the net structure N .

The above properties are related among them as shown in the following partial order diagram:

$$\begin{array}{ccccc} \text{sSMO} & \longrightarrow & \text{uSMO} & \longrightarrow & \text{SMO} \\ \downarrow & & \downarrow & & \downarrow \\ \text{sMO} & \longrightarrow & \text{uMO} & \longrightarrow & \text{MO} \end{array}$$

Here, say, $\text{sMO} \longrightarrow \text{uMO}$ means that if a net N is sMO then $\langle N, M_0 \rangle$ is uMO for all initial markings M_0 . By means of simple counterexamples it is possible to prove that properties not in a partial order relationship are uncorrelated.

4 Properties analysis

In this section we discuss in detail the observability problem. In particular we provide necessary and sufficient conditions to characterize the properties defined above and we also prove that all these properties are decidable.

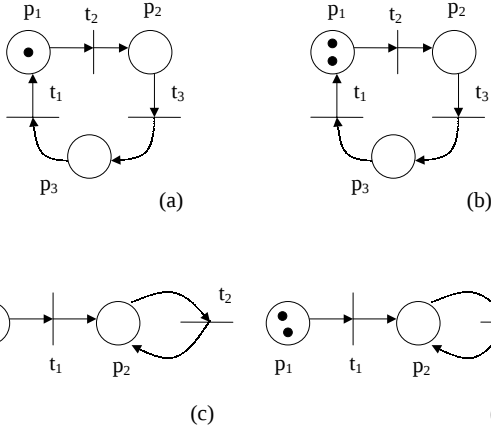


Figure 1: Place/Transition nets used in the examples.

4.1 Word completeness

A necessary and sufficient condition for completeness of a word was given in [3] in terms of languages.

Proposition 21 ([3]). *A word $w \in L(N, M_0)$ is marking complete iff $\forall \bar{M}_0 < M_0 : w \notin L(N, \bar{M}_0)$.*

Example 22. Let us consider the net system in figure 1.a. The word $w = t_2$ is marking complete. On the contrary, it is not marking complete for the net system in figure 1.b since $t_2 \in L(N, \bar{M}_0)$ with $\bar{M}_0 = [1 \ 0 \ 0] < M_0 = [2 \ 0 \ 0]$. A complete word for the net system in figure 1.b is $w = t_2 t_2$. It can be proved with proposition 21. ■

Theorem 23. *Let $\langle N, M_0 \rangle$ be a net system and w a word in $L(N, M_0)$. It is decidable whether w is marking complete wrt to $\langle N, M_0 \rangle$.*

Proof: It follows from proposition 21 because it only requires to check if w can be fired from a finite set of initial markings. □

4.2 Observability

A characterization based on the net language for both marking and strongly marking observability were given in [3], where it was proven that these properties are decidable.

We will not discuss here this point but present some examples.

Example 24. All net systems in figure 1 are marking observable, as it can be read in table 1 that summarizes all observability properties of P/T nets in figure 1.

In all cases there exist at least one complete word. In the case of figure 1.a (b), the word $w = t_2$ ($w = t_2 t_2$) is complete since its firing removes all tokens in place p_1 . Analogously, $w = t_1$ and $w = t_1 t_1$ are complete words for the net systems in figure 1.c and d, respectively.

On the contrary, only net systems in figure 1.a and c are SMO (in one step). The net systems in figure 1.b and d are not SMO. In fact, in both cases there exist arbitrarily long sequences that are enabled at the initial marking and that are not complete. In the case of figure 1.b, $w = (t_2 t_3 t_1)^i$ is not marking complete $\forall i \in \mathbb{N}$. Analogously, the net system in figure 1.d is not SMO since $\forall i \in \mathbb{N}$, $w = t_1 (t_2)^i$ is not marking complete. ■

	MO	SMO	uMO	uSMO	sMO	sSMO
(a)	×	×	×	×	×	—
(b)	×	—	×	—	×	—
(c)	×	×	×	×	—	—
(d)	×	—	—	—	—	—

Table 1: Observability properties of the nets in figure 1.

4.3 Uniform observability

In this section we first provide necessary and sufficient conditions for both uniform MO and uniform SMO. Then we prove the decidability of both these properties.

Let us first demonstrate an important lemma.

Lemma 25. *Let $\langle N, M_0 \rangle$ be a net system. A place $p \in P$ is observable in $\langle N, M_0 \rangle$ iff at least one element in the semi-linear set*

$$\mathcal{A}_p = \{M \in \mathbb{N}^m \mid M(p) = 0\} \cup \left(\bigcup_{t \in p^\bullet} \{M \in \mathbb{N}^m \mid M(p) = \text{Pre}(p, t), M \geq \text{Pre}(\cdot, t)\} \right) \quad (3)$$

is reachable.

