Adaptive NN Control of Dynamic Systems with Unknown Dynamic Friction

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Abstract

In this paper, based on the dynamic LuGre friction model, adaptive NN controllers are presented by using neural networks to parameterize the unknown characteristic function $\alpha(x, \dot{x})$ or the unknown dynamic friction bounding function respectively. Using Lyapunov synthesis, the adaptive control algorithms are designed to achieve globally asymptotic tracking of the desired trajectory and guarantee the boundedness of all the signals in the closed-loop. Intensive simulations are carried out to verify the effectiveness of the proposed methods.

1 Introduction

In servo-motion control systems, friction causes many undesired phenomena such as tracking errors, limit cycles, and the undesired stick-slip motion of friction can lead to overall system performance degradation or instability. Accordingly, it is important to compensate for the effects of friction. Adaptive friction compensation techniques based on different friction models have been proposed in the literature [1, 2, 4, 6]. Friction is usually modeled as a static map between velocity and friction force (or torque) that depends on the sign of velocity, which includes the Static, Coulomb, and Viscous friction components. However, in applications with high precision positioning and with low velocity tracking, friction compensation based on static models is not always satisfactory. Several interesting properties observed in systems with friction cannot be explained by static models because the internal dynamics of friction are not considered. All these static and dynamic characteristics of friction are captured by the dynamic LuGre model proposed in [2].

Based on the LuGre model, several nice model-based controllers have been developed. Under the assumption of known system parameters and functions, a model-based controller was presented in [2] with the unmeasurable friction state being estimated by an observer which is driven by the tracking error. Two globally stable model-based adaptive friction compensation schemes were presented in [4] for "structured" variations by assuming that changes in friction are mainly due to either changes in the normal force that only affects proportionally the static friction characteristics, or temperature variations affecting uniformly both static and dynamic parameters. However, the resulting schemes adapt only one single parameter. In [11], using a dual-observer structure for the internal friction state estimation, two elegant controllers were presented for the following two cases: (i) unknown friction coefficients with known inertia parameter and known friction characteristic function, and (ii) known friction characteristic function with unknown multiplying coefficient for handling non-uniform variation in the friction force and normal force variations. Case (ii) is an alternative to the same solution given in [4].

Due to its uniform approximation ability to any continuous nonlinear functions, there has been considerable research interest in neural network (NN) control of nonlinear systems, and some are applied to systems with friction which is usually difficult to model. In [3], an adaptive NN controller was proposed, where the NN was used to parameterize the nonlinear characteristic function of the dynamic friction model which may be a function of both position and velocity with known system parameters. In [9], a reinforcement learning based NN adaptive control scheme was applied to compensate for stick-slip friction for tracking control of 1-DOF mechanical system with guaranteed high precision and smoothness of motion. In [8], both model-based and NN based adaptive controllers were investigated for dynamic friction, where the friction model for controller design is given in an easy-to-use linear-in-the-parameters (LIP) form.

In this paper, new adaptive controllers are presented by combining NN parameterization, dual-observer for state estimation/stability and adaptive control techniques together. Two cases are considered based on the uncertainty levels of the system: (a) unknown characteristic function $\alpha(x, \dot{x})$ of the dynamic friction model,

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and (b) full set of unknown parameters. For Case (a), an adaptive NN controller is proposed by using a neural network to parameterize the unknown $\alpha(x, \dot{x})$. For Case (b), by dividing the friction model into two portions: (i) the viscous friction with unknown constant coefficient, and (ii) the unknown dynamic friction which is a function of the unmeasurable internal friction state $z(t)$, and bounded by a function which is independent of $z(t)$, an adaptive NN controller is developed based on NN parameterization of the unknown dynamic friction bounding function. Using Lyapunov synthesis, adaptation algorithms are designed to achieve globally asymptotic tracking of the desired trajectory and guarantee the boundedness of all signals in the closed-loop. Simulation results are presented to show the effectiveness of the proposed approaches.

2 Dynamic Friction Modeling

Friction is a multifaceted phenomenon, exhibits the well-known classical Coulomb and viscous friction, nonlinearity at low velocity, and the elasticity of the contact surfaces. The classical models cannot sufficiently describe all the dynamic effects of friction, such as the pre-sliding displacement, the frictional lag, the stribeck effect, all occur in the so-called "low-velocity" and the "pre-sliding" regions. All of these observed static and dynamic characteristics of friction can be accurately captured by the dynamic LuGre-model proposed in [2].

