Control of Arm Movement Using Population of Neurons

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Abstract

Movements of human arm in a horizontal plane are very stereotyped in the sense that the corresponding paths are mainly straight lines and the velocity profiles are "bell-shaped like" functions. A dynamics of two link model of the human arm has been studied with the goal of synthesizing the torques which accomplish the desired transfer. For that purpose a set of parameters which describes the desired transition (initial position, final position, peak velocity, etc.) is chosen randomly according to a certain distribution. The parameters of the desired trajectory as well as the system variables (angles and angular velocities) are encoded using populations of different number of neurons, usually $100 - 150$. The underlying mathematics including integration, differentiation and other algebraic relationships, has been done at the level of neuronal activity. Finally, the driving torques are generated from the corresponding activities using an optimal decoding rule.

1 Introduction

It has been experimentally verified that humans tend to reach from one point in a horizontal plane to another in a stereotyped fashion, that is, the path of a human wrist is primarily a straight line, while the corresponding velocity profile is a bell-shaped function. Moreover, the peak velocity and the distance traveled by a wrist are not independent, i.e. the longer the distance, the higher the peak velocity, which implies that the total time of transfer remains fairly constant over different experimental trials, see [1].

2 Mathematical Model

The model of a human arm is represented as a two link rigid body (no muscles are assumed at this point), see Fig. 1. Using standard tools from analytical mechanics (kinetic and potential energy, Lagrangian and Lagrange equations of motion) the nonlinear

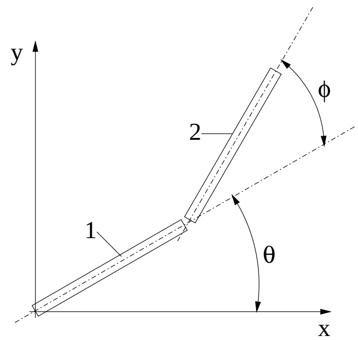

Figure 1: Two link system in horizontal plane.

model of the two link system has been obtained:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $x = [\theta, \phi, \dot{\theta}, \dot{\phi}]^T$. Equation (1) can be rewritten as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = -T^{-1}(x)C(x)\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + T^{-1}(x)\begin{bmatrix} \tau_s \\ \tau_e \end{bmatrix} \tag{2}$$

where

$$T(x) = \begin{bmatrix} t_1 + t_2 + 2t_3\cos(x_2) & t_2 + t_3\cos(x_2) \\ t_2 + t_3\cos(x_2) & t_2 \end{bmatrix}$$

represents the inertial matrix, and

$$C(x) = \begin{bmatrix} -t_3\sin(x_2)x_4 & -t_3\sin(x_2)(x_3 + x_4) \\ t_3\sin(x_2)x_3 & 0 \end{bmatrix}$$

is the matrix of Coriolis and centripetal terms. Since the problem is defined in a horizontal plane, there are no gravity forces in the model (2). The friction forces have been neglected for this analysis. The terms t_1, t_2 and t_3 are defined as follows:

$$t_1 = m_1(\tfrac{l_1}{2})^2 + m_2 l_1^2 + J_1 \qquad t_2 = m_2(\tfrac{l_1}{2})^2 + J_2$$

$$t_3 = m_2 l_1 \frac{l_2}{2}$$

where m_i is the mass of the $i-th$ link, l_i is its length and J_i represents its moment of inertia with respect to its center of gravity ($i = 1, 2$).

3 Control

The vector of external inputs $u = [\tau_s, \tau_e]^T$ contains the two torques τ_s (shoulder) and τ_e (elbow), both applied in a horizontal plane, which are to be synthesized by a population of neurons. The torques are first found analytically using *feedback linearization*, a procedure for stabilizing certain class of nonlinear systems, see [2]. This provides an elegant solution i.e. the synthesized torques depend both on the desired parameters (via desired angles and desired velocities) and the actual position and velocity, and thus can be viewed as a combination of feedforward/feedback signal. The feedback linearization technique is based on a local change of coordinates which define so called *normal form* of the system (2). Let us first define two output equations for the system (2), where y_1 and y_2 are chosen such that the system has defined the vector *relative degree* $r = [r_1, r_2]$, see [2]:

