A Stability Analysis Based on Economic Principles for the Control of the Cotton Aphid

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Abstract

The cotton aphid is an important pest insect affecting the profitability of cotton production in the Southwest. In this paper, we study the problem of the optimal timing of pesticide application to control the aphid. The problem is complicated by the presence of a significant predator insect. The predator serves as a natural control of the aphid and is adversely affected by application of pesticide. We determine optimal state dependent rules for application of pesticide. We show that first application of pesticide is a switching time between two dynamic systems.

1 Introduction

The optimal application of pesticide is a problem made difficult by the very short half-lives of modern pesticides. At the present time, all approved pesticides deteriorate very quickly in the natural environment. Usually the half-life is less than two days. With pesticides which deteriorate quickly in the natural environment, the optimization becomes dynamic and the timing of the multiple application becomes important. By contrast, the optimization problem for pesticides with long half-lives (e.g. DDT) is simply one of dosage, since a single dose is usually sufficient for an entire growing season. Unfortunately, we have learned that pesticides with long half-lives travel through the ecosystem and can have disastrous effects on the upper end of the food chain - the brown pelican, American bald eagle, etc.

In this paper, we study the problem of the optimal timing of pesticide application of a farmer who seeks to maximize seasonal profits. This problem is complicated by the presence of a significant predator insect. The predator serves as a natural control of the aphid and is adversely affected by application of pesticide. The short half-life of the pesticide implies that the aphids and the predator insects are killed in great numbers for a short period of time and after which the kill from the pesticide is negligible. The recovery rate of the predator insect is much slower than that of the aphid.

The productivity of a cotton plant and the price of cotton depend on many factors and there exist very sophisticated and complete models of cotton production and output price. However, in order to emphasize the problem of application of pesticide, we use a very simple model of cotton plant, aphid and predator interaction. We allow all other factors to be viewed as random perturbations to the model. The farmer's problem is to maximize profits through selective application of pesticide. Pesticides are costly to apply and first application of pesticide radically affects the model dynamics - i.e. first application of pesticide determines the switching time between two dynamics systems.

In the absence of the predator insect, the problem is much simpler. We would simply find an evenly spaced pattern of application, such that, the marginal cost of spraying (here the cost of application) was exactly equal to the expected increase in discounted revenue at harvest. This solution has been found be many authors (Hall and Norgaard (1973), Talpaz and Borosh (1974), Talpaz et. al. (1978)). The idea that the impact on the predator insect must be explicitly recognized is given by Longworth and Rudd (1975). Harper and Zilberman (1989) incorporate a predator insect in their model; however, their predator acts in a minor way for controlling the pest and its recovery rate is similar to the pest.

Our problem is more complicated, in that, the first application of pesticide alters the dynamics of the model by effectually eliminating the predator population. That is spraying removes a natural control fro the aphid population. Hence, the relevant marginal cost now includes both the direct cost of spraying today and the increase in future cost associated with more frequent spraying in the future. The increase in future cost of spraying is directly related to the proportion of predators in existence. If the proportion of predators is very small, we are very close to the dynamics system after spraying and the future additional cost is small.

The decision problem of our farmer will include a state dependent decision rule for first spraying and a decision rule for determining the frequency of spraying after the first application. In this paper, we determine when first spraying should be postponed and under what conditions (states) the first application of pesticide should be undertaken. We will demonstrate that when the first application occurs is critical. If applied too early insecticide will have to be applied more times during the growing season increasing the cost. If applied too late, the cotton plant itself may be so damaged that the profitability of the crop will be compromised.

2 Model

The goal of the mathematical model is to develop a hybrid control model for the interaction. We include in the model pesticide application. The pesticide has the potential to radically affect the dynamics of the basic model and actually introduces a new model that is effective for a few days. Thus the application time of the pesticide is a switching time between two dynamical systems. Stability is not asked in this situation.

We describe a simple predator prey model of the aphid and predator insect interaction. For this to be meaningful we need to, at the same, time develop a model of the cotton plant and its interaction with the cotton aphid. We will include in the model a representation of the effect of the application of insecticide. The insecticide acts on both the aphid and the predator insect. The model will be set up to provide input for a cost and production model and the model will allow for quantitative predictions. The primary assumptions are: 1) the aphids are harmful to the cotton plant and have an effect on the total amount of cotton that can be harvested from a given field. 2) There are predator insects that feed primarily on the cotton aphid. 3) Under some conditions the cotton plant, the aphid population and the predator insect population with reach a mutually stable equilibrium. 4) Insecticide application is costly and the application effects both the aphid and predator insect population.