The LuGre model considers the dynamic effects of friction as arising out of the defection of bristles which model the asperities between two contacting surfaces, and is given by

$$F = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{x} \tag{1}$$

$$\dot{z} = \dot{x} - \alpha(\dot{x})|\dot{x}|z \tag{2}$$

where F is the friction force, z denotes the average deflection of the bristles, which is not measurable, $\sigma_0, \sigma_1, \sigma_2$ are friction force parameters that can be physically explained as the stiffness of bristles, damping coefficient, and viscous coefficient, and the nonlinear friction characteristic function $\alpha(\dot{x})$ is a finite positive function which can be chosen to describe different friction effects. One of the parameterization of $\alpha(\dot{x})$ to characterize the Stribeck effect is given in [2]

$$\alpha(\dot{x}) = \frac{\sigma_0}{f_c + (f_s - f_c)e^{-(\dot{x}/\dot{x}_s)^2}} \tag{3}$$

where f_c is the Coulomb friction level, f_s is the level of the stiction force and $\dot{x}_s$ is the constant stribeck velocity.

Remark 1 *In the above model, there are no terms which explicitly account for position dependence of the friction force. However, there may exist some applications where the function $\alpha(\cdot)$ in the LuGre model also depends on the actual position, or on a more complex combination of position and velocity [3]. Therefore, we assume that $\alpha(x, \dot{x})$ is an upper and lower bounded positive smooth function of x and $\dot{x}$, and consider the LuGre model in the following form:*

$$F = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{x} \tag{4}$$

$$\dot{z} = -\alpha(x, \dot{x})|\dot{x}|z + \dot{x} \tag{5}$$

Assumption 1 *There exist positive constants α_{min} and α_m such that $0 < \alpha_{min} \le \alpha(x, \dot{x}) \le \alpha_m, \forall (x, \dot{x}) \in R^2$.*

Lemma 1 *Noting Assumption 1, if $|z(0)| \le 1/\alpha_{min}$ then $|z(t)| \le 1/\alpha_{min}, \forall t \ge 0$ [2].*

In control engineering, a NN is usually taken as a function approximator which emulates a given nonlinear function up to a small error tolerance. It has been proven [5] that any continuous functions can be uniformly approximated by a linear combination of Gaussian radial basis function (RBF) if the size is large enough. The RBF NN is a particular network architecture and can be described as

$$f_{nn}(W, x, \dot{x}) = W^T S(x, \dot{x}) \tag{6}$$

where $x, \dot{x}$ are the input variables, $W \in R^l$ is the weight vector, and

$$S(x, \dot{x}) = [s_1(x, \dot{x}), s_2(x, \dot{x}), \cdots, s_l(x, \dot{x})]^T \in R^l \tag{7}$$

is the basis function vector having the form of $s_i(x, \dot{x}) = exp(-\frac{(x-\mu_{1i})^2 + (\dot{x}-\mu_{2i})^2}{\sigma^2}), i = 1, \cdots, l$ with $\sigma \in R$ being the variance and $[\mu_{1i}, \mu_{2i}]^T \in R^2$ being the centre vector. For simplicity, RBF NNs shall be used for function approximation though other neural networks can also be used without any difficulty.

3 Adaptive Friction Compensation

The servo mechanism under study is modeled as a simple mass system with dynamic friction, described by

$$m\ddot{x} + F = u \tag{8}$$

where m is the mass, x is the displacement, u is the control force, and F is the friction force given by the dynamic friction model (4).

Assumption 2 *States x and $\dot{x}$ are measurable for feedback controller design.*

Assumption 3 *The desired trajectory x_d, and its first and second derivations $\dot{x}_d, \ddot{x}_d$ are continuous and bounded.*

The control objective is to design an adaptive controller to track the given desired trajectory. In this paper, we shall present two novel adaptive compensation schemes for dynamic friction using the concept of dual-observer and neural network based parameterization under different situations: (a) unknown nonlinear function $\alpha(x,\dot{x})$, or (b) unknown full set of related parameters and nonlinear function of the dynamic friction model.

