Neural Net Backlash Compensation With Hebbian Tuning By Dynamic Inversion [*]

Rastko R. Selmic and Frank L. Lewis

Automation and Robotics Research Institute, The University of Texas at Arlington,
7300 Jack Newell Blvd. South, Fort Worth, Texas 76118-7115, USA.
Email: rselmic@arrirs04.uta.edu
flewis@controls.uta.edu

Abstract

Neural network compensation scheme is presented for the class of nonlinear systems with backlash nonlinearity. The compensator uses the backstepping technique with neural networks (NN) for inverting the backlash nonlinearity in the feedforward path. Instead of a derivative, which cannot be implemented, a filtered derivative is used. Full rigorous stability proofs are given using filtered derivative. Compared with adaptive backstepping control schemes, we do not require the unknown parameters to be linear parametrizable. No regression matrices are needed. The technique provides a general procedure for using NN to determine the dynamic preinverse of an invertible dynamical system. A modified Hebbian algorithm is presented for NN tuning which yields a stable closed-loop system. Using this method yields a relatively simple adaptation structure and offers computational advantages over gradient descent based algorithms.

1 Introduction

Recently, in seminal work several rigorously derived adaptive schemes have been given for actuator nonlinearity compensation [23]. Backlash compensation is considered in [22]. For the dynamic system in Lagrangian form, deadzone compensation using NN is given in [19]. It is not required for deadzone to be symmetric, and the function outside the dead-band may not be linear. NN approximation of the discontinuous functions with application to friction compensation is given in [17].

Dynamic inversion using NN is presented in [5], [7], [12] where NN is used for cancellation of the inversion error. A compensated inverse dynamics approach using adaptive and robust control techniques is presented in [21].

In this paper an application of dynamic inversion to backlash compensation is given. A general model of backlash is used, which is not required to be symmetric. We use a backstepping approach [4] with a filtered derivative to derive a compensator that has a neural network in the feedforward loop. A *rigorous closed-loop system stability proof* that guarantees small tracking error and bounded NN weights is presented. We show detailed analysis using a real filter needed to calculate the derivative of the signals used in inverse dynamics design. The NN is used for compensating the dynamic inversion error. A modified Hebbian tuning algorithm for the NN is designed in order to simplify the computational burden in multi-axes mechanical systems with backlash. The proposed method can be applied for compensation of a large class of invertible dynamical nonlinearities. The nonlinear system here is assumed to be in Brunovsky form. Simulation results show that NN backlash compensator can significantly reduce degrading effect of backlash nonlinearity.

2 Background

NN have been used extensively in feedback control systems. Most applications are ad hoc with no demonstrations of stability. The stability proofs that do exist rely almost invariably on the universal approximation property for NN [1], [10], [16].

The two-layer NN in Figure 2.1 consists of two layers of tunable weights and has a hidden layer and an output layer. The hidden layer has L neurons, and the output layer has m neurons. The multilayer NN is a nonlinear mapping from input space $\Re^n$ into output space $\Re^m$.

By collecting all the NN weights into matrices, the NN equation with linear output activation function may be written in terms of vectors as

$$y = W^T\sigma(V^T x) \ . \tag{2.1}$$

For notational convenience we define the matrix of all the weights as

$$Z = \begin{bmatrix} W & \\ & V \end{bmatrix} . \tag{2.2}$$

There are many different ways to choose the activation functions $\sigma(\cdot)$, including sigmoid, hyperbolic tangent, etc. We use sigmoid activation function.

Many well-known results say that any sufficiently smooth function f can be approximated arbitrarily closely on

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a compact set using a two-layer NN with appropriate weights, i.e.

$$f(x) = W^T \sigma(V^T x) + \varepsilon(x) \tag{2.3}$$

where the $\varepsilon(x)$ is the NN approximation error, and $\|\varepsilon(x)\| \le \varepsilon_N$ on a compact set S. The approximating weights V and W are ideal target weights, and it is assumed that they are bounded such that $\|V\|_F \le V_M$, $\|W\|_F \le W_M$, or $\|Z\|_F \le Z_M$.

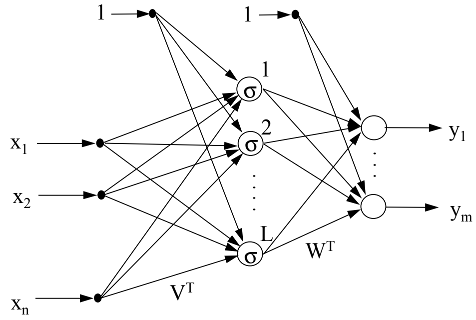

Figure 2.1 Two-layer NN.