2.1 The Predator-Prey-Plant Model

The model presented here is based on the masters thesis of David Hogan of the Department of Mathematics and Statistics at Texas Tech University. The model is based on simple concepts of birth and death models. We assume that the number of aphids present in a cotton field at time $t+h$ is dependent on the number at time t, the number hatched in the time interval h, the number that have died in the time interval h. We will assume that the increase due to emigration is negligible and we will assume that the number that have immigrated is likewise negligible. We lump the predator insects into one group. We realize that different predator insects such a lacewings and lady beetles have quite different life cycles but to include all of the possibilities would be a daunting task. Since we are striving for simplicity we make the assumption that we have one class of predator insects with a somewhat average life cycle and feeding pattern. The model for the cotton plant is somewhat more problematic. The rate of growth of the cotton plant is not a function time but a complicated function of the environment. Our approach to the problem allows us to incorporate all factors which are outside the framers control (i.e. growth rate of cotton conditional on the aphid population, price process for cotton, heat days, water, etc.) into the underlying probability structure. To make our model more realistic, we will calibrate average cotton yield and average cost of spraying to more complex cotton models in existence. We are now able to completely describe the evolution of our system in the following equations:

$$
\begin{aligned}
A_{t+h} &= A_t + ®_{1t}A_t \ ¡ \ ±_{1t}A_t \\
P_{t+h} &= P_t + ®_{2t}P_t \ ¡ \ ±_{2t}P_t \\
C_{t+h} &= C_t + °_t C_t \ ¡ \ ´_t C_t
\end{aligned}
$$

where A_t is number of aphids at time t, P_t is the number of predators at time t and C_t represents the amount t of foliage on the cotton plant at time t. The functions $®_{it}$ are birth rates as are the functions $±_{it}$ death rates. The function $°_t$ is the growth rate of the cotton and $´_t$ is rate at which the plant is loosing foliage. We make a very important assumption about the rate functions. We assume that:

Assumption: The rate functions are proportional to the time interval h, i.e. the number of aphids that are hatched in a short time interval h is given by h*® where ® is independent of h.

This assumption allows us to pass to the limit as h becomes small to form di¤erential equations instead of di¤erence equations. It is then a matter of convenience whether we solve the model in discrete or continuous time. We now consider the coe¢cients of the di¤erential equations.

The growth rate of aphids is proportional to the food supply and hence at time t is proportional to the amount of foliage in the …eld. The basic birthrate can be recovered from the doubling time of the aphid population when undisturbed. The aphid produces about 25 o¤spring in three days or about 8 per day so that the population doubles in about 6 hours or about .25 days. Using the formula $2 = e^{f®tg}$ and solving for ® with t=.25 we have ® = 4 ¤ ln2 . Thus we arrive at a representation of $®_{h+t} = (¸C_t) + 4\ln 2)h$. The point is that h is a factor in each of the coef…cients. Using this we can pass to the limit as h becomes small and reduce the di¤erence equations to di¤erential equations.

We can likewise examine the death coe¢cient. It is a function of the natural death rate of the aphids, the number consumed by the predator insects and most importantly of the application of the pesticide. We will assume that the pesticide kills on the order of 99% plus of the aphids. However, the problem arises in that it kills the same percentage of predators as it does aphids. Since the doubling time of the predators is measured in days instead of hours we see that an application of pesticide has the e¤ect of removing the natural controls from the picture. The aphids then display exponential growth instead of bounded growth that one would expect in a well-balanced environment.

3 Agent's Problem

We model the cotton framer as a pro…t maximizing, risk neutral agent. That is, we assume separability in the farmer's production and consumption decisions. To elaborate further, the farmer in this model cares about cotton and pesticide only to the extent to which they a¤ect the outcome in terms of end of period net pro…t. We then obtain risk neutrality by thinking of the farmer as having access to a complete set of contingent claims over the relevant states. For the general principle of hedging risk, see Duffie (1996). For a specific problem of hedging risk in cotton production, see Gardner (1989). As a result, the farmer simply maximizes the total expected end of season return.

A common complaint of the separability assumption is that many farmers are dedicated environmentalists who cares about the effect pesticides have on the environment and should therefore incorporate the environmental impact of pesticides in their decision to spray. It is a simple matter to add such concern into the utility function of the farmer and therefore his decision rule; however, it is a difficult task to have this enter in the model in a way which is neither trivial nor dominating. As an alternative, we assume that the environmental effects of the pesticide are appropriately reflected in the price of the pesticide which the farmer faces. This causes the farmer to act as if we explicitly enter care for the environment into his utility function.

We assume the intensity of spraying is fixed. That is we assume the farmer decides when to spray but cannot adjust the amount of pesticide used. We justify this assumption by assuming there exists a single optimal level of spraying. For concreteness, think of having to employ a crop duster to spray the cotton. The primary cost of application then is the cost of flying the airplane. Therefore, once the decision to fly has been made it is optimal to spay heavily as the marginal cost of the extra pesticide is negligible.

