Control of LEO Satellite Clusters.

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Abstract

In this paper we discuss the application of the perceptive control theory to the control of satellites flying in formation. We present the algorithms used and discuss some of their properties.

1 Introduction

The concept of a Distributed Aperture Radar, or a sparse thinned-array radar, has been under investigation for at least 20 years. The advantages of such a system in terms of increased performance, more efficient use of resources, enhanced survivability, and increased reliability are readily accepted. It is typical for good conceptual ideas to be resurrected periodically and tested against current implementation technologies to determine their practical feasibility and whether it is possible to achieve and demonstrate the full potential of the concept. The current state of technology, in terms of radar capability, onboard processing, miniaturization, satellite positioning, satellite control, and intersatellite communication, appears ready to make a serious assault on the demonstration and utilization of the Distributed Aperture Radar concept. Although the component technologies may be ready, the management and control of a Distributed Aperture Radar constellation is still a major area to be addressed. The applicability of satellite constellation is, of course, not limited to radar. See, for example, [5] for a description of the use of a satellite cluster to perform measurements of the Earth's magnetosphere.

Smaller, simpler, cheaper satellites, which are the hallmark of the distributed aperture concept, are more economical than large satellites. By being physically separated, the distributed array constellation is less vulnerable to being taken out by a single weapon, meteor, or collision with spacecraft debris. This requires that all satellites be identical, that they all perform similar tasks (forming a network of peers and not a master/slave configuration), and that all distributed operations be readily reconfigurable in case of failure.

These requirements pose exciting new challenges to the controls community. One of these challenges is to determine how simple identical control systems for individuals in the cluster can be chosen to achieve a given collective behavior. Although this area is currently being researched, the systems and objectives considered so far have been rather simple. The work we present in this paper will further this research to the stage where it can be successfully applied to real-world, complex problems.

This paper is organized as follows. In Section 2 we will briefly discuss a possible configuration for the cluster (for more discussion on this topic see [4]). In Section 3 we present the basic elements of the perceptive control theory. Finally, in Section 4 we show how it can be applied to the control of the satellite constellation.

2 Cluster Description

To achieve the DAR mission, the satellites in the cluster have to cover a certain surface with a more or less uniform distribution. Given the severe fuel constraints, the satellites also have to be in "natural" orbits while on formation (i.e., orbits that theoretically can be maintained without the use of thrusters). For computation purposes in this paper, we have selected a formation of 16 satellites. The orbits we selected are similar to the ones introduced in [6]. Although the orbits we use may not be the most efficient for carrying out the DAR mission, they capture the essence of satellite formation control.

First, we determine a nominal orbit for the center of the cluster. This orbit will be circular. For our computations, we selected a polar orbit with a radius of 7016 km (or an altitude of 10% of the

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Earth's radius).

To locate the other satellites in the constellation, we will use the linearized equations of motion around a circular orbit of period Ω. Let x be the coordinate along the velocity vector, z the coordinate along the vertical and let y complete the direct referential. The linearized equations (known as Hill's equations), have the form given in (1).

$$\begin{aligned} \ddot{x} &= -2\Omega z \\ \ddot{y} &= -\Omega^2 y \\ \ddot{z} &= 2\Omega\dot{x} + 3\Omega^2 z \end{aligned} \tag{1}$$

The unforced solution to these equations is given in (2):

$$\begin{aligned} x &= 2A\sin(\Omega t + \phi) \\ y &= B\sin(\Omega t + \psi) \\ z &= -A\cos(\Omega t + \phi) \end{aligned} \tag{2}$$

For the projection of these solutions on the xy plane to be a circle, we need to choose $B = 2A$. To set the n satellites at equal spacing around this circle we choose for the ith satellite:

$$\begin{aligned} \psi &= \frac{2\pi(i-1)}{n} \\ \phi &= \psi + \frac{\pi}{2} \end{aligned}$$