Learning in the biological nervous system was explained by Hebb [3] through the correlation of a presynaptic signal with a postsynaptic activity. Suppose the NN is to associate the input pattern X to the output vector Y. Define the output error as e=Y-y, with y the output when x=X. Then, a continuous-time tuning rule based on the Hebbian philosophy is given by

$$\dot{W} = \eta \sigma(V^T x) e^T \ , \tag{2.4}$$

$$\dot{V} = \eta X \left(\sigma(V^T x)\right)^T \ . \tag{2.5}$$

The Hebbian tuning algorithm has not been shown to converge. It has rarely been used for feedback control algorithms [24], and not to our knowledge in backlash compensation with proofs of stability.

3 Backlash Nonlinearity

Here is assumed a general model of the backlash that is not required to be symmetric. We use a backstepping approach to derive a compensator which has a neural network in the feedforward loop. The proposed method can be applied for compensation of a large class of invertible dynamical nonlinearities.

To focus on backlash compensation, we assume the system is in Brunovsky form. The generality of the method and its applicability to a broad range of nonlinear functions make this approach a useful tool for compensation of actuator nonlinearities such as backlash, hysteresis, etc. Backlash compensation is done using dynamic inversion, where NN is used for the dynamic inversion compensation [7], [12].

3.1 Backlash Nonlinearity and Backlash Inverse

The backlash nonlinearity is shown in Figure 3.1, and a mathematical model is given by (3.1) [23].

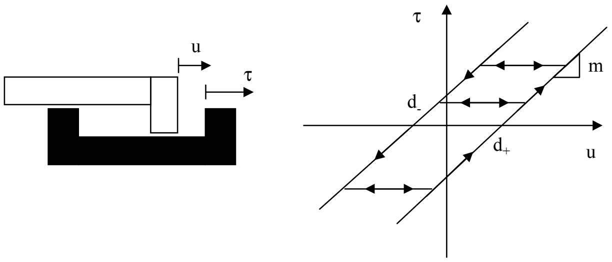

Figure 3.1 Backlash nonlinearity.

$$\dot{\tau} = B(\tau, u, \dot{u}) = \begin{cases} m\dot{u}\,, \text{ if } \dot{u} > 0 \text{ and } \tau = mu - md_+ \\ \quad\ \text{if } \dot{u} < 0 \text{ and } \tau = mu - md_- \\ 0\,, \ \text{otherwise} \end{cases} \tag{3.1}$$

One can see that backlash is a first-order velocity-driven dynamic system, with inputs u and $\dot{u}$, and state τ. It contains its own dynamics, therefore its compensation requires the design of dynamic compensator.

Whenever the motion u(t) changes its direction, the motion $\tau(t)$ is delayed from motion of u(t). The objective of a backlash compensator is to make this delay as small as possible, i.e. to make the $\tau(t)$ to closely follow u(t). In order to cancel the effect of backlash in the system, the backlash precompensator needs to generate the inverse of the backlash nonlinearity. The backlash inverse function is shown in Figure 3.2.

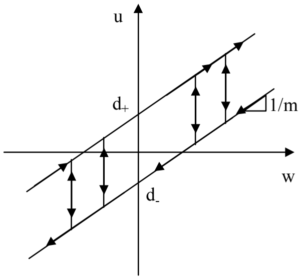

Figure 3.2 Backlash inverse.

The dynamics of the NN backlash compensator is given by

$$\dot{u} = B_{inv}(u, w, \dot{w}) \ . \tag{3.2}$$

4 NN Controller With Backlash Compensator

The NN backlash compensator is designed using the backstepping technique [4]. In this section we will show how to tune or learn the weights of the NN on-line so that the tracking error is guaranteed small and all internal states (e.g. the NN weights) are bounded. It is assumed that the actuator output $\tau(t)$ is measurable.

The dynamics of a large class of single-input nonlinear systems can be written in the Brunovsky form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= f(x) + \tau_d + \tau \\ y &= x_1 \end{aligned} \tag{4.1}$$

where the output is y(t), the state is $x \equiv [x_1 \ x_2 \dots x_n]^T$, τ_d is a disturbance, τ(t) is actuator output, and function f(x) represents system nonlinearities like friction, etc. The actuator output τ(t) is related to the control input u(t) through the backlash nonlinearity (3.1). Therefore overall dynamics of the system consists of (4.1) and backlash dynamics (3.1).