$$\begin{aligned} y_1 &= h_1(x) \\ y_2 &= h_2(x) \end{aligned} \tag{3}$$

where $h_1(x) = x_1$ and $h_2(x) = x_2$. Note that the definition of y_1 and y_2 is only for the purpose of exact feedback realization, and is chosen arbitrarily. However the choice of the functions $h_1(x)$ and $h_2(x)$ is sensible, as $x_1 = \theta$ and $x_2 = \phi$ are readily available for measurements (feedback). Thus the system (2) can be rewritten as:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \\ \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} &= -T^{-1}(x)C(x)\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + T^{-1}(x)\begin{bmatrix} \tau_s \\ \tau_e \end{bmatrix} \\ y_1 &= h_1(x) \\ y_2 &= h_2(x) \end{aligned} \tag{4}$$

For this system it is easily found that:

$$L_{g_1}h_1 = 0 \qquad L_{g_2}h_1 = 0$$

and

$$L_{g_1}L_f h_1 \neq 0 \qquad L_{g_2}L_f h_1 \neq 0$$

Likewise

$$L_{g_1}h_2 = 0 \qquad L_{g_2}h_2 = 0$$

and

$$L_{g_1}L_f h_2 \neq 0 \qquad L_{g_2}L_f h_2 \neq 0$$

where $L_f h \doteq \frac{\partial h(x)}{\partial x} f(x)$ represents the *derivative of the vector field h along f*. Therefore the system (4) has defined vector relative degree $r = [2, 2]$ at some initial point x°. This, together with the fact that $rank(g(x^\circ)) = 2$ necessarily imply the solvability of exact linearization problem i.e. there exists a change of coordinates defined locally around x°:

$$\begin{aligned} z_1 &= h_1 &= x_1 \\ z_2 &= L_f h_1 &= x_3 \\ z_3 &= h_2 &= x_2 \\ z_4 &= L_f h_2 &= x_4 \end{aligned}$$

and a smooth feedback law

$$u = -A^{-1}(x)b(x) + A^{-1}(x)v \tag{5}$$

where

$$A(x) = \begin{bmatrix} L_{g_1}L_f h_1(x) & L_{g_2}L_f h_1(x) \\ L_{g_1}L_f h_2(x) & L_{g_2}L_f h_2(x) \end{bmatrix} = T^{-1}(x)$$

$$b(x) = \begin{bmatrix} L_f{}^2 h_1(x) \\ L_f{}^2 h_2(x) \end{bmatrix} = -T^{-1}(x)C(x)\begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$$

such that the obtained system is in *Brunowsky canonical from*

$$\begin{aligned} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} &= \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} z_1 \\ z_2 \\ \hline z_3 \\ z_4 \end{bmatrix} + \left[\begin{array}{cc} 0 & 0 \\ 1 & 0 \\ \hline 0 & 0 \\ 0 & 1 \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \\ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} &= \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \end{array}\right] \begin{bmatrix} z_1 \\ z_2 \\ \hline z_3 \\ z_4 \end{bmatrix} \end{aligned} \tag{6}$$

Note that the system (6) is completely decoupled i.e. it can be considered as two second order systems, which are completely controllable and completely observable, but are not asymptotically stable. Therefore the asymptotic tracking problem can easily be solved, since the poles of the two systems can be arbitrarily assigned. Introducing the new feedback law $v_i = v_i{}^* + F_i Z_i$, see [3], each of the decoupled second order linear systems become

$$\dot{Z}_i = \begin{bmatrix} 0 & 1 \\ -f_{i1} & -f_{i2} \end{bmatrix} Z_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_i{}^* \qquad i = 1, 2$$

$$y_i = \begin{bmatrix} 1 & 0 \end{bmatrix} Z_i \tag{7}$$

where

$$F_i = \begin{bmatrix} -f_{i1} & -f_{i2} \end{bmatrix} \quad \text{and} \quad Z_i = \begin{bmatrix} z_{2i-1} \\ z_{2i} \end{bmatrix}$$