Substituting friction dynamics (4) into system equation (8), we have

$$m\ddot{x} = u - (\sigma_1+\sigma_2)\dot{x} - \sigma_0 z + \alpha(x,\dot{x})\sigma_1|\dot{x}|z \qquad (9)$$

Define $e = x - x_d$, $\dot{x}_r = \dot{x}_d - \lambda e$ and $r = \dot{e} + \lambda e$, where $\lambda > 0$ and r is the filtered tracking error. Then the tracking error dynamics is transformed into

$$m\dot{r} = u - (\sigma_1+\sigma_2)\dot{x} - \sigma_0 z + \alpha(x,\dot{x})\sigma_1|\dot{x}|z - m\ddot{x}_r \qquad (10)$$

3.1 Controller Design for Unknown $\alpha(x,\dot{x})$

Assuming that the parameters $m, \sigma_0, \sigma_1, \sigma_2$ are known while the nonlinear function $\alpha(x,\dot{x})$ is unknown. Note that the parameterization given in (3) is not exclusive. Rather than to find another analytical description of $\alpha(x,\dot{x})$ for better or for worse, we shall use the RBF NN $\hat{\alpha}(x,\dot{x}) = W^T S(x,\dot{x})$ to approximate the unknown $\alpha(x,\dot{x})$. Since $\alpha(x,\dot{x})$ is a continuous function, according to the general approximation ability of neural networks [7], the following function approximation holds over a compact set $\Omega \subset R^2$

$$\alpha(x,\dot{x}) = W^{*T}S(x,\dot{x}) + \epsilon,\ \forall (x,\dot{x}) \in \Omega \qquad (11)$$

where W^* is the optimal weight vector.

Assumption 4 *The neural network approximation error ϵ is bounded over the compact set Ω, i.e., $|\epsilon| \le \epsilon_b$, where ϵ_b is a small positive constant.*

Define an auxiliary variable $\delta = z + \frac{m}{\sigma_1}r$ for the estimate of the internal friction state z. Its derivate is then given by

$$\dot{\delta} = \dot{z} + \frac{m}{\sigma_1}\dot{r} = (\dot{x} - \alpha(x,\dot{x})|\dot{x}|z) + \frac{m}{\sigma_1}\dot{r} \qquad (12)$$

Substituting (10) and $z = \delta - \frac{m}{\sigma_1}r$ into (12) yields

$$\dot{\delta} = \dot{x} - \frac{\sigma_0}{\sigma_1}\delta + \frac{m\sigma_0}{\sigma_1^2}r + \frac{1}{\sigma_1}(u - (\sigma_1+\sigma_2)\dot{x} - m\ddot{x}_r) \qquad (13)$$

To estimate the unmeasurable friction state δ, consider the following dual-observer

$$\dot{\hat{\delta}}_1 = \dot{x} - \frac{\sigma_0}{\sigma_1}\hat{\delta}_1 + \frac{m\sigma_0}{\sigma_1^2}r + \xi - r \qquad (14)$$

$$\dot{\hat{\delta}}_2 = \dot{x} - \frac{\sigma_0}{\sigma_1}\hat{\delta}_2 + \frac{m\sigma_0}{\sigma_1^2}r + \xi + \sigma_1 r\dot{x} \qquad (15)$$

where $\hat{\delta}_1$ and $\hat{\delta}_2$ are the estimates of friction state δ, $\xi = \frac{1}{\sigma_1}(u - (\sigma_1+\sigma_2)\dot{x} - m\ddot{x}_r)$. Combining (13) with the dual-observer (14)-(15), we can obtain the internal friction state error dynamics as follows

$$\dot{\tilde{\delta}}_1 = -\frac{\sigma_0}{\sigma_1}\tilde{\delta}_1 + r \qquad (16)$$

$$\dot{\tilde{\delta}}_2 = -\frac{\sigma_0}{\sigma_2}\tilde{\delta}_2 - \sigma_1 r\dot{x} \qquad (17)$$

where $\tilde{\delta}_1 = \delta - \hat{\delta}_1$ and $\tilde{\delta}_2 = \delta - \hat{\delta}_2$.