The following assumptions are needed. They are typical assumptions commonly made in the literature and hold for many practical systems.

<u>**Assumption 1**</u> **(Bounded disturbance).** The unknown disturbance satisfies $\|\tau_d\| \le \tau_M$, with $\tau_M(t)$ a known positive constant.

<u>**Assumption 2**</u> **(Bounded estimation error).** The nonlinear function f(x) is assumed to be unknown, but a fixed estimate $\hat{f}(x)$ is assumed known such that the functional estimation error, $\tilde{f}(x) = f(x) - \hat{f}(x)$, satisfies $\|\tilde{f}(x)\| \le f_M(x)$, for some known bounding function $f_M(x)$.

This is not unreasonable [7], as in practical systems the bound $f_M(x)$ can be computed knowing the upper bound on payload masses, frictional effects, and so on.

To design a motion controller that causes the system output, y(t), to track a smooth prescribed trajectory, $y_d(t)$, we define the desired state as

$$x_d(t) = \begin{bmatrix} y_d & \dot{y}_d & \dots & y_d^{(n-1)} \end{bmatrix}^T \tag{4.2}$$

with $y_d^{(n-1)}$ the (n-1)-st derivative. We define the *tracking error* by

$$e = x - x_d \tag{4.3}$$

and the *filtered tracking error* by

$$r = \begin{bmatrix} \lambda_1 & \lambda_2 & \dots & \lambda_{n-1} & 1 \end{bmatrix} e \equiv \Lambda^T e \equiv \begin{bmatrix} \Lambda^T & 1 \end{bmatrix} e \ , \tag{4.4}$$

with Λ a gain parameter vector selected so that $e(t) \to 0$ exponentially as $r(t) \to 0$. Then, (4.4) is stable system so that e(t) is bounded as long as controller guaranties that the filtered error r(t) is bounded.

Differentiating (4.4) and invoking (4.1), it can be seen that the dynamics are expressed in terms of filtered error as

$$\dot{r} = f(x) + Y_d + \tau_d + \tau \ , \tag{4.5}$$

where

$$Y_d = -y_d^{(n)} + \begin{bmatrix} 0 & \Lambda^T \end{bmatrix} e \tag{4.6}$$

is known function of the desired trajectory and actual states.

<u>**Assumption 3**</u> **(Bounded desired trajectory).** The desired trajectory is bounded so that $\|x_d(t)\| \le X_d$, where X_d is a known constant.

4.1 Backstepping Controller Design with NN Backlash Compensation

A robust compensation scheme for unknown terms in f(x) is provided by selecting the tracking controller

$$\tau_{des} = -K_V r - \hat{f}(x) - Y_d + v_1 \ , \tag{4.7}$$

with $\hat{f}(x)$ an estimate for the nonlinear terms f(x), and $v_1(t)$ a robustifying term to be selected for the disturbance rejection. The feedback gain matrix $K_V>0$ is often selected diagonal. The estimate $\hat{f}(x)$ is fixed in this paper and will not be adapted, as is common in robust control techniques [7]. If f(x) in (4.1) is unknown, it can be estimated using adaptive control techniques [7], or the neural network controller in [10].

<u>**Theorem 1**</u> **(Control law for outer tracking loop).** Given the system (4.1) and Assumptions 1-2, select the tracking control law (4.7). Choose the robustifying signal v_1 as

$$v_1(t) = -(f_M(x) + \tau_M)\frac{r}{\|r\|} \ . \tag{4.8}$$

Then the filtered tracking error r(t) is UUB and it can be kept as small as desired by increasing the gains K_v.

<u>**Proof**</u>: See [18].

4.2 NN Backlash Compensation using Dynamic Inversion

In the Theorem 1 is given the control law which ensures stability in terms of the filtered tracking error. In the presence of the unknown backlash nonlinearity, the desired and actual value of the control signal τ will be different. Following the idea of dynamic inversion where neural network is used for compensation of the inversion error, originally given by Calise et al. [7], [12], we give a rigorous analysis of the closed-loop system stability. In the backstepping and dynamic inversion approach there is deficiency that the derivative of the desired input is required. It usually assumes the pure differentiation. We solve this problem by applying the real filter instead of pure differentiator, and still rigorously prove the system stability.

In order to find the complete system error dynamics define the error between the desired and actual actuator outputs as

$$\tilde{\tau} = \tau_{des} - \tau \ . \tag{4.9}$$

Differentiating one has

$$\begin{aligned} \dot{\tilde{\tau}} &= \dot{\tau}_{des} - \dot{\tau} \\ &= \dot{\tau}_{des} - B(\tau, u, \dot{u}) \end{aligned} \tag{4.10}$$

which together with (4.5) and involving (4.7) represent the complete system error dynamics.