Our final assumption is that decisions this season affect only this seasons crop. In particular, this implies that aphids do not develop resistance to pesticide over time. Again we assume any tie ins between periods is accounted for in the price of pesticide. Regev et. al. (1983) consider the problem when this assumption is dropped.

Our approach to the problem allows us to incorporate all factors which are outside the farmer's control (i.e. growth rate of cotton conditional on the aphid population, price process for cotton, heat days, water, etc.) into the underlying probability structure. In reality, the farmer can control some of the determinates we take as exogenous. For instance, the farmer can affect water levels by choosing to irrigate. However, so long as we consider the decision process for these actions as independent of both the evolution of aphids and their interaction with the cotton crop, the affect of these action on production is incorporated into the law of motion for cotton. The cost of these actions is accounted for by an adjustment in the price of cotton - i.e. we consider the only the net price of cotton.

We are now prepared to write the farmer's decision problem:

$$\max_{\{S_t\}} E \left\{ \sum_{t=0}^{T-1} \beta^{t} (-P_t^s S_t) + \beta^{T} P_T^C C_T \right\}$$

$$\begin{aligned} & \text{s.t.} \\ A_{t+h} &= A_t + \alpha_{1t} A_t - \delta_{1t} A_t \\ P_{t+h} &= P_t + \alpha_{2t} P_t - \delta_{2t} P_t \\ C_{t+h} &= C_t + \gamma_t C_t - \eta_t C_t \\ S_t &\in \{0, 1\} \end{aligned}$$

T is the harvest date. We assume a fixed growing season for cotton. $\{P^c; P^s\}$ are the price of spraying and cotton. We assume the price of cotton is zero for all dates prior to T. This ensures early harvest is never optimal. Sampling the aphid predator population in a field of cotton is expensive and time consuming. To reflect this, we choose a decision period of one week. However, to make the model realistic we update the evolution equations much more frequently. Therefore, in the computations, we set T equal to 16 reflecting a four month growing season.

The problem will be easier to analyze as a T period Bellman equation:

$$V(A, P, C, t) = \max_{S} \{ -P^{s} S + \beta E V(A', P', C', t+1) \}$$
$$V(A, P, C, T) = P^{c} C_{T}$$

Where the Bellman is subject to the above constraints. We will solve this problem by backward recursion on the Bellman equation.

Which dynamics system we are in depends on the level of P. If P is large, there exists a natural control of the aphid population and we consider ourselves to be in the pre-spraying dynamic system. If P is small, there is no natural control of the aphid population and we are in the post-spraying dynamics system. The doubling time of the predator population is such that we never move from a post to a pre spraying system. That is once the predators are reduced by spraying they do not recover. If P_0 is small we consider the first application of pesticide to have been in period $t = -1$.

We can see immediately, the solution to the problem. In the post-spraying dynamic system, the farmer will spray at regular intervals with the time between spraying determined solely by the cost of spraying and the growth rate of the aphids. In the pre-spraying system, the farmer will postpone spraying so long as the predators are acting as a significant control for the aphids. In the next section, we give the results of our model.

4 Results

To emphasize our main result, we have chosen not to display results for alternative specifications of the underlying difference equations. Instead, we take the difference equations as a given and explore the implications of different initial conditions - e.g. when will a farmer optimally spray if he starts with twice as many predators as aphids. We also explore the implications for different relative price of spraying. Recall that the farmer only cares about end of period profit; hence, we can double the price of spraying and the price of cotton without effecting the implications of the model. Below, we give the exact difference equations we use to estimate the model:

$$A_{t+1} = A_t + .0001 A_t C_t + .75 \ln 2 - .025 A_t P_t - .015 A_t^2$$
$$P_{t+1} = P_t + .0001 P_t A_t + .2 \ln 2 - .01 P_t^2$$
$$C_{t+1} = C_t + .03 - .0005 A_t - .001 C_t^2$$

To fix in the readers mind the implications of this set of equations, we simulate the equations from several different initial conditions and print the results in Figure 1. Notice, these simulations do not include any effects for spraying. The length of the simulation is 112 periods. This corresponds to daily updating for the length of the growing season - recall, the farmer is making weekly decisions; hence we have sixteen decision periods. We can see, for our equations, the most important factor for control of the aphids is starting with a relatively large number of predators. If we start with too small a predator population, they do not serve as an e¤ective control on the predators until it is too late to prevent damage to the cotton crop.