The second equation of the system (7) can be rewritten by back substitution as

$$\ddot{\theta} = -f_{11}\theta - f_{12}\dot{\theta} + {v_1}^* \tag{8}$$

for $i = 1$, and likewise

$$\ddot{\phi} = -f_{21}\phi - f_{22}\dot{\phi} + {v_2}^* \tag{9}$$

for $i = 2$. Thus, v_1^* and v_2^* have to be chosen so that the error differential equation has the following form

$$\ddot{\epsilon} + f_{i2}\dot{\epsilon} + f_{i1}\epsilon = 0 \tag{10}$$

where $\epsilon = \theta_d - \theta$ or $\epsilon = \phi_d - \phi$ and the subscript d stands for desired. For asymptotic stability, we need $f_{i1}, f_{i2} > 0$. From (8), (9) and (10) readily follows

$$\begin{aligned} {v_1}^* &= \ddot{\theta}_d + f_{12}\dot{\theta}_d + f_{11}\theta_d \\ {v_2}^* &= \ddot{\phi}_d + f_{22}\dot{\phi}_d + f_{21}\phi_d \end{aligned} \tag{11}$$

Hence, the torque pair which will cause the system (4) to follow the desired trajectory parameterized by (θ_d, ϕ_d) and the desired velocity profile parameterized by $(\dot{\theta}_d, \dot{\phi}_d)$ is given by

$$\begin{bmatrix} \tau_s \\ \tau_e \end{bmatrix} = C \begin{bmatrix} \dot{\theta} \\ \dot{\phi} \end{bmatrix} + T \begin{bmatrix} \ddot{\theta}_d + f_{12}(\dot{\theta}_d - \dot{\theta}) + f_{11}(\theta_d - \theta) \\ \ddot{\phi}_d + f_{22}(\dot{\phi}_d - \dot{\phi}) + f_{21}(\phi_d - \phi) \end{bmatrix}$$

The similar result can be obtained using so called *inverse dynamics* approach, as the synthesized torques yield a cancellation in the dynamics of an arm, see [4]. The block diagram of the system has been shown in the figure below.

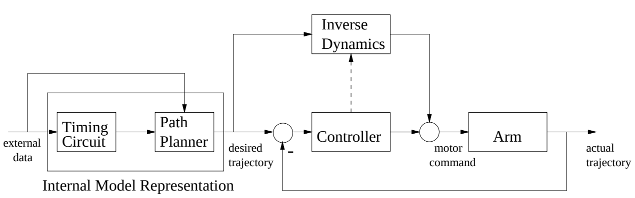

Figure 2: Block diagram of an arm control system, showing both a feedback and an adaptive (error modulated) feedforward control. Internal dynamics is a dynamical system comprising a timing circuit as well as a path planner.

4 Neural Dynamics

The point of departure in synthesizing the controlling torques using a population of neurons is a *path plan*; straight line path and bell-shaped velocity profile with known parameters such as the distance, direction and

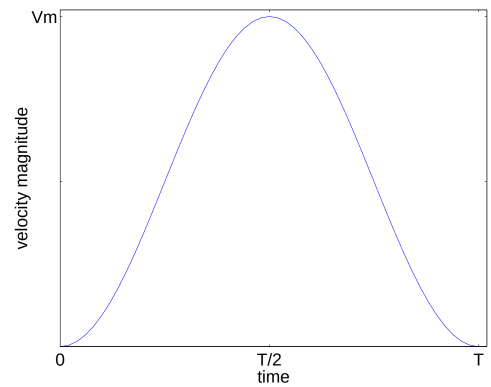

Figure 3: The velocity profile plot; V_m– peak velocity, T– total time of transfer.

peak velocity of the movement (Fig. 3), clearly constrain the desired angular velocities.

The initial position of the arm has been randomized by choosing the initial angles from a uniform distribution. The final position is also random in the sense that it is determined by a human decision (where we want to reach). The peak velocity is chosen randomly but with high correlation to the total distance, which is known once the initial and final position have been specified, see Fig. 4. This assumption has a biological relevance; if we want to reach farther, we do it with higher velocity so that the total reaching time remains fairly constant over different trials. The average time of the transfer is assumed to be 0.5 (*sec*). Using the obvious kinematic