The dynamics of the backlash nonlinearity can be written as

$$\dot{\tau} = \varphi \tag{4.11}$$

$$\varphi = B(\tau, u, \dot{u}) \tag{4.12}$$

where φ(t) is pseudo-control input [7]. In the case of known backlash, the ideal backlash inverse is given by

$$\dot{u} = B^{-1}(u, \tau, \varphi) \ . \tag{4.13}$$

Since the backlash and therefore its inverse are not known, one can only approximate the backlash inverse

$$\dot{\hat{u}} = \hat{B}^{-1}(\hat{u}, \tau, \hat{\varphi}) \ . \tag{4.14}$$

The backlash dynamics can now be written as

$$\begin{aligned}\dot{\tau} &= B\left(\tau, \hat{u}, \dot{\hat{u}}\right) = \hat{B}\left(\tau, \hat{u}, \dot{\hat{u}}\right) + \widetilde{B}\left(\tau, \hat{u}, \dot{\hat{u}}\right) \\ &= \hat{\varphi} + \widetilde{B}\left(\tau, \hat{u}, \dot{\hat{u}}\right)\end{aligned} , \qquad (4.15)$$

where $\hat{\varphi} = \hat{B}\left(\tau, \hat{u}, \dot{\hat{u}}\right)$ and therefore its inverse $\dot{\hat{u}} = \hat{B}^{-1}\left(\tau, \hat{u}, \hat{\varphi}\right)$. The unknown function $\widetilde{B}\left(\tau, \hat{u}, \dot{\hat{u}}\right)$, which represents the backlash inversion error, will be approximated using neural network.

In order to design the stable closed-loop system with backlash compensation, one selects a nominal backlash inverse $\dot{\hat{u}} = \hat{\varphi}$ and pseudo-control input $\hat{\varphi}$ as

$$\hat{\varphi} = K_b \widetilde{\tau} + \xi - y_{nn} + v_2 , \qquad (4.16)$$

where ξ is the filter output, $v_2(t)$ is a robustifying term detailed later, and y_{nn} is the output of the neural network.

The dynamics of the filter is given by

$$\dot{\xi} + a\xi = a\dot{\tau}_{des} , \qquad (4.17)$$

where a is the filter pole. Output of the filter is then given by

$$\xi = \dot{\tau}_{des} - \frac{1}{a}\dot{\xi} , \qquad (4.18)$$

which gives the input $\hat{\varphi}$ as

$$\hat{\varphi} = K_b \widetilde{\tau} + \dot{\tau}_{des} - \frac{1}{a}\dot{\xi} - y_{nn} + v_2 . \qquad (4.19)$$

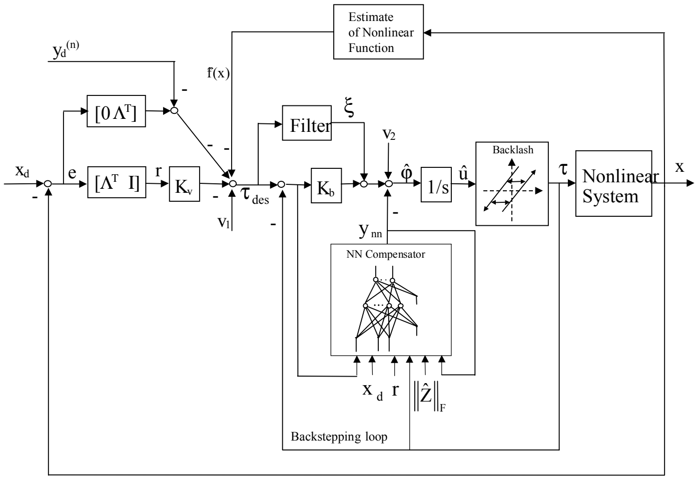

Figure 4.1 NN Backlash Compensator

Using the proposed controller (4.16), the error dynamics (4.10) can be written as

$$\begin{aligned}\dot{\widetilde{\tau}} &= \dot{\tau}_{des} - \hat{\varphi} - \widetilde{B}\left(\tau, \hat{u}, \dot{\hat{u}}\right) \\ &= -K_b \widetilde{\tau} + \frac{1}{a}\dot{\xi} + y_{nn} - v_2 - \widetilde{B}\left(\tau, \hat{u}, \dot{\hat{u}}\right)\end{aligned} . \qquad (4.20)$$

Figure 4.1 shows the closed-loop system with NN backlash compensator.