Since the orbit of any satellite can be derived from that of any other by a change in the origin of time, all orbits are geometrically identical, and thus all have the same energy and eccentricity. To compute the eccentricity of the orbit, first we compute the ratio λ of apogee to perigee:

$$\lambda = \frac{R_{\max}}{R_{\min}} = \frac{(R_o + A)^2 + 4A^2}{(R_o + A)^2 + 4A^2}$$

The eccentricity ϵ will be given by:

$$\epsilon = \frac{\lambda - 1}{\lambda + 1}$$

For a cluster with nominal altitude 7016 km and 200 m in diameter, we will have

$$\begin{aligned} A &= 0.050km \\ \lambda &= 1 + 2.851 \times 10^{-5} \\ \epsilon &= 1.425 \times 10^{-5} \end{aligned}$$

3 Perceptive Control Theory (PCT)

The basis for our formation layer control algorithms will be the perceptive control theory [2, 7]. This theory has been successfully applied to formation control of multiple autonomous agents [1, 3, 9]. In the perceptive control theory, the key component of a system, such as a formation of multiple satellites, is defined to be the *perceptive action reference.* This action reference is computed online, based on sensory measurements. The online planner of the system generates the desired state values for the system, according to the computed action reference. In addition, the action reference is calculated near or at the same rate as the feedback control. In other words, the action plan is adjusted at a high rate, which enables the planner to handle unexpected or uncertain discrete/continuous events or planned system reconfiguration. Furthermore, unlike traditional methods that require controller replanning to complete the task once the event or reconfiguration is over, the perceptive control in this case does not require any replanning.

More important, with respect to the perceptive reference frame, the discrete and continuous control commands can be easily modeled in a unified system model. This provides a novel and convenient framework for modeling hybrid dynamic systems. The issues related to the integration of logic and continuous control such as operation mode switch can be easily addressed in this framework.

A formation of multiple vehicle spacecraft has its special action reference according to which the tasks are synchronized. Traditionally, this action reference is the time. A task schedule or action plan can be described with respect to time. It is convenient to use time as the action reference, since it is easy to obtain and can be referenced by different entities of a systems. Humans, however, do not necessarily act by referencing a time frame. Human actions are usually based on human perception. Therefore, the development of an action reference directly related to the sensory measurement and the mission task is the key to the perceptive frame. A perceptive frame will be a mathematical abstraction of these action references. It is a projection from the state $X \in \mathcal{X}$ to a reference $s \in \mathcal{S}$,

$$\Pi : \mathcal{X} \mapsto \mathcal{S}$$

Based on this projection, an action plan of a multi-vehicle formation in the perceptive frame is parameterized by the corresponding action reference parameter. Since the action reference is a function of the real-time sensory information, the desired quantities generated by the action plan are also directly related to the measured data. This creates a mechanism to provide information in the form of numerical values for the base plan using the measurements, and the information is interpreted through a perceptive frame to determine the control input value. Thus, the planning becomes a closed-loop real-time process. The plan (desired action) can be perceived as an "abstract" entity, like a function. It only has a "real" value when measurements are made within the given perceptive frame.

4 Application of PCT to Cluster Formation Flying

The perceptive control theory is a general method for controller design of formations of multiple autonomous agents. A simplified schematic of the control architecture is shown in Figure 1.

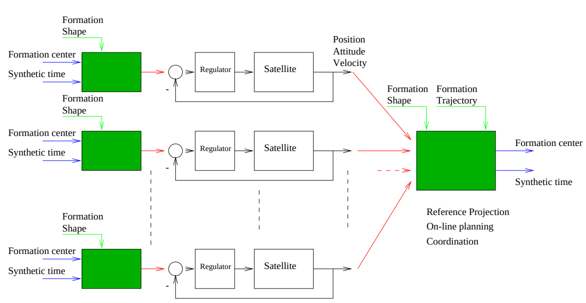

Figure 1: Formation Control Layer Architecture

To develop a formation control scheme for a multiple-vehicle spacecraft, the perceptive control method will be generalized to a broader sensing-based perceptive context. The first step is to find an appropriate perceptive action reference that can efficiently represent and transfer the output measurements of each vehicle to all action planners. The second step is to develop a computational scheme for determining the perceptive reference based on real-time sensor measurements. The third step is the design of a general-purpose controller with respect to a perceptive reference frame.