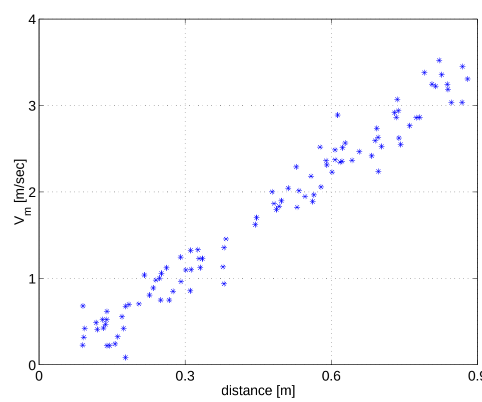

Figure 4: Correlation between the peak velocity V_m and the distance.

relationship, the velocity of the wrist can be expressed as

$$\vec{V}(t) = \frac{V_m}{2}(1 - \cos(\frac{2\pi t}{T}))\frac{\vec{r_f} - \vec{r_i}}{\delta} \tag{12}$$

where $\vec{r_i}$ and $\vec{r_f}$ are the vectors of the initial and final positions of the arm, $\delta = \| \vec{r_f} - \vec{r_i} \|$, and $\frac{V_m}{2}(1 - \cos(\frac{2\pi t}{T}))$ is a mathematical fit of the bell shaped function described earlier, which can be thought of as a solution to the second order differential equation

$$\ddot{u} + \omega^2 u = \omega^2 \qquad (13)$$

with zero initial conditions, where $\omega = 2\pi t/T$. The equation (13) is solved using a population of neurons approach, where the neurons are assumed to be responsible for encoding analog variables/vectors using their activities. The neuronal activity is a frequency in its nature and represents the instantaneous firing rate associated with the neurons. The firing rates of neurons are assumed to be piecewise linear, positive semidefinite functions of analog *meta* variables, see [5]. In the case of scalar variables taking both positive and negative values, the concept of so called on/off cell has been used see Fig. 5. For vectors, a preferred direction is what determines the extent of population response. Therefore the analog variable u is being encoded by a

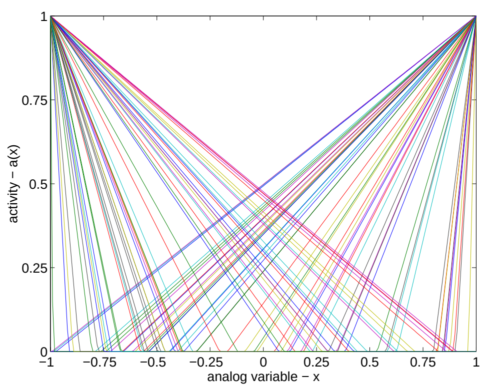

Figure 5: The normalized activity $a(x)$ as a function of an analog variable which takes values between -1 and 1.

population of neurons using a nonlinear transformation $u \rightarrow a(u)$ where the shape of activity function depends on a choice of the firing neuron model, and is chosen as a piecewise linear function in our study i.e.

$$a_i(u) = [\alpha_i u + \beta_i]_+$$

where $[\ \ ]_+$ stands for a rectification symbol, as the negative activities are not physically meaningful. On the other hand, the analog variable u can be reconstructed form the activities by using a *linear decoding rule*, i.e.

$$u^{est}(u) = \sum_{i=1}^{N} X_i a_i(u)$$

where X_i represent *optimal decoding weights* (see [6]) in the sense that they minimize the error defined as

$$Error = \frac{1}{u_{max} - u_{min}} \left\langle \int_{u_{min}}^{u_{max}} [u - u^{est}(u)]^2 du \right\rangle_\eta$$

and η is an additive zero mean Gaussian noise with the variance σ. The symbol $< \ >_\eta$ stands for an average over noise η (see [7]), which is incorporated in the model via

$$u^{est}(u) = \sum_{i=1}^{N} X_i [a_i(u) + \eta_i]$$

where η_i are independent identically distributed random variables $\eta_i \sim \mathcal{N}(0, \sigma^2)$ and N is the total number of neurons within a population. Likewise, there is another population of M neurons which encodes $\dot{u}$ via a nonlinear transformation i.e.