Remark 2 *Two friction state observers are introduced here for different purposes. Observers (14) and (15) driven by $-r$ and $\sigma_1 r\dot{x}$ are introduced to cancel the cross-coupling terms $-\sigma_0 r\tilde{\delta}_1$ and $\sigma_1 r\alpha(x,\dot{x})|\dot{x}|\tilde{\delta}_2$ respectively in (24), the derivative of the Lyapunov function candidate, for closed-loop stability.*

Using $z = \delta - \frac{m}{\sigma_1}r$, equation (10) can be rewritten as

$$\begin{aligned} m\dot{r} &= -\sigma_0\delta + \sigma_1\alpha(x,\dot{x})|\dot{x}|\delta + \frac{m\sigma_0}{\sigma_1}r - m\alpha(x,\dot{x})|\dot{x}|r \\ &\quad + u - (\sigma_1+\sigma_2)\dot{x} - m\ddot{x}_r \end{aligned} \qquad (18)$$

Consider the control law

$$u = (\sigma_1+\sigma_2)\dot{x} + m\ddot{x}_r + u_{ar} \qquad (19)$$

where u_{ar} is the adaptive robust control term given by

$$\begin{aligned} u_{ar} &= -c_1 r + \sigma_0\hat{\delta}_1 - \sigma_1\hat{\alpha}(x,\dot{x})|\dot{x}|\hat{\delta}_2 - \frac{m\sigma_0}{\sigma_1}r \\ &\quad - \frac{\sigma_1^2\sigma_2\alpha_m r\dot{x}^2}{\sigma_0} - k\sigma_1|\dot{x}||\hat{\delta}_2|sgn(r) \end{aligned} \qquad (20)$$

with $\hat{\alpha}(x,\dot{x}) = W^T S(x,\dot{x})$, constants $c_1 > 0$ and $k \ge \epsilon_b$. Substituting (20) into (18), we have

$$\begin{aligned} m\dot{r} &= -c_1 r - \sigma_0\tilde{\delta}_1 + \sigma_1\alpha(x,\dot{x})|\dot{x}|\tilde{\delta}_2 \\ &\quad + \sigma_1(\tilde{W}^T S(x,\dot{x}) + \epsilon)|\dot{x}|\hat{\delta}_2 - m\alpha(x,\dot{x})|\dot{x}|r \\ &\quad - \frac{\sigma_1^2\sigma_2\alpha_m r\dot{x}^2}{\sigma_0} - k\sigma_1|\dot{x}||\hat{\delta}_2|sgn(r) \end{aligned} \qquad (21)$$

where $\tilde{W} = W^* - W$.

Remark 3 *The presence of k is for robust closed-loop stability because of the existence of $\epsilon \ne 0$. The particlar choice of the dynamic gain, $k\sigma_1|\hat{\delta}_2||\dot{x}|$, has the following advantages: (i) the value of k is only needed to be large enough to suppress the bounded approximation error ϵ, (ii) the gain is a function of $|\hat{\delta}_2||\dot{x}|$ so that it is zero whenever $|\hat{\delta}_2| = 0$ and/or $|\dot{x}| = 0$, and decreases as $|\hat{\delta}_2||\dot{x}|$ diminishes.*

Theorem 1 *Consider the closed-loop system consisting of system (8) with dynamic friction given by (4) and (5), dual-observer (14)-(15), and adaptive controller*

(19) and (20). If the weights of NN approximation to $\alpha(x,\dot{x})$ are updated by

$$\dot{\hat{W}} = \Gamma S(x,\dot{x})\sigma_1|\dot{x}|\hat{\delta}_2 r \tag{22}$$

where $\Gamma = \Gamma^T > 0$ is a dimensionally compatible constant matrix, then the tracking error converges to zero and all the signals in the closed-loop are bounded.

Proof: Consider the Lyapunov function candidate

$$V = \frac{1}{2}mr^2 + \frac{1}{2}\sigma_0\tilde{\delta}_1^2 + \frac{1}{2}\alpha_m\tilde{\delta}_2^2 + \frac{1}{2}\tilde{W}^T\Gamma^{-1}\tilde{W} \tag{23}$$

The time derivative of V along (21) is

$$\begin{aligned} \dot{V} &= mr\dot{r} + \sigma_0\tilde{\delta}_1\dot{\tilde{\delta}}_1 + \alpha_m\tilde{\delta}_2\dot{\tilde{\delta}}_2 + \tilde{W}\Gamma^{-1}\dot{\tilde{W}} \\ &= -c_1r^2 - \sigma_0 r\tilde{\delta}_1 + \sigma_1 r\alpha(x,\dot{x})|\dot{x}|\tilde{\delta}_2 \\ &\quad +\sigma_1 r(\tilde{W}^T S(x,\dot{x}) + \epsilon)|\dot{x}|\hat{\delta}_2 - m\alpha(x,\dot{x})|\dot{x}|r^2 \\ &\quad -\frac{\sigma_1^2\sigma_2\alpha_m r^2\dot{x}^2}{\sigma_0} - k\sigma_1|\dot{x}||\hat{\delta}_2|rsgn(r) + \sigma_0\tilde{\delta}_1\dot{\tilde{\delta}}_1 \\ &\quad +\alpha_m\tilde{\delta}_2\dot{\tilde{\delta}}_2 + \tilde{W}^T\Gamma^{-1}\dot{\tilde{W}} \end{aligned} \tag{24}$$