In our approach, we will compute a perceptive reference variable—synthetic time—that is communicated to the individual vehicles, which use it to generate local guidance commands. Given the current state of the set of vehicles $s_1(t), s_2(t), \ldots, s_n(t)$ and their preplanned trajectory indexed in a parameter τ, $\sigma_1(\tau), \sigma_2(\tau), \ldots, \sigma_n(\tau)$, the reference variable θ is computed as

$$\theta = \arg\min_{\tau}(\|(s_1(t), s_2(t), \ldots, s_n(t)) - (\sigma_1(\tau), \sigma_2(\tau), \ldots, \sigma_n(\tau))\|_P)$$

It can be shown that for norms P which meet a certain set of conditions, the system represented in Figure 1 is stable in the sense of Lyapunov if the individual components are stable. Once θ is determined, each individual satellite regulates to the command $\sigma_i(\theta)$. The nominal trajectories can be stored in each vehicle relative to a cluster center. In this way, the trajectory is separated into two components, one describing the overall behavior of the cluster and one describing its shape.

4.1 Virtual Leader Reference Projection

For our example, we use as a reference variable the position of the "center" of the constellation. Since the trajectories we are considering are symmetric, we computed the center by taking the average of the current positions. Let P_i be the absolute position of the i^{th} satellite at time t and $P_{ni}(\theta)$ a parameterization of the nominal trajectory of the i^{th} satellite. The measured center at time t is then given by

$$C(t) = \frac{1}{n}\sum_{1}^{n} P_i$$

and a parameterization of the nominal trajectory for the constellation center is

$$C_n(\theta) = \frac{1}{n}\sum_{1}^{n} P_{ni}(\theta)$$

The reference projection is then obtained by finding the point in the nominal trajectory for the constellation center closest to the current center. Figure 2 shows graphically how this is done for the case of a constellation of two satellites. The reference variable will be the time index θ that minimizes the distance between the nominal center at time θ and the current position of the center. We denote this projection by Π:

$$\theta = \Pi(C) = \arg\min\left(\|C(t) - C_n(\theta)\|\right)$$

This approach has the advantage of not requiring a master slave architecture to be set up. We are thus able to have a completely symmetric formation control layer. The benefits of this are a simpler architecture and more tolerance to faults.

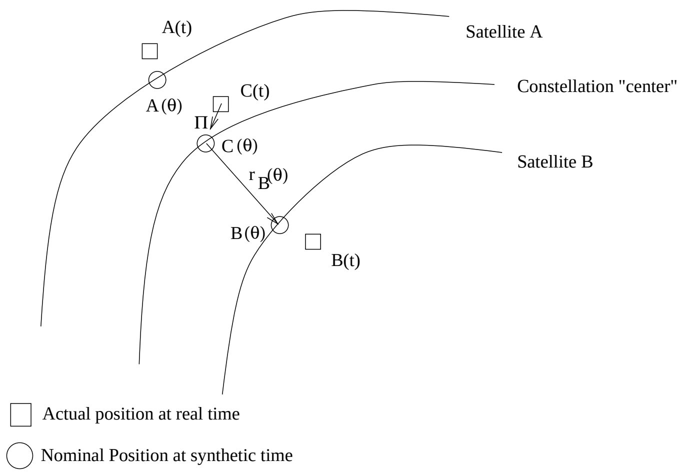

Figure 2: Reference projection using constellation center.