Based on the NN approximation property, the backlash inversion error plus the part of the filter dynamics can be represented as

$$\widetilde{B}\left(\tau, \hat{u}, \dot{\hat{u}}\right) - \frac{1}{a}\dot{\xi} = W^T \sigma\left(V^T x_{nn}\right) + \varepsilon(x_{nn}) , \qquad (4.21)$$

where NN input vector is chosen as $x_{nn} = \left[1\ \ r^T\ \ x_d{}^T\ \ \widetilde{\tau}^T\ \ \tau\ \ y_{nn}\ \ \|\hat{Z}\|_F\right]^T$, and $\varepsilon(x_{nn})$ represents the NN approximation error. Note that the NN output is also the NN input. This requires the fixed-point solution assumption, which holds for bounded sigmoidal activation functions [5], [12].

Define $\hat{V}$, $\hat{W}$ as estimates of the ideal NN weights, which are given by the NN tuning algorithms. Define the weight estimation errors as

$$\widetilde{V} = V - \hat{V}, \quad \widetilde{W} = W - \hat{W}, \quad \widetilde{Z} = Z - \hat{Z} , \qquad (4.22)$$

and the hidden-layer output error for a given x as

$$\widetilde{\sigma} = \sigma - \hat{\sigma} \equiv \sigma\left(V^T x_{nn}\right) - \sigma\left(\hat{V}^T x_{nn}\right) . \qquad (4.23)$$

Substituting matrix expressions for NN, the error dynamics (4.20) can be written as

$$\dot{\widetilde{\tau}} = -K_b \widetilde{\tau} + \hat{W}^T \sigma\left(\hat{V}^T x_{nn}\right) - v_2 - W^T \sigma\left(V^T x_{nn}\right) - \varepsilon(x_{nn}) \qquad (4.24)$$

More appropriately, the error dynamics can be written as

$$\dot{\widetilde{\tau}} = -K_b \widetilde{\tau} - \widetilde{W}^T \hat{\sigma} - W^T \widetilde{\sigma} - v_2 - \varepsilon(x_{nn}) , \qquad (4.25)$$

with the NN input is bounded by

$$\|x_{nn}\| \le c_0 + \|r\| + X_d + \|\widetilde{\tau}\| + c_1 \|\widetilde{Z}\|_F , \qquad (4.26)$$

The next theorem shows how to tune the neural network weights so the tracking errors r(t) and $\widetilde{\tau}(t)$ achieve small values while the NN weights $\hat{V}$, $\hat{W}$ are close to V, W, i.e. the weight estimation errors defined by (4.22) are bounded.

Theorem 2 **(Hebbian tuning law for backstepping loop).** Let assumptions 1, 2 hold. Let the desired trajectories be bounded. Select the control input as (4.16). Choose the robustifying signal v_2 as

$$v_2 = K_{Z_1}\left(\|\hat{Z}\|_F + Z_M\right)\left(\widetilde{\tau} + \|r\|\frac{\widetilde{\tau}}{\|\widetilde{\tau}\|}\right) + K_{Z_2}\|r\|\frac{\widetilde{\tau}}{\|\widetilde{\tau}\|} + K_{Z_3}\left(\|\hat{Z}\|_F + Z_M\right)^2 \frac{\widetilde{\tau}}{\|\widetilde{\tau}\|} , \qquad (4.27)$$

where $K_{Z_1} > \sqrt{L}$, $K_{Z_2} > 1$ and $K_{Z_3} > c_1\sqrt{L}$, where the L is the number of hidden layer nodes. Let the estimated NN weights be provided by the NN tuning algorithm

$$\dot{\hat{V}} = -T\|\widetilde{\tau}\| x_{nn} \hat{\sigma}^T - kT\|\widetilde{\tau}\|\hat{V} , \qquad (4.28)$$

$$\dot{\hat{W}} = -S\hat{\sigma}\widetilde{\tau}^T - kS\|\widetilde{\tau}\|\hat{W} , \qquad (4.29)$$

with any constant matrices $S=S^T>0$, $T=T^T>0$, and $k>0$ small scalar design parameter. Then the filtered tracking error r(t), error $\tau(t)$ and NN weight estimates $\hat{V}$, $\hat{W}$ are UUB. Moreover, the error $\tau(t)$ can be made arbitrarily small by increasing the gain K_b.