Substituting the observer error dynamics (16) and (17) into (24) and noting $\alpha(x,\dot{x}) \leq \alpha_m$, we have

$$\begin{aligned} \dot{V} &\leq -c_1r^2 - m\alpha(x,\dot{x})|\dot{x}|r^2 - \frac{\sigma_0^2}{\sigma_1}\tilde{\delta}_1^2 - \frac{\alpha_m\sigma_0}{\sigma_2}\tilde{\delta}_2^2 \\ &\quad +2\sigma_1\alpha_m|r||\dot{x}||\tilde{\delta}_2| - \frac{\sigma_1^2\sigma_2\alpha_m r^2\dot{x}^2}{\sigma_0} \\ &\quad +\sigma_1|\epsilon||\dot{x}||r||\hat{\delta}_2| - k\sigma_1|\dot{x}||\hat{\delta}_2|rsgn(r) \\ &\quad +\tilde{W}^T(\sigma_1 rS(x,\dot{x})|\dot{x}|\hat{\delta}_2 + \Gamma^{-1}\dot{\tilde{W}}) \end{aligned} \tag{25}$$

Using the fact that

$$\begin{aligned} 2\sigma_1\alpha_m|r||\dot{x}||\tilde{\delta}_2| &\leq \frac{(2\sigma_1\alpha_m|r||\dot{x}|)^2}{4\alpha_m\sigma_0/\sigma_2} + \frac{\alpha_m\sigma_0}{\sigma_2}\tilde{\delta}_2^2 \\ &= \frac{\sigma_1^2\sigma_2\alpha_m r^2\dot{x}^2}{\sigma_0} + \frac{\alpha_m\sigma_0}{\sigma_2}\tilde{\delta}_2^2 \end{aligned} \tag{26}$$

and substituting adaptation law (22) into (25), and noting $|\epsilon| \leq k$, we have the following inequality

$$\dot{V} \leq -c_1r^2 - m\alpha(x,\dot{x})|\dot{x}|r^2 - \frac{\sigma_0^2}{\sigma_1}\tilde{\delta}_1^2 \leq 0 \tag{27}$$

for c_1, σ_0, σ_1 and $\alpha(\cdot)$ are positive.

From the definition of Lyapunov function V in (23) and $\dot{V} \leq 0$, the global uniform boundedness of the tracking error r, the observer errors $\tilde{\delta}_1, \tilde{\delta}_2$, and the parameter estimation errors $\tilde{W}$ are guaranteed. From the definition of r and Assumption 3, it can also be concluded that the tracking error e is bounded. The boundedness of control u is apparent from (19). Q.E.D.

Since $r \in L_2, e \in L_2 \cap L_\infty$, e is continuous and $e \to 0$ as $t \to \infty$, and $\dot{e} \in L_2$. By noting that $r \in L_2$ and $x_d, \dot{x}_d, \ddot{x}_d \in L_\infty$, it is concluded that $\dot{r} \in L_\infty$ from equation (21). Using the fact that $r \in L_2$ and $\dot{r} \in L_\infty$, thus $r \to 0$ as $t \to 0$. Hence $\dot{e} \to 0$ as $t \to 0$. Q.E.D.

3.2 Controller Design for Full Set of Unknown Parameters

In practice, friction compensation becomes more challenging when the related parameters and functions of the system are unknown. In [10], using the boundedness property of the internal friction state $z(t)$, a simple linearly parameterized friction model was firstly presented and then adaptive control was developed based on the simplified model. In this section, motivated by the work in [8, 10], the dynamic friction is firstly separated into two parts: (i) the viscous friction with unknown constant coefficient, and (ii) the unknown dynamic friction which is a function of the unmeasurable internal friction state $z(t)$ and is bounded by a function which is independent of $z(t)$. Then an RBF NN is applied to approximate this unknown bounding function. Based on Lyapunov synthesis, adaptation algorithms for both the NN weights and the unknown system and friction parameters are presented.