This property will have important implications for the decision problem. A large uncontrolled aphid population early in the season has large e¤ects on harvested cotton. The aphid population grows so quickly, that even in case where it is eventually brought under control by incre3asing predator, the cotton foliage never recovers. Hence, in the decision problem, there exist may con…gurations of initial conditions for which the farmer will always spray. Now we must translate the evolution equations into a decision problem for the farmer.

Figures 2-5 give the solution of the stopping problem for di¤erent levels of predator insects, di¤erent levels of spray price and at di¤erent dates. We can see from these …gures that they do indeed meet our expectations on many levels. The decision to spray is increasing in the number of aphids, increasing in the amount of cotton foliage, and decreasing in the number of predators. In …gures 2-4, the heading max predators refers to the maximum sustainable level of predators. The heading medium predators refers to one half the maximum sustainable level. While the heading minium predators refers to the level of predators which exists after applying pesticide. In …gure 5, the exact number of predators considered is listed at the top of each column. The time indicator along the left side of each …gure refers to the number of weeks into the growing season. Harvest occurs after week 16.

From these …gures, we can see the existence of the two dynamic systems to which we have been referring. Notice in the early weeks, the rule for spraying when the minimum number of predators exist is to spray any time the aphids grow appreciably. The rule takes this form because the aphids are e¤ectively an uncontrolled population in this state. The aphids grow so quickly that the probability for cotton damage is very high. The reason we see less spraying toward the end of the season is that the aphids do to have as much time to damage the crop as they do earlier. The aphids in this model do not destroy the cotton as much as prevent it from growing.

These results also have strong implications for integrated pest management (IPM) strategies - see Ward et al. (1990). IPM consists of many factors. One of the important factors is to transplant or attract predator insects at the beginning of the growing season. In our model, these strategies are going to have to be carefully implemented in order to be e¤ective. First, we can see that importing predators is likely to be a successful strategy only if the price of spraying is relatively high. For low price of spraying, the farmer applies pesticide for all levels of predator insects unless the aphid population is particularly small. Even with higher prices for spraying, if either the aphid population is large or the predator in‡ux is small. The optimal strategy is to spray in the early periods, e¤ectively negating the e¤ect of this aspect of IPM. However, our model does not go against IPM. Our model is simply a tool informed the farmer as to which type of IPM is likely to be cost e¤ective. Before importing predators, the farmer can make a judgment as to whether the cost of importing the predators will be recovered form lower spraying costs. The di¤erence between Figures 4 and 5 gives a good indication that this will be more likely when spraying costs are high.

5 Conclusion

In this paper, we have demonstrated the importance of using state dependent decision rules in order to determine the frequency of pesticide application for the control of the cotton aphid. The key result is that the farmer should consider not only the aphid population but should also consider the predator population. The existence of a signi…cant predator can decrease the cost of spraying signi…cantly. We have also shown that for many cases - those where the predator population is small relative to the aphid population - the solution is equivalent to the case with no predators in existence. In these cases, our model yields equivalent solutions to the older pesticide application papers which ignore the predator population.

Our paper has used a very stylized model of predator prey interaction However, the techniques of this paper are quite robust to more complicated models. For realistic decision rules one need only incorporate more realistic laws of motion. However, the power of this technique is that all factors which are outside of the farmer's control or do not interact with the aphid and predator populations can be sussumed into the law of motion. That is we do not need to model whether or not the farmer uses dry or wet land techniques or whether the year is sunny or cloudy. All of these factors can be incorporated into the probability structure of the model.

Another important factor which we do not consider here is that the initial predator population can be partially controlled by the farmer. That is the farmer may import predators or use a particular plant around his …eld in order to attract predator insects. However, these actions do not a¤ect our analyses; they simply a¤ect the initial conditions for the model. We can see in the simulations above, the number of predator insects the farmer must attract or import for this action to a¤ect the optimal spraying.