Proof: Select the Lyapunov function candidate

$$L = L_1 + \frac{1}{2}\widetilde{\tau}^T\widetilde{\tau} + \frac{1}{2}\mathrm{tr}\left(\widetilde{W}^T S^{-1}\widetilde{W}\right) + \frac{1}{2}\mathrm{tr}\left(\widetilde{V}^T T^{-1}\widetilde{V}\right) , \qquad (4.30)$$

which weights both errors r(t) and $\tau(t)$, and NN weights estimation errors. Taking derivative

$$\dot{L} = \dot{L}_1 + \widetilde{\tau}^T\dot{\widetilde{\tau}} + \mathrm{tr}\left(\widetilde{W}^T S^{-1}\dot{\widetilde{W}}\right) + \mathrm{tr}\left(\widetilde{V}^T T^{-1}\dot{\widetilde{V}}\right) , \qquad (4.31)$$

and using (4.5), (4.25) one has

$$\begin{aligned}\dot{L} &= r^T\left(f(x)+Y_d+\tau_d+\tau\right)\\ &\quad+\tilde{\tau}^T\left(-K_b\tilde{\tau}-\tilde{W}^T\hat{\sigma}-W^T\tilde{\sigma}-v_2-\varepsilon(x)\right), \qquad (4.32)\\ &\quad+\mathrm{tr}\left(\tilde{W}^TS^{-1}\dot{\tilde{W}}\right)+\mathrm{tr}\left(\tilde{V}^TT^{-1}\dot{\tilde{V}}\right)\end{aligned}$$

$$\begin{aligned}\dot{L} &= r^T\left(f(x)+Y_d+\tau_d+\tau_{des}\right)-r^T\tilde{\tau}+\tilde{\tau}^T\left(-K_b\tilde{\tau}-v_2-\varepsilon(x)\right)\\ &\quad-\tilde{\tau}^TW^T\tilde{\sigma}+\mathrm{tr}\left[\tilde{W}^T\left(S^{-1}\dot{\tilde{W}}-\hat{\sigma}\tilde{\tau}^T\right)\right]+\mathrm{tr}\left(\tilde{V}^TT^{-1}\dot{\tilde{V}}\right)\end{aligned}$$

Applying (4.7) and tuning rules yields

$$\begin{aligned}\dot{L} &= r^T\left(\tilde{f}(x)+\tau_d-K_Vr+v_1\right)-r^T\tilde{\tau}\\ &\quad+\tilde{\tau}^T\left(-K_b\tilde{\tau}-v_2-\varepsilon(x)\right)\\ &\quad-\tilde{\tau}^TW^T\tilde{\sigma}+k\|\tilde{\tau}\|\mathrm{tr}\left[\tilde{W}^T\left(W-\tilde{W}\right)\right] \qquad , \qquad (4.33)\\ &\quad+k\|\tilde{\tau}\|\mathrm{tr}\left[\tilde{V}^T\left(V-\tilde{V}\right)\right]-\|\tilde{\tau}\|\mathrm{tr}\left[\tilde{V}^Tx_{nn}\hat{\sigma}^T\right]\end{aligned}$$

Using (4.8) and (4.27), expression (4.33) can be bounded as

$$\begin{aligned}\dot{L} &\le -K_{V\min}\|r\|^2-\|r\|(f_M+\tau_M)+\|r\|\left\|\tilde{f}+\tau_d\right\|\\ &\quad+k\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F\left(Z_M-\left\|\tilde{Z}\right\|_F\right)-r^T\tilde{\tau}-K_{b\min}\|\tilde{\tau}\|^2-\tilde{\tau}^T\varepsilon(x)\\ &\quad-\tilde{\tau}^TK_{Z_1}\left(\left\|\hat{Z}\right\|_F+Z_M\right)\left(\tilde{\tau}+\|r\|\frac{\tilde{\tau}}{\|\tilde{\tau}\|}\right)-\tilde{\tau}^TK_{Z_2}\|r\|\frac{\tilde{\tau}}{\|\tilde{\tau}\|}\\ &\quad-\tilde{\tau}^TK_{Z_3}\left(\left\|\hat{Z}\right\|_F+Z_M\right)^2\frac{\tilde{\tau}}{\|\tilde{\tau}\|}-\tilde{\tau}^TW^T\tilde{\sigma}-\|\tilde{\tau}\|\mathrm{tr}\left[\tilde{V}^Tx_{nn}\hat{\sigma}^T\right]\end{aligned}$$