$$b_j(\dot{u}) = [\gamma_j \dot{u} + \delta_j]_+$$

with the corresponding decoding rule

$$\dot{u}^{est}(\dot{u}) = \sum_{j=1}^{M} Y_j b_j(\dot{u})$$

The differential equation (13) can now be translated at the level of activities

$$\frac{da_n(t)}{dt} = -\frac{1}{\tau}\left\{ a_n(t) - \left[\sum_{i=1}^{N} \omega_{ni}^{(int)} a_i(t) + \tau \sum_{j=1}^{M} \omega_{nj}^{(ext)} b_j(t) + \beta_n \right]_+ \right\}$$

$$\frac{db_m(t)}{dt} = -\frac{1}{\tau}\left\{ b_m(t) - \left[\sum_{j=1}^{M} \omega_{mj}^{(int)} b_j(t) + \tau(\omega^2 - \omega^2 \sum_{i=1}^{N} \omega_{mi}^{(ext)} a_i(t)) + \delta_m \right]_+ \right\}$$

where the coupling weights for the oscillator (13) are given by

$$\begin{array}{ll} \omega_{ni}^{(int)} = \alpha_n X_i & \omega_{nj}^{(ext)} = \alpha_n Y_j \\ \omega_{mj}^{(int)} = \gamma_m Y_j & \omega_{mi}^{(ext)} = \gamma_m X_i \end{array}$$

The solution u and $\dot{u}$ and the corresponding activities $a(t)$ and $b(t)$ are given in Fig. 6 and 7, respectively. Once the timing circuit function has been generated, it is used as a driving signal for the path planner which indeed provides the desired angles and their first and second derivatives. Using (12) as well as basic kinematics of the system, the path planner is described by the nonlinear differential equation

$$\begin{bmatrix} \dot{\theta}_d \\ \dot{\phi}_d \end{bmatrix} = M^{-1}(\theta_d, \phi_d) \frac{V_m}{2} (1 - \cos(\frac{2\pi t}{T})) \frac{\vec{r_f} - \vec{r_i}}{\delta} \qquad (14)$$

where

$$M(\theta_d, \phi_d) = \begin{bmatrix} -l_1 \sin(\theta_d) - l_2 \sin(\theta_d + \phi_d) & -l_2 \sin(\theta_d + \phi_d) \\ l_1 \cos(\theta_d) + l_2 \cos(\theta_d + \phi_d) & l_2 \cos(\theta_d + \phi_d) \end{bmatrix}$$

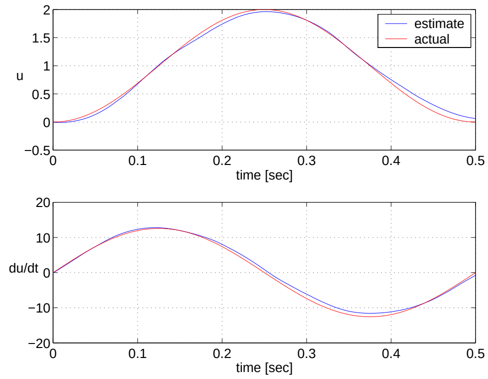

Figure 6: The solution of the oscillator (timing circuit) given by (13).

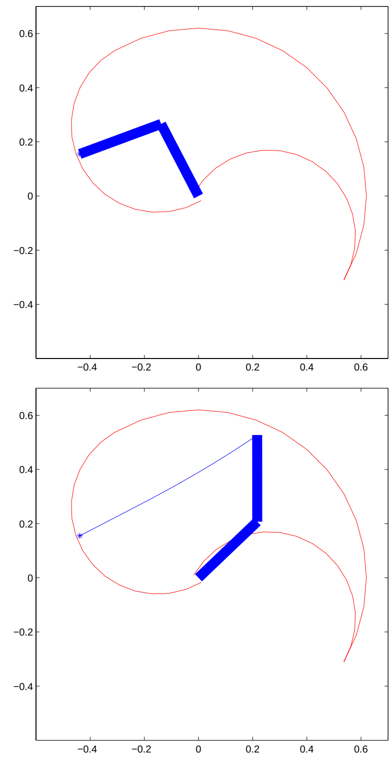

Figure 8: The initial (left) and final position of the arm together with the path (right).