Proof: (if) Let w be a word in $L(N, M_0)$. Let us consider two subcases.

- i) If $M_w \in \{M \in \mathbb{N}^m \mid M(p) = 0\}$, then $0 = M_w(p) \geq \mu_w(p) \geq 0$, thus $M_w(p) = \mu_w(p)$.
- ii) If $M_w \in \{M \in \mathbb{N}^m \mid M(p) = \text{Pre}(p, t), M \geq \text{Pre}(\cdot, t)\}$ where $t \in p^\bullet$, then t may fire at M_w and since $M_w(p) = \text{Pre}(p, t)$ the updated estimate is $\mu'_w(p) = M_w(p)$, hence $M_{wt}(p) = \mu_{wt}(p)$.

(only if) We prove this by contradiction.

If no marking with $M(p) = 0$ is reachable, then $M_w(p) > 0 \forall w \in L(N, M_0)$, thus the initial place estimation error is strictly positive. It may decrease only during step 4 of algorithm 9. However, if $\forall w$ and $\forall t \in p^\bullet$, $M_w(p) > \text{Pre}(p, t)$, then $\mu'_w(p) < M_w(p)$, thus the place estimation error keeps positive. □

By virtue of the previous lemma, the study of uniform marking observability reduces to the study of m home space problems.

Proposition 26. *A net system $\langle N, M_0 \rangle$ is uniformly marking observable iff the semi-linear set $\mathcal{A}_p$ given by eq. (3) is a home space $\forall p \in P$.*

Proof: It follows from the previous lemma and the fact that a net system $\langle N, M_0 \rangle$ is uniformly marking observable iff each place $p \in P$ is observable in $\langle N, M \rangle$, $\forall M \in R(N, M_0)$, i.e., iff the semi-linear set (3) is a home-space $\forall p \in P$. □

Let us now consider the uniform SMO property. We first demonstrate, as an intermediate result, that the repeated firing of a repetitive sequence does not decrease the estimation error.

Lemma 27. *Let $\langle N, M_0 \rangle$ be a net system and let us assume that there exists a firing sequence w' that enables a repetitive sequence w , i.e., $M_0[w']M_{w'}[w]M_{w'w}$ with $M_{w'w} \geq M_{w'}$. Then $\forall i > 1$, $e(M_{w'w^i}, \mu_{w'w^i}) = e(M_{w'w}, \mu_{w'w})$.*

Proof: While observing a sequence w , the error may decrease only during step 4 of algorithm 9, i.e., when we compute the updating estimate.

Let t be the first transition in the sequence w . If t fires after $w'w^i$, in step 4 of algorithm 9 we have

$$\mu'_{w'w^it}(p) \geq \text{Pre}(p, t), \quad \forall p \in P.$$

Using proposition 14 it is easy to show that for all $i \geq 1$

$$(M_{w'w^{i+1}} - \mu_{w'w^{i+1}}) \leq (M_{w'w^i} - \mu'_{w'w^it}).$$

thus

$$(M_{w'w^{i+1}} - M_{w'w^i}) + \mu'_{w'w^it} \geq \mu'_{w'w^it} \geq \text{Pre}(p, t).$$

Therefore, $\mu'_{w'w^{i+1}t} = \mu_{w'w^{i+1}}$, i.e., the estimate is not updated and the error remains constant each time w is repeated after it has fired once. $\square$

Proposition 28. A net system $\langle N, M_0 \rangle$ is uniformly strongly marking observable only if its reachability set is finite.

Proof: If the reachability set is not finite, then (by fact 3) there exist words w' and w such that $M_0[w']M_{w'}[w]M_{w'w}$ with $M_{w'w} \not\geq M_{w'}$, thus $w \in L(N, M_{w'}) \cap L(N, M_{w'w})$ is not marking complete wrt $\langle N, M_{w'w} \rangle$ (by proposition 21). Also, we can have words of infinite length w^i (for all $i > 1$) that are not marking complete (by lemma 27) thus the system $\langle N, M_{w'w} \rangle$ is not SMO and finally $\langle N, M_0 \rangle$ is not uSMO. $\square$

Example 29. The net systems in figure 1.a, b and c are uMO. In fact, in the first two cases, $\forall p \in P$ the set $\{M \in \mathbb{N}^m \mid M(p) = 0\}$ is always a home space. In the third case the sets $\{M \in \mathbb{N}^m \mid M(p_1) = 0\}$ and $\{M \in \mathbb{N}^m \mid M(p_2) = 1, M(p_1) = 0\}$ are home space. On the contrary, the net system in figure 1.d is not uMO since the net system $\langle N, M \rangle$ is not MO at $M = [02] \in R(N, M_0)$.