4.2 Use of Relative Position Sensing

Relative position between satellites can be obtained with much higher accuracy than absolute position. Since we are tracking a nominal trajectory given in absolute position for each satellite it may seem that we will suffer from these larger errors. However, by expressing the nominal trajectories for each satellite relative to the constellation center, and by computing the constellation center using both relative and absolute position, we can easily work around this problem. In this case, each satellite will compute a reference projection. As we shall see, this will not significantly affect the shape of the formation. If P_i is the absolute position of the i^{th} satellite the relative position between the i^{th} and j^{th} satellites will be given by:

$$P_{ij} = P_j - P_i$$

Let ϵ_i be the error in absolute position for the i^{th} satellite. It can then compute the measured center of the constellation as:

$$C_{mi} = \frac{1}{n}\left(n\left(P_i + \epsilon_i\right) + \sum_{i=2}^{n} P_{ij}\right)$$

The satellite now computes its position command P_i^c by summing the desired relative position from nominal center to the measured (noisy) position of the center:

$$P_i^c = r_i\left(\Pi\left(C + \epsilon_i\right)\right) + C + \epsilon_i$$

We can approximate this expression by linearizing the projection:

$$P_i^c \approx r_i\left(\Pi\left(C\right)\right) + \nabla r_i\left(\Pi\left(C\right)\right)\epsilon_i + C + \epsilon_i$$

The measured tracking error $\overline{e}$ will now be:

$$\overline{e} = P_i^c - P_{im} \approx e + C(t) - C\left(\Pi(C)\right) + \nabla r_i\left(\Pi\left(C\right)\right)\epsilon_i$$

where e is the actual tracking error. Forcing the measured tracking error to be zero,

$$\overline{e} = 0 \Rightarrow e = -\left(C(t) - C\left(\Pi(C)\right) + \nabla r_i\left(\Pi\left(C\right)\right)\epsilon_i\right)$$

The real tracking error thus has two components: one that does not depend on the satellite, and thus does not affect the constellation shape, and one that depends on the measurement error and the gradient of the nominal trajectory. Since this will be a small number, it will dampen the effect of the measurement errors.

4.3 Simulation Results

All the simulations were carried out using the development environment developed for Air Force Research Labs (AFRL) and Techsat21 by Princeton Satellite Systems (PSS). The constellation control algorithms were written as an "Agent Skill" and added to the simulation developed by PSS to demonstrate Team Agent to AFRL. The simulation model included the first three harmonics for the Earth's potential. The nominal trajectories were planned using only Hill's equations. This was done to demonstrate the ability of the control algorithm to ignore perturbations that affect the constellation as a whole. The reference projection was done in orbital element space. The center of the constellation was projected into a circular orbit with the same semiaxis, longitude of line of nodes, and inclination as the measured center. Let Orbel(C_m) be the classical orbital elements associated with the position and velocity of C_m:

$$[a, i, \Omega, \omega, e, M] = \text{Orbel}(C_m)$$

where a is the semimajor axis, i is the inclination, Ω is the longitude of the line of nodes, ω is the argument of periapsis, e is the eccentricity, and M is the mean anomaly. Also let n denote the mean motion of the orbit. We then define the projection Π as

$$\Pi(C_m) = \theta = n(M + \omega)$$

the projected center of the constellation will be.

$$[a, i, 0, 0, 0, M + \omega] = \text{Orbel}(C_n(\Pi(C_m)))$$

Relative position with respect to this center was then computed by propagating the initial conditions from the equator (nominal apogee) to the current mean anomaly (synthetic time) as described in Section 2. The commanded position for the i^{th} satellite is

$$\bar{S}_i = S_i^{\text{nom}}(\Pi(C_m)) - C_n(\Pi(C_m)) + C_m$$

Each satellite controlled to its commanded position by solving a linear program to minimize the fuel required to achieve the new orbit.

Orbit corrections were performed every 800 sec, allowing 600 sec for the correction. Figures 3 and 5 show the behavior of the constellation with no control. Figures 4 and 6 show how the control restores the original relative shape of the constellation, even in the presence of significant position noise (40 m average). The dotted line in Figure 4 show the position of a single satellite with the same initial conditions as the constellation under the same gravitational model. The proposed algorithm thus ignores the effects of unmodeled gravitational effects on the constellation, as was desired. Note that a satellite orbiting a point-mass potential would have drifted several kilometers from this position in the given time.) The use of initial conditions such as those developed in [8] can be easily accommodated and further enhances the performance of the algorithm.