In particular, the dynamic friction (4) can be further written as

$$\begin{aligned} F &= \sigma_1\dot{x} + \sigma_2\dot{x} + \sigma_0 z - \sigma_1\alpha(x,\dot{x})|\dot{x}|z \\ &= \theta\dot{x} + F_z(x,\dot{x},z) \end{aligned} \tag{28}$$

where $\theta = \sigma_1 + \sigma_2$, $\theta\dot{x}$ represents the viscous friction force and $F_z(x,\dot{x},z) = \sigma_0 z - \sigma_1\alpha(x,\dot{x})|\dot{x}|z$ is the dynamic friction force which depends on z.

From Lemma 1, we know that F_z is bounded by

$$\begin{aligned} |F_z(x,\dot{x},z)| &= |(\sigma_0 - \sigma_1\alpha(x,\dot{x}))||z(t)| \\ &\leq \frac{\sigma_0 + \sigma_1\alpha(x,\dot{x})}{\alpha_{min}} = F_{zm}(x,\dot{x}) \end{aligned} \tag{29}$$

where $F_{zm}(x,\dot{x})$ is the bounding function of $F_z(x,\dot{x},z)$ and is independent of the unmeasurable internal friction state z. In sequel, an RBF NN can be applied to approximate $F_{zm}(x,\dot{x})$, and similarly there exists the following function approximation

$$F_{zm}(x,\dot{x}) = W^{*T}S(x,\dot{x}) + \epsilon \tag{30}$$

with W^* being the optimal weight vector, and the NN approximation error ϵ being bounded by a small positive constant ϵ_d, i.e., $|\epsilon| \leq \epsilon_d$.

System tracking error dynamics (10) can be rewritten as

$$\begin{aligned} m\dot{r} &= u - (\sigma_1+\sigma_2)\dot{x} - \sigma_0 z + \alpha(x,\dot{x})\sigma_1|\dot{x}|z - m\ddot{x}_r \\ &= u - \theta\dot{x} - F_z(x,\dot{x},z) - m\ddot{x}_r \end{aligned} \tag{31}$$

Consider the following controller

$$u = -c_1 r + \hat{\theta}\dot{x} + \hat{m}\ddot{x}_r - \hat{F}_{zm}(x,\dot{x})sgn(r) - ksgn(r) \tag{32}$$

where constant $c_1 > 0$, $\hat{\theta}$ and $\hat{m}$ are the estimates of unknown θ and m respectively, $\hat{F}_{zm}(x,\dot{x}) = W^T S(x,\dot{x})$

is the RBF NN approximation of function bound $F_{zm}(x, \dot{x})$, and $k > \epsilon_d$.

Substituting (32) into (31) yields

$$\begin{aligned} m\dot{r} &= -c_1 r - \tilde{\theta}\dot{x} - \tilde{m}\ddot{x}_r - \hat{F}_{zm}(x,\dot{x})sgn(r) \\ &\quad -ksgn(r) - F_z(x,\dot{x},z) \end{aligned} \quad (33)$$

where $(\tilde{*}) = (*) - (\hat{*})$ denotes the unknown parameter estimation errors.

Adding and subtracting $F_{zm}(x,\dot{x})sgn(r)$ in (33) and noting equation (30), we have

$$\begin{aligned} m\dot{r} &= -c_1 r - \tilde{\theta}\dot{x} - \tilde{m}\ddot{x}_r + (\tilde{W}^T S(x,\dot{x}) + \epsilon)sgn(r) \\ &\quad -F_z(x,\dot{x},z) - F_{zm}(x,\dot{x})sgn(r) - ksgn(r) \\ &= -c_1 r - \tilde{\theta}\dot{x} + \tilde{W}^T S(x,\dot{x})sgn(r) - (k-\epsilon)sgn(r) \\ &\quad -F_z(x,\dot{x},z) - F_{zm}(x,\dot{x})sgn(r) - \tilde{m}\ddot{x}_r \end{aligned} \quad (34)$$

where $\tilde{W} = W^* - W$.