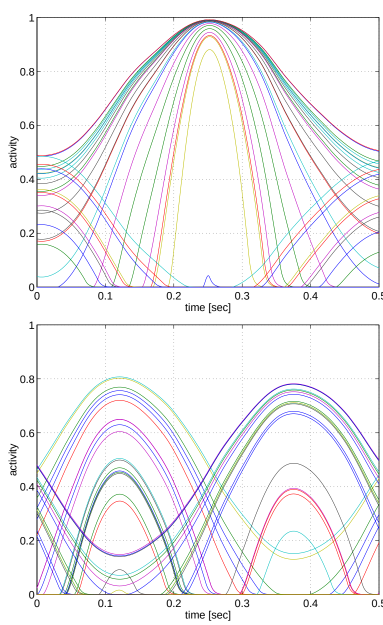

Figure 7: The neuronal activities as functions of time. The activities $a(u(t))$ (left) and $b(\dot{u}(t))$ (right).

The equation (14) is to be solved by another population of neurons, where $\overrightarrow{r_i}$ and $\overrightarrow{r_f}$ are encoded using a rule of *preferred direction*. This can be thought of as *internal dynamics*, see [8], and represents a feedforward path in the synthesis of the control torques (Fig. 2). Likewise, the actual position and velocities can be encoded using another populations of neurons, which comes from the human sensory system and is included in the structure of the torques as a feedback path. The neuronal activities are often referred to as *explicit space*, because biologists can make direct measurements on them, see [5]. Finally, the two torques are encoded by ensembles of neurons, whose activities (firing rates) involve the whole set (vector) of analog variables e.g. the desired and actual angles, desired and actual velocities, etc. The torques are then obtained from the activities using optimal decoding rule, in the same way it was described earlier. Note that this procedure represents the reconstruction of analog variable/vector as a weighted sum of suitably chosen basis functions, see [6], and where the weights are chosen in such a way that the reconstruction error is minimized. In this case, the basis functions are actually the neuronal activities, and the desired torques are simply linear (weighted) combination of these functions.

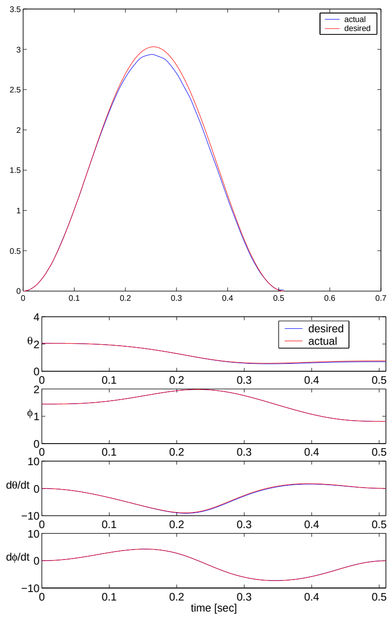

Figure 9: The desired and actual velocity profiles (left) and the desired and actual angles and angular velocities (right).

5 Simulation Results

The procedure described above has been tested by simulations, and the deviations of the actual path from the desired one (straight line) are negligible, see Fig. 8. Also the specified velocity profile and angles have been followed quite accurately, Fig. 9 which is not surprising as a population of $100 - 150$ neurons does fairly well in terms of the precision of representation.

6 Conclusion

The experimental evidence shows that the movements of human arm in a horizontal plane are very simple. Very often in a task of reaching from one point to another, we use a straight line path and a bell shaped velocity profile. The question of interest is how do we generate the torques which will make the arm accomplish the desired transfer while following the constraints imposed on it? In robotics, tracking the desired trajectory is a very standard problem, which can easily be solved using feedback linearization. This procedure is elegant in the sense that the generated control (torques) are synthesized through feedforward/feedback subsystems, where the feedforward action can be attributed to a specific neural circuitry such as the cerebellum. On the other hand, the source of the feedback is yet to be specified, even though there is no doubt about its existence. We assumed it originates from the sensory system, and is incorporated in the model with a delayed action, as any other feedback law would be. It might be even coming from a visual system, although the delays would be much higher in that case. Of course the combination of the two is the most realistic scenario. The future work would be in the direction of making the more realistic model of human arm, using muscles and their dynamics. The activities will then be driving the muscles, which will respond by exerting forces and torques, consequently. Also, the other types of movements can be studied, not necessarily in a horizontal plane, which certainly is a harder and more challenging problem.

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