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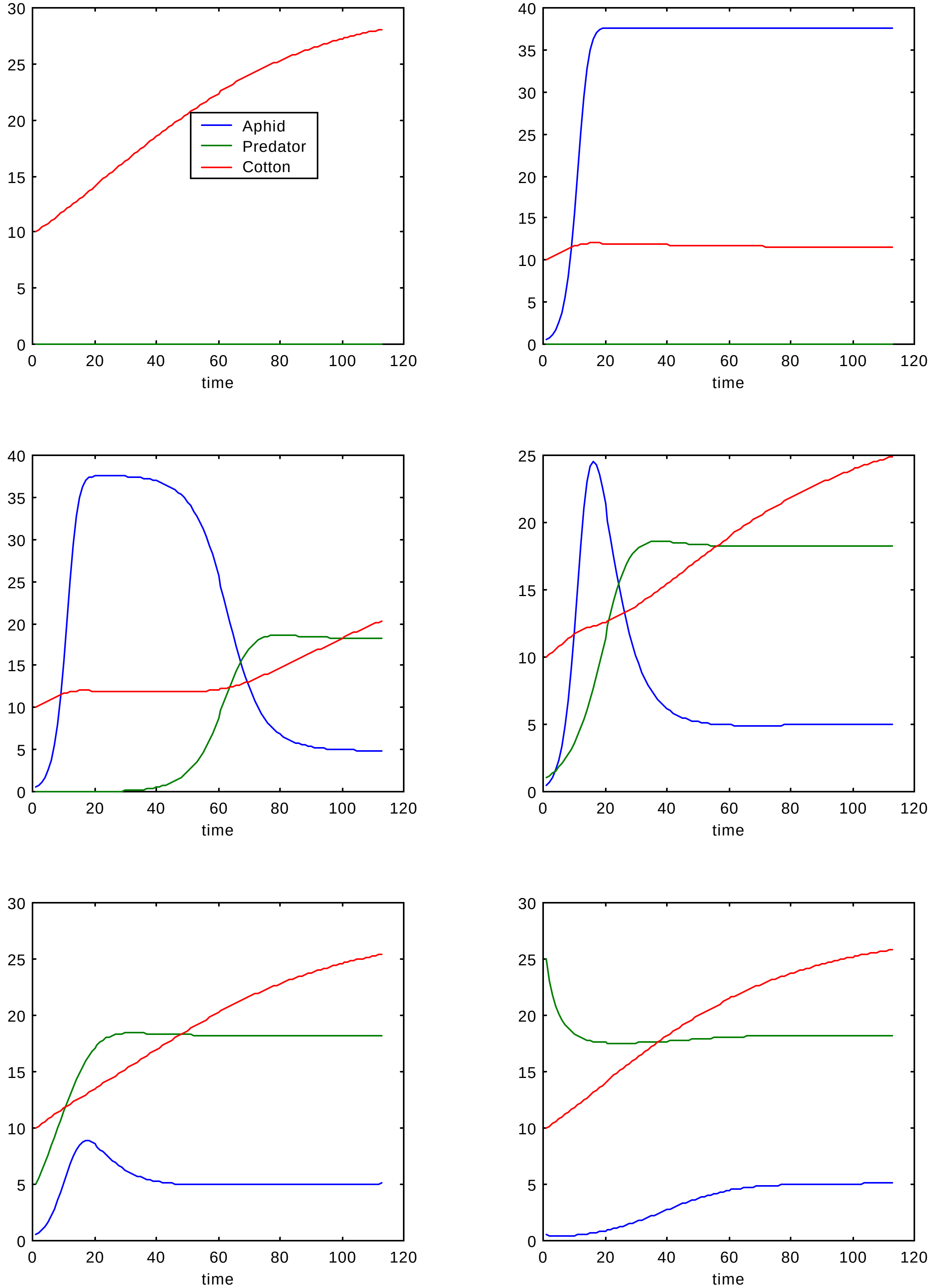