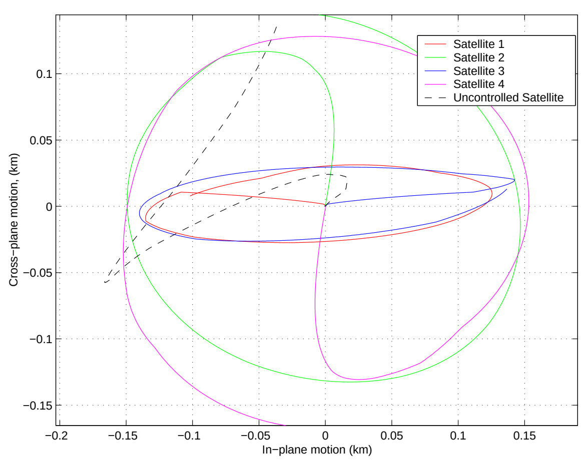

Figure 3: Satellites' motion relative to constellation center (no constellation control).

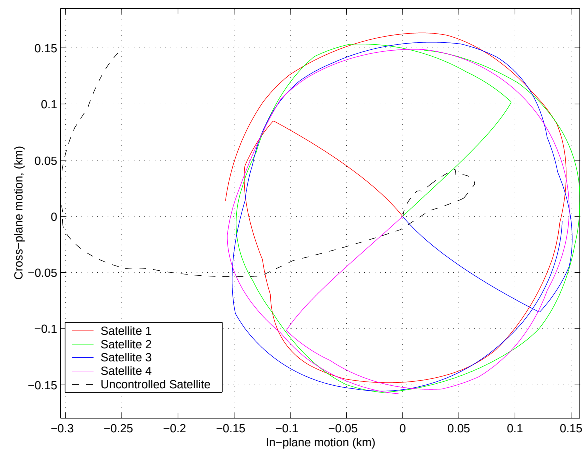

Figure 4: Satellites' motion relative to constellation center (controlled constelation).

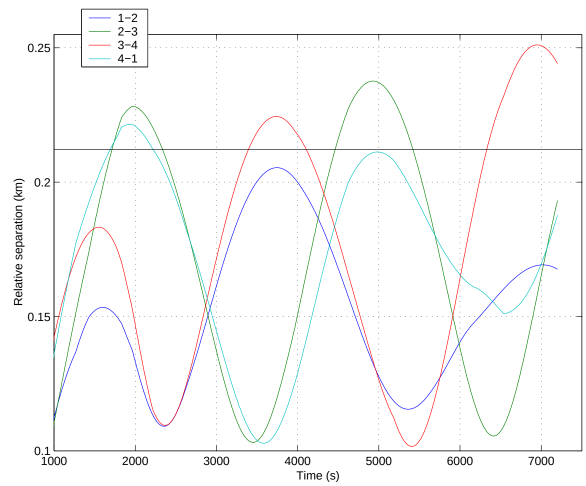

Figure 5: Satellites relative distance to each other. Uncontrolled Constelation.

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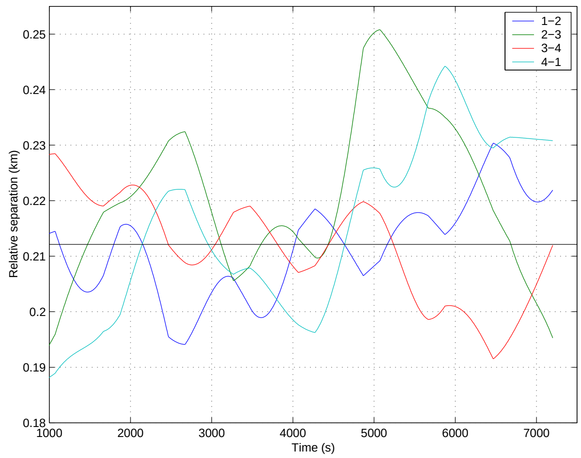

Figure 6: Satellites relative distance to each other. Controlled Constelation.

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