Theorem 2 *Consider the closed-loop system consisting of system (8) with dynamic friction given by (4) and (5), and adaptive controller (32). If the parameters $\hat{\theta}, \hat{m}$ and NN weight W are updated by*

$$\dot{\hat{\theta}} = -\eta_\theta \dot{x} r \quad (35)$$

$$\dot{\hat{m}} = -\eta_m \ddot{x}_r r \quad (36)$$

$$\dot{W} = \Gamma S(x,\dot{x})|r| \quad (37)$$

where η_θ and η_m are positive constants, $\Gamma = \Gamma^T > 0$ is a dimensionally compatible constant matrix, then the tracking error converges to zero and all the signals in the closed-loop are bounded.

Proof: Consider the candidate Lyapunov function candidate

$$V = \frac{1}{2}mr^2 + \frac{1}{2\eta_\theta}\tilde{\theta}^2 + \frac{1}{2\eta_m}\tilde{m}^2 + \frac{1}{2}\tilde{W}^T\Gamma^{-1}\tilde{W} \quad (38)$$

The time derivative of V along (34) is

$$\begin{aligned} \dot{V} &= m\dot{r}r + \frac{1}{\eta_\theta}\tilde{\theta}\dot{\tilde{\theta}} + \frac{1}{\eta_m}\tilde{m}\dot{\tilde{m}} + \tilde{W}^T\Gamma^{-1}\dot{\tilde{W}} \\ &= -c_1 r^2 - \tilde{\theta}\dot{x}r - \tilde{m}\ddot{x}_r r + \tilde{W}^T S(x,\dot{x})rsgn(r) \\ &\quad +\epsilon rsgn(r) - F_z(x,\dot{x},z)r - F_{zm}(x,\dot{x})rsgn(r) \\ &\quad -krsgn(r) + \frac{1}{\eta_\theta}\tilde{\theta}\dot{\tilde{\theta}} + \frac{1}{\eta_m}\tilde{m}\dot{\tilde{m}} + \tilde{W}^T\Gamma^{-1}\dot{\tilde{W}} \end{aligned} \quad (39)$$

Re-arranging the related items in (39) yields

$$\begin{aligned} \dot{V} &= -c_1 r^2 - \tilde{\theta}(\dot{x}r - \frac{1}{\eta_\theta}\dot{\tilde{\theta}}) - \tilde{m}(\ddot{x}_r r - \frac{1}{\eta_m}\dot{\tilde{m}}) \\ &\quad +\tilde{W}^T(S(x,\dot{x})rsgn(r) + \Gamma^{-1}\dot{\tilde{W}}) \\ &\quad +\epsilon|r| - k|r| - F_z(x,\dot{x},z)r - F_{zm}(x,\dot{x})|r| \end{aligned} \quad (40)$$

Noting that $(\dot{\tilde{*}}) = -(\dot{\hat{*}})$ and substituting adaptation laws (35)-(37) into (40), we have

$$\dot{V} = -c_1 r^2 + \epsilon|r| - k|r| - F_z(x,\dot{x},z)r - F_{zm}(x,\dot{x})|r| \quad (41)$$

Because $|\epsilon| \leq k$ and $|F_z(x,\dot{x},z)| \leq F_{zm}(x,\dot{x})$, (41) can be further simplified as $\dot{V} \leq -c_1 r^2 \leq 0$. From the definition of Lyapunov function V in (38) and $\dot{V} \leq 0$, the global uniform boundedness of the tracking error r, the parameter estimation errors $\tilde{\theta}, \tilde{m}$, and the NN weight estimation error $\tilde{W}$ are guaranteed. Obviously, the estimates $\hat{\theta}, \hat{m}$ and weight W are bounded. From the definition of r and Assumption 3, it can also be concluded that the tracking error e is bounded. The boundedness of the control u is apparent from (32). Since $r \in L_2, e \in L_2 \cap L_\infty$, e is continuous and $e \to 0$ as $t \to \infty$, and $\dot{e} \in L_2$. By noting that $r \in L_2$ and $x_d, \dot{x}_d, \ddot{x}_d \in L_\infty$, it is concluded that $\dot{r} \in L_\infty$ from equation (33). Using the fact that $r \in L_2$ and $\dot{r} \in L_\infty$, thus $r \to 0$ as $t \to 0$. Hence $\dot{e} \to 0$ as $t \to 0$. Q.E.D.