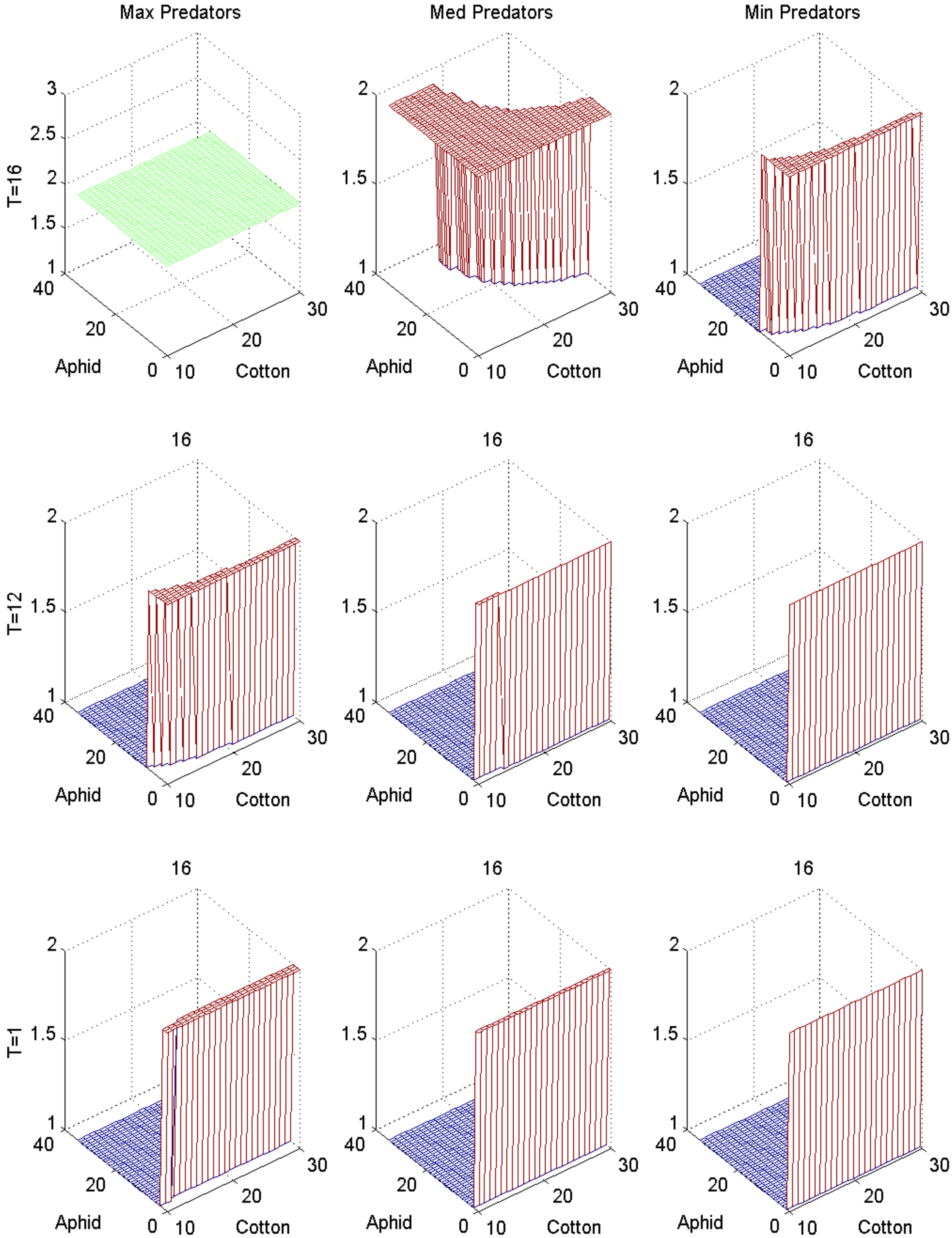
Figure 1: Equation Simulation no Spraying