The net systems in figure 1.a and c are uSMO. Obviously, the net system in figure 1.b is not uSMO, being not SMO. Analogously, being the net system in figure 1.d not uMO, it is also not uSMO. $\blacksquare$

Theorem 30. It is decidable if a net system $\langle N, M_0 \rangle$ is uniformly and strongly uniformly marking observable.

Proof: Let us first prove the decidability of uniform marking observability. Because of proposition 26 it is sufficient to prove that the home-space property for the set $\mathcal{A}_p$ is decidable. Let us observe that $\forall p \in P$ the semi-linear set in eq. (3) is given by the finite union of linear sets having the same periods. In fact, if we consider a generic place $p_k \in P$,

$$\{M \in \mathbb{N}^m \mid M(p_k) = 0\} = \left\{ \sum_{i \neq k} a_i \vec{e}_i \mid a_i \in \mathbb{N} \right\}$$

$$\{M \in \mathbb{N}^m \mid M(p_k) = \text{Pre}(p_k, t), M \geq \text{Pre}(\cdot, t)\} = \left\{ \text{Pre}(\cdot, t) + \sum_{i \neq k} b_i \vec{e}_i \mid b_i \in \mathbb{N} \right\}$$

where $\vec{e}_i$ is the i -th canonical basis vector of dimension m . Thus, the decidability of the home-space property for $\mathcal{A}_p$ immediately follows by theorem 7.

Secondly, let us prove the decidability of strongly uniform marking observability. Let us observe that if the necessary requirement stated by proposition 28 is satisfied, then the reachability set is finite and the uniform strongly marking observability can be verified by proving the strongly marking observability — that is decidable [3] — for a finite set of initial markings. $\square$

4.4 Structural observability

In this subsection we provide necessary and sufficient conditions for both structural and strongly structural marking observability and we prove the decidability of these properties.

Proving structural observability, requires the study of the system properties for all possible initial markings. Next two lemmas show that to prove that a place is observable for all initial markings in $\mathbb{N}^m$, just a finite subset of $\mathbb{N}^m$ needs to be checked.

Lemma 31. If a place $p \in P$ is observable in $\langle N, M \rangle$ then it is also observable in $\langle N, \overline{M} \rangle \forall \overline{M} \geq M$ with $\overline{M}(p) = M(p)$.

Proof: A place p is observable in $\langle N, M \rangle$ if and only if $\exists w \in L(N, M)$ such that $M[w]M'$ and $\mu_w(p) = M'(p)$. In this case $\forall \overline{M} \geq M$ with $\overline{M}(p) = M(p)$, $w \in L(N, \overline{M})$ (by fact 1) and $\overline{M}[w]\overline{M}'$ with $\overline{M}'(p) = M'(p) = \mu_w(p)$ (by fact 2), i.e., p is also observable in $\langle N, \overline{M} \rangle$. $\square$

Lemma 32. Let N be a Petri net and let $r_p = \max_{t \in T} \text{Pre}(p, t)$. Let

$$M_i^p = \begin{cases} M_i^p(p') = 0 & \text{if } p' \neq p \\ M_i^p(p) = i. & \end{cases} \quad (4)$$

A place $p \in P$ is observable in $\langle N, M_i^p \rangle \forall i \in \mathbb{N}$, iff p is observable in $\langle N, M_i^p \rangle$ for $i = 1, \dots, r_p + 1$.

Proof: If p is observable in $\langle N, M_{r_p+1}^p \rangle$ then $\exists w$ and $t \in p^\bullet$ such that $M_{r_p+1}^p[w]\overline{M}$, $\overline{M}(p) = \text{Pre}(p, t)$, i.e., the firing of the word w reduces the number of tokens in p . This implies that for all M_i^p with $i > r_p + 1$ the word w may also fire until we reach a marking $\overline{M}'$ such that $\overline{M}' \geq M_{r_p}^p$ and $\overline{M}' = \varrho_i \leq r_p$. Since p is observable in $\langle N, M_{r_p}^p \rangle$, then it is also observable in $\langle N, \overline{M}' \rangle$ by lemma 31. $\square$

Proposition 33. A Petri net N is structurally marking observable iff $\forall p \in P$, p is observable in $\langle N, M_i^p \rangle$, where M_i^p is defined as in equation (4) and $i = 1, \dots, r_p + 1$.