Including (4.26) and applying some norm properties, one has

$$\begin{aligned}\dot{L} &\le -K_{V\min}\|r\|^2-\|r\|(f_M+\tau_M)+\|r\|\left\|\tilde{f}+\tau_d\right\|\\ &\quad+k\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F\left(Z_M-\left\|\tilde{Z}\right\|_F\right)-K_{b\min}\|\tilde{\tau}\|^2+\|\tilde{\tau}\|\|r\|+\|\tilde{\tau}\|\|\varepsilon(x)\|\\ &\quad+2\|\tilde{\tau}\|Z_M\sqrt{L}+\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F\left\|x_{nn}\hat{\sigma}^T\right\|-K_{Z_1}\|\tilde{\tau}\|^2\left\|\tilde{Z}\right\|_F\\ &\quad-K_{Z_1}\|\tilde{\tau}\|\|r\|\left\|\tilde{Z}\right\|_F-K_{Z_2}\|\tilde{\tau}\|\|r\|-K_{Z_3}\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F^2\end{aligned}$$

$$\begin{aligned}\dot{L} &\le -K_{V\min}\|r\|^2+k\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F\left(Z_M-\left\|\tilde{Z}\right\|_F\right)-K_{b\min}\|\tilde{\tau}\|^2+\|\tilde{\tau}\|\|r\|\\ &\quad+C_0\|\tilde{\tau}\|+C_1\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F+\sqrt{L}\|\tilde{\tau}\|\|r\|\left\|\tilde{Z}\right\|_F+\sqrt{L}\|\tilde{\tau}\|^2\left\|\tilde{Z}\right\|_F\\ &\quad+c_1\sqrt{L}\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F^2-K_{Z_1}\|\tilde{\tau}\|^2\left\|\tilde{Z}\right\|_F-K_{Z_1}\|\tilde{\tau}\|\|r\|\left\|\tilde{Z}\right\|_F-K_{Z_2}\|\tilde{\tau}\|\|r\|\\ &\quad-K_{Z_3}\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F^2\end{aligned}$$

where $C_0=\varepsilon_N+2Z_M\sqrt{L}$, $C_1=\sqrt{L}c_0+X_d$. Therefore, one has the following inequality

$$\begin{aligned}\dot{L} &\le -K_{V\min}\|r\|^2-K_{b\min}\|\tilde{\tau}\|^2-k\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F^2+kZ_M\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F+C_0\|\tilde{\tau}\|\\ &\quad+C_1\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F+\|\tilde{\tau}\|\|r\|+\sqrt{L}\|\tilde{\tau}\|\|r\|\left\|\tilde{Z}\right\|_F+\sqrt{L}\|\tilde{\tau}\|^2\left\|\tilde{Z}\right\|_F\\ &\quad+c_1\sqrt{L}\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F^2-K_{Z_1}\|\tilde{\tau}\|^2\left\|\tilde{Z}\right\|_F-K_{Z_1}\|\tilde{\tau}\|\|r\|\left\|\tilde{Z}\right\|_F-K_{Z_2}\|\tilde{\tau}\|\|r\|\\ &\quad-K_{Z_3}\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F^2\end{aligned}$$

Taking $K_{Z_1}>\sqrt{L}$, $K_{Z_2}>1$, and $K_{Z_3}>c_1\sqrt{L}$ yields

$$\begin{aligned}\dot{L} &\le -K_{V\min}\|r\|^2-K_{b\min}\|\tilde{\tau}\|^2-k\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F^2+kZ_M\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F\\ &\quad+C_0\|\tilde{\tau}\|+C_1\|\tilde{\tau}\|\left\|\tilde{Z}\right\|_F\end{aligned}$$

Completing the squares yields

$$\begin{aligned}&\dot{L}\le -K_{V\min}\|r\|^2\\ &-\|\tilde{\tau}\|\left[K_{b\min}\|\tilde{\tau}\|+k\left(\left\|\tilde{Z}\right\|_F-\left(\frac{Z_Mk+C_1}{2k}\right)\right)^2-k\left(\frac{Z_Mk+C_1}{2k}\right)^2-C_0\right].\end{aligned}$$

Thus, the $\dot{L}$ is negative as long as

$$\|\tilde{\tau}\|>\frac{k\left(\frac{Z_Mk+C_1}{2k}\right)^2+C_0}{K_{b\min}}, \qquad (4.35)$$

$$\left\|\tilde{Z}\right\|_F>\frac{Z_Mk+C_1}{2k}+\sqrt{\left(\frac{Z_Mk+C_1}{2k}\right)^2+\frac{C_0}{k}}. \qquad (4.36)$$

■

The first terms of (4.28), (4.29) are modified versions of the standard Hebbian tuning law. The k terms correspond to the *e*-modification [13], to guarantee bounded parameter estimates. These algorithms do not require computation of the sensitivity matrix, nor Jacobian matrix. The tuning rules provide simple NN design and a great computational advantage especially when the net size is large.

