

# High Resolution Sensing and Anisotropic Segmentation for SAR Imagery <sup>1</sup>

Tryphon T. Georgiou <sup>2</sup> and Allen Tannenbaum <sup>3</sup>

## Abstract

The purpose of this note is to introduce new methodologies for high resolution image processing and knowledge-based segmentation for SAR imagery. These techniques could also have a major impact on problems in radar and remote sensing where modern mathematical tools and optimization techniques are likely to advance significantly the current state of the art.

## 1 Introduction

In this talk we will present certain novel methodologies for image analysis, primarily for the purposes of SAR (synthetic aperture radar) imaging.

SAR imaging is a mature subject with a history and technological developments of over 40 years. A classical text on the subject is [4] to which we refer the interested reader for a detail discussion of enabling technology and a wide range of applications. Extensive amount of SAR data, documentation and technology can also be found on the web, e.g., [1]. The purpose of this talk is to address two particular aspects of SAR processing where mathematical and software developments may permit significant improvement of imaging. In particular, we will focused on (i) high resolution spectral analysis and image reconstruction and (ii) knowledge based image analysis and segmentation. To this end we briefly outline the origin of SAR data and the nature of the problems we address.

SAR data are typically collected by a flying system (satellite, plane) as depicted in Figure 1, see [4, 10, 1]. The target area is illuminated by a series of electromagnetic pulses. The recorded echoes are analyzed into components according to the distance of the responsible scatterers (e.g., using matched filtering). The echoes from a particular distance range contain information about the profile and reflectivity of the relevant scatterers at that range. The information is encoded in the amplitude and the doppler shift of the reflected

signal. E.g., positive doppler shift indicates a scatterer which is positioned more aft, etc. Spectral analysis of the echoed signal into Fourier components produces the amplitude and position of scatterers at each distance range—thereby generating a SAR image of the targeted area.

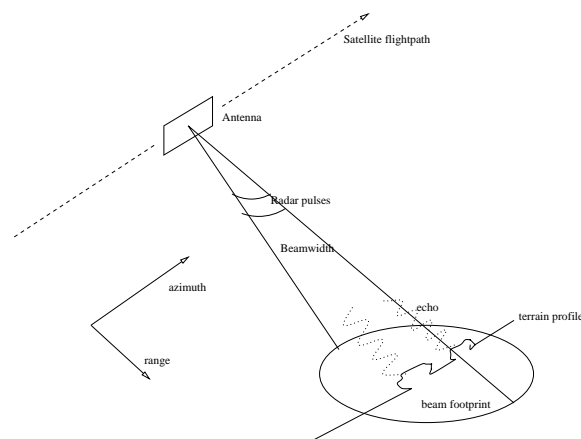


Figure 1. Synthetic Aperture Radar (SAR) system

Analogous issues arise in sonar or radar applications which involve array of sensors [8]. A target area is illuminated and the direction of the reflected wave, as it impinges upon an antenna array, reveals the location of the scatterers. The spatial harmonics of the received signal across the elements of the antenna carry the useful information which may allow resolving the location of nearby scatterers.

Section 2 outlines a new methodology for very high resolution spectral analysis and discusses its relevance and performance when applied to SAR data. Section 3 highlights a new framework for the subsequent processing of SAR images using knowledge based segmentation and anisotropic smoothing.

## 2 Signal processing: Very High Resolution (VHR) spectral analysis

Traditional Fourier transform based techniques are limited by the “uncertainty principle”, whereas the “modern spectral estimation techniques”—Maximum-entropy-method (MEM), Capon maximum likelihood, MUSIC, ESPRIT—which were developed in the 1980’s, are able

<sup>1</sup>Supported by grants from the AFOSR and the NSF

<sup>2</sup>Department of Electrical and Computer Engineering, University of Minnesota, MN 55455; tryphon@ece.umn.edu

<sup>3</sup>Department of Electrical and Computer Engineering, Georgia Tech, GA 30332-0250; tannenba@ece.gatech.edu

to exceed the uncertainty limit. However such benefits can only be drawn in high signal-to-noise conditions and the techniques are quite sensitive to the nature of the noise. Our recent work [5, 6, 2, 3] has given rise to a new framework. The algorithms we have developed are akin to the aforementioned techniques (MEM, etc.) in that they rely on analytical properties of the power spectrum and its relation to covariance statistics. Yet, they differ in that they use non-traditional covariance statistics and generalized analytic interpolation to provide superior resolution and robustness.

The new approach is based on the observation that the state-covariance of a linear filter provides *analytic interpolation constraints* for the spectral density functions of the input. More precisely, if  $\Phi_u(\theta)$  with  $\theta \in [-\pi, \pi]$  is the spectral density of the input process  $u_n$ ,

$$x_{n+1} = Ax_n + bu_n, \quad n = 0, 1, \dots \quad (1)$$

is the state equation of the filter, and  $P = E\{x_n x_n'\}$  is its state-covariance, then

$$\Phi_u(\theta) = \text{Re}f(e^{j\theta})$$

where  $f$  is a positive real function which satisfies

$$f(A) = W, \quad (2)$$

with  $WE + EW' = P$  and  $E$  being the grammian of the system ( $E = bb' + AEA'$ ). Equation (2) along with the analytic constraints on  $f$  limit the allowable spectra  $\Phi_u$  for the input process. Classical analytic interpolation theory can be used to characterize all admissible spectra, while more recent developments ([2, 3]) allow restriction on the dimension of the relevant spectra.

We mention one of the possible techniques based (2): To start with, the input-to-state filter (1) can be selected with a pass-band on any harmonic interval where high resolution is desired. After estimating  $P$  from the observation record, a unique canonical power spectrum which consists of a minimal number of (complex) sinusoids can be constructed using singular-value-decomposition of  $P$  [6]. The frequency of such sinusoids reveal individual scatterers within the pass-band. An example of the method is shown below. Figures 2a and 2b are reconstructed from SAR data. They show two nearby buildings in a city. Figure 2a has been constructed using traditional Fourier transform techniques. The performance of modern techniques (MEM, MUSIC, ESPRIT etc.) is not acceptable due to the large number of scatterers within the field of view (some of it not shown). Figure 2b has been constructed using the aforementioned canonical spectral analysis and a suitable filter following [6]. The lower left corner of both images is shown in Figures 3a and 3b, respectively. Comparison of the two shows that the ‘‘wall structure’’ is highlighted more clearly in 3b with a distinct series of individual scatterers lined up across the image. This indicates a

significant improvement in resolution over traditional methods.