Proof: By definition a Petri net N is sMO iff $\forall p \in P$, p is observable in $\langle N, M \rangle \forall M \in \mathbb{N}^m$. By lemma 31 and lemma 32 it is however sufficient to check that each p is observable for the finite number of initial makings given in the statement. $\square$

Proposition 34. A Petri net N is strongly structurally marking observable iff

- (a) N has no repetitive sequences;
- (b) $\forall p \in P, \exists t \in T$ such that

$$\text{Pre}(p', t) = \begin{cases} 1 & \text{if } p' = p \\ 0 & \text{if } p' \neq p \end{cases}.$$

Proof: (if) We will prove that (a) and (b) imply that for any initial marking M_0 , in a finite number of steps the net loses all its tokens: this is a sufficient condition for SMO of $\langle N, M_0 \rangle$ by lemma 25. In fact, if no repetitive sequences exist, for any initial marking the length of all words firable is bounded, i.e., after a finite number of firings the net reaches a dead marking. Furthermore, if assumption (b) is verified, for each place $p \in P$ there exists a transition t whose single input is p and the corresponding arc weight is unitary, i.e., if t cannot fire then place p must be empty. Thus the dead marking must be the zero marking.

(only if) We prove this by contradiction.

Let us first assume that (a) is violated, i.e., $\exists M \in \mathbb{N}^m$, $\exists w \in L(N, M)$ such that $M[w]M'$ with $M' \geq M$. Thus w is not marking complete wrt $\langle N, M' \rangle$ (by proposition 21). Also, we can have words of infinite length w^i (for all $i > 1$) that are not marking complete (by lemma 27) thus the system $\langle N, M' \rangle$ is not SMO.

Secondly, we assume that (a) is verified while $\exists p \in P$ such (b) is violated. We first observe that we can exclude the existence of transitions with no input arcs, because this would violate condition (a). Then it is obvious that given the marking M_1^p as in equation (4) (that contains one token in p and zero tokens elsewhere) no transition is enabled, thus the marking of p cannot be observed. $\square$

Theorem 35. *It is decidable if a Petri net N is structurally marking observable and structurally strongly marking observable.*

Proof: To prove that N is sMO it is sufficient to prove (by propositions 33) that all places are observable in $\langle N, M_0 \rangle$ for a finite number of initial markings M_0 . The property of being observable for a place is decidable because of theorem 5 and of the characterization given by lemma 25.

To prove that N is sSMO it is sufficient to check by propositions 34 that no repetitive sequences exist (and this may be checked with linear algebraic tools given the net incidence matrix) and that the net structure satisfies condition (b) (this may be checked by inspection). $\square$

Example 36. Being sMO and sSMO structural properties of the net, the same conclusions can be drawn for nets in figure 1.a, b and c, d, respectively.

In particular, the net in figure 1.a is sMO by proposition 33. On the contrary, it is not sSMO. In fact, if we consider the initial marking in figure 1.b the net system is not SMO.

The net N in figure 1.c is not sMO (thus it is also not sSMO). In fact, if we consider $M_0 = [0\ 2]$, $\langle N, M_0 \rangle$ is not MO. $\blacksquare$

A final remark regards the classes of nets that are sSMO. Although this property is rather easy to prove, the class of nets that satisfy this property is of little practical interest (they must become empty and deadlock in a finite number of steps). The property of structural MO, on the contrary, is more difficult to prove but is satisfied by a wider (more interesting) class of nets.

5 Conclusions

In this paper we dealt with the problem of estimating the marking of a Place/Transition net based on event

observation, assuming that the net structure is known while the initial marking is unknown.

We defined several observability properties and showed that they are decidable. In particular we considered two main properties: *marking observability* and *strongly marking observability*. The first one means that there exists at least one word that is complete, while the second one means that all words can be completed in a finite number of steps to a complete word.

We investigated the possibility that the two properties above are satisfied by a net N starting from an initial marking M_0 , by a net N starting from any marking M reachable from an initial marking M_0 (*uniform observability*), or by a net N starting from any marking in $\mathbb{N}^m$ (*structural observability*) where m is the number of places of the net.

We finally showed that many observability properties can be proved by reducing them to other decision problems (e.g., home-space properties, marking reachability, existence of repetitive sequences) that can be checked using algorithms well known from the literature.

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