5 Simulation of NN Backlash Compensator

To illustrate the performance of the NN backlash compensator, we consider the nonlinear system

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -\frac{1}{T}x_2+Ma\,x_2^2\sin(x_1)+Mga\cos(x_1)+\tau\end{aligned}$$

(5.1)

which represents a mechanical motion of robot-like system with one link. We selected *T*=1 s; *M*=1 kg; *a*=2.5m. The input t is passed through the additional backlash nonlinearity given by (3.1). The parameters of the backlash are d_+=10, d_-=-10.5, m=1.

The NN weight tuning parameters are chosen as $S=10I_{11}$, $T=10I_6$, k=0.001, where I_N is NxN identity matrix. The robustifying signal gains are K_{z1}=5, K_{z2}=2, K_{z3}=7. The controller parameters are chosen as Λ=10, K_v=10, K_b=50.

The NN has L=10 hidden-layer nodes with sigmoidal activation functions. The first-layer weights V are initialized randomly. They are uniformly randomly distributed between -1 and 1. Second-layer weights W are initialized at zero. Note that this weight initialization will not affect system stability since the weights W are initialized at zero, and therefore there is initially no input to the system except for the PD loop. Filter that generates the signal $\dot{\tau}_{des}$ is implemented as $\frac{s}{s+100}$.

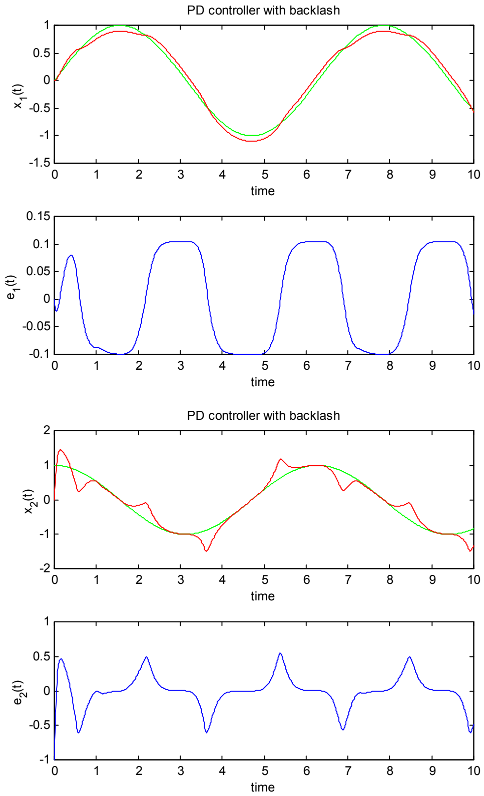

Figure 5.1 State $x_1(t)$ and tracking error $e_1(t)$ (above) and state $x_2(t)$ and tracking error $e_2(t)$ (below) without backlash compensation.

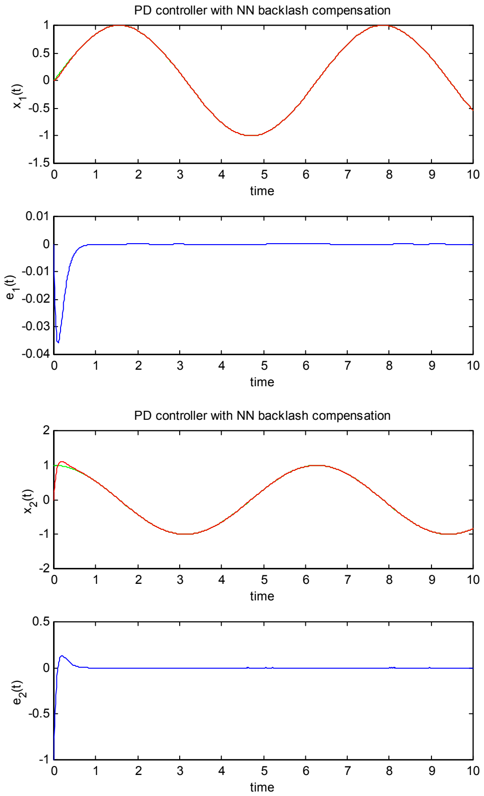

Figure 5.2 State $x_1(t)$ and tracking error $e_1(t)$ (above), and $x_2(t)$ and tracking error $e_2(t)$ (below) with NN backlash compensation.

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