Spray Price = 1; Value of 1 Indicates Spray for that State

Figure 2

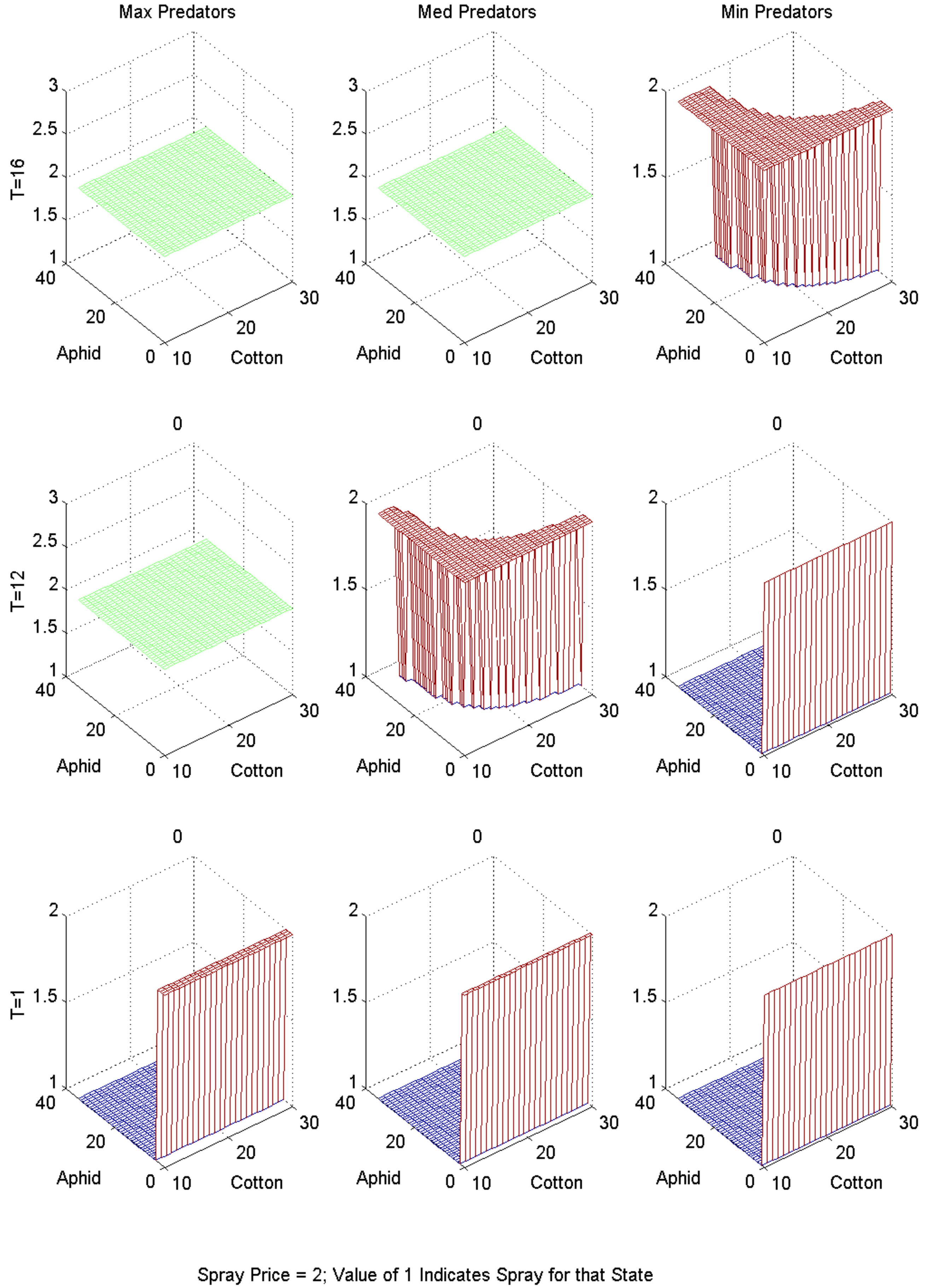
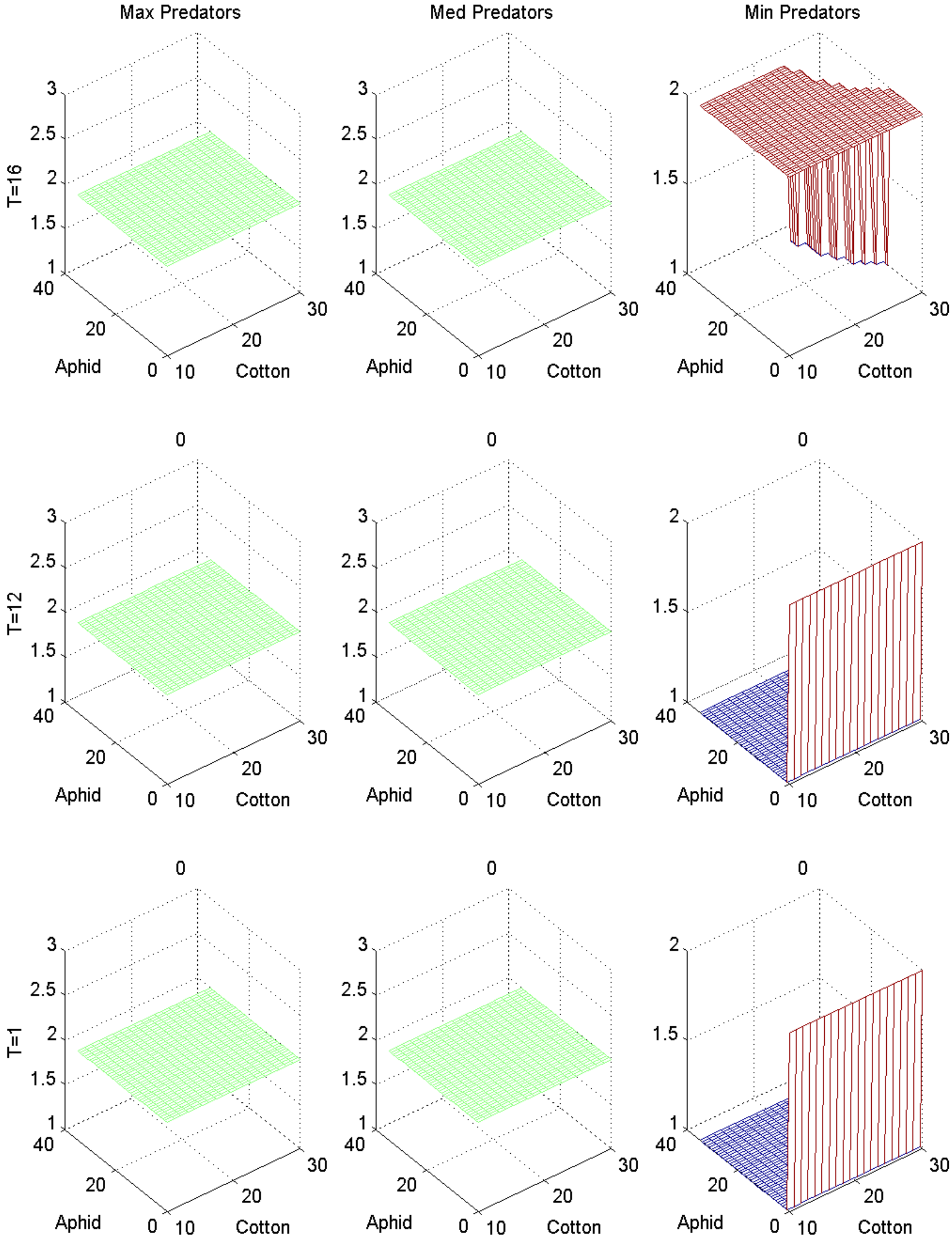
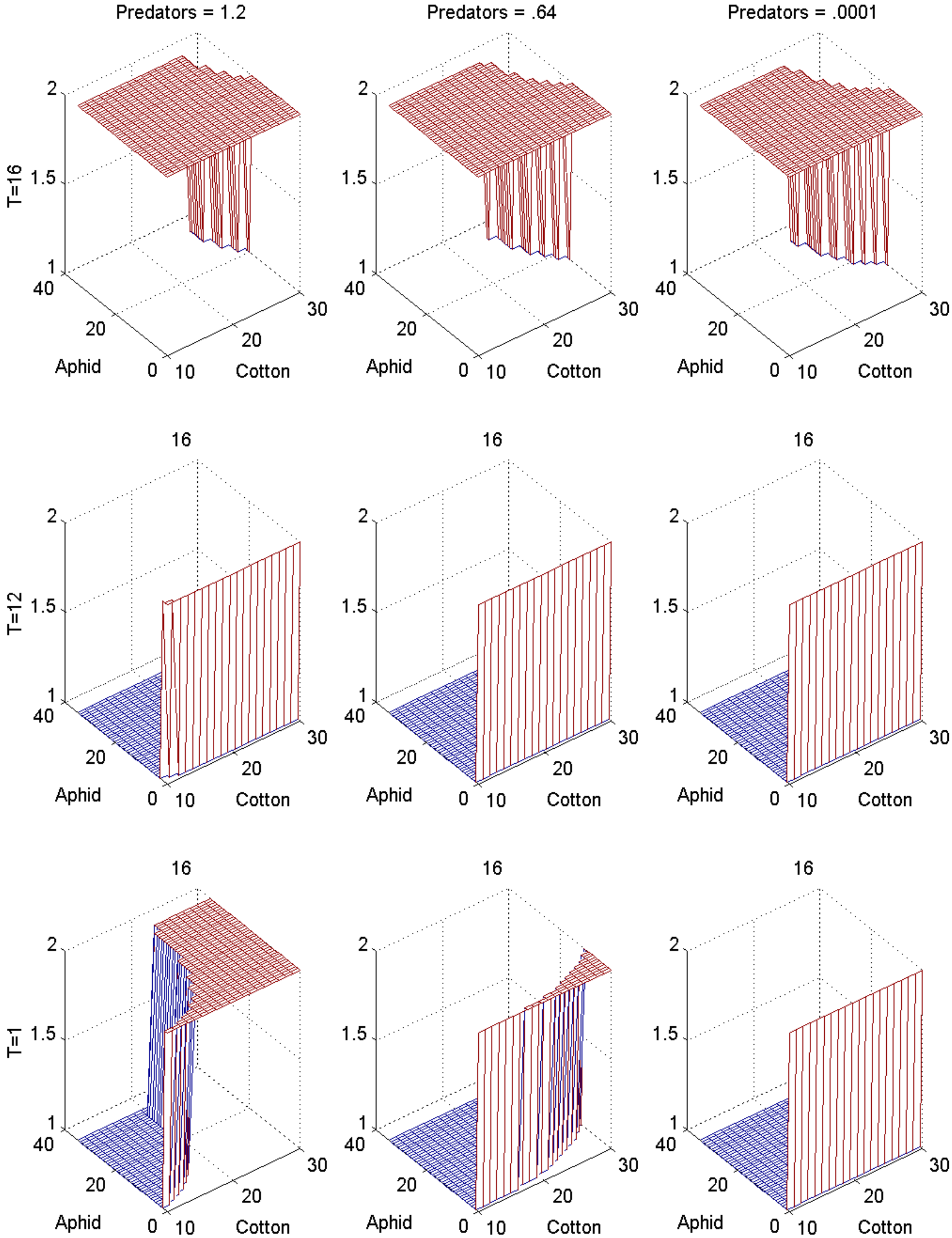

Max Predators
Med Predators
Min Predators
T=16
T=12
T=1
Aphid
Cotton
0
Spray Price = 2; Value of 1 Indicates Spray for that State

Figure 3



Spray Price = 3; Value of 1 Indicates Spray for that State

Figure 4



Spray Price = 3; Policies for Small Number of Predators

Figure 5