4 Simulation Studies

The dynamic friction model given by (2) - (3) is used in the simulation, and the parameters are chosen as $\sigma_0 = 10^5, \sigma_1 = \sqrt{10^5}, \sigma_2 = 0.4, \dot{x}_s = 0.001, f_c = 1$ and $f_s = 1.5$ [8]. For simplicity, the system parameter is chosen as $m = 1$. The control objective is to make the output $x(t)$ track a desired trajectory $x_d(t) = 0.5sin(2\pi t)$. The initial states are $[x(0), \dot{x}(0)]^T = [0.1, 0]^T$. The filtered tracking error $r = \dot{e} + \lambda e, \lambda = 5, e = x - x_d$, and $\ddot{x}_r = \ddot{x}_d - \lambda\dot{e}$. For case (a), the control parameters are chosen as $c_1 = 30, k = 10$, $\alpha_m = 10^5$, $\Gamma = 10$. For case (b), the control parameters are chosen as $c_1 = 50, k = 10$. The adaptation gains are chosen as $\Gamma = 10, \eta_\theta = 10$ and $\eta_m = 10$. Figure 1 and 4 show that good position tracking performances are achieved. Figure 2 and 5 indicate the bounded control signal u. Figure 3 and 6 present the boundedness of NN weights.

5 Conclusion

In this paper, new adaptive friction compensation schemes have been presented by combining NN parameterization, dual-observer for state estimation/stability and adaptive control techniques together. Based on Lyapunov synthesis, the adaptation algorithms were designed to achieve globally asymptotic tracking of the desired trajectory and guarantee the boundedness of all the signals in the closed-loop. Simulation results have shown that the proposed approaches were effective for dynamic friction compensation in servo mechanisms.

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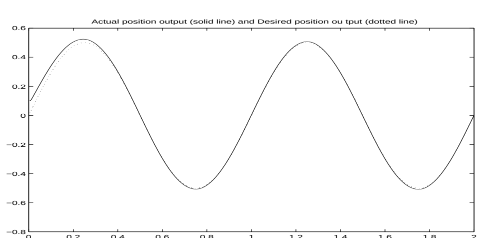

Figure 1: Position tracking performance: Case 1

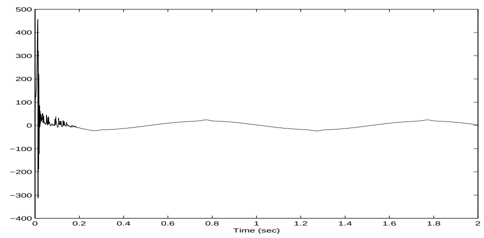

Figure 2: Control input: Case 1

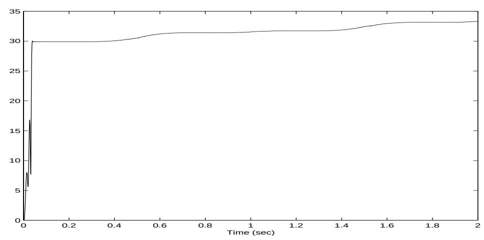

Figure 3: Norm of estimated weights $\|W\|$: Case 1

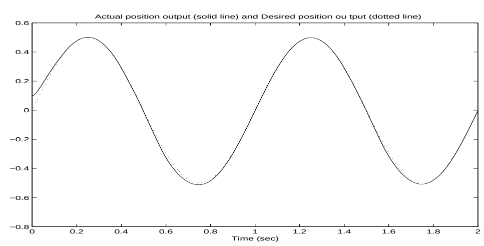

Figure 4: Position tracking performance: Case 2

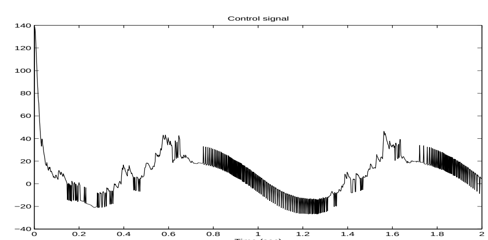

Figure 5: Control input: Case 2

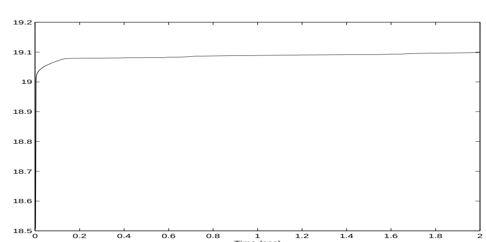

Figure 6: Norm of estimated weights $\|W\|$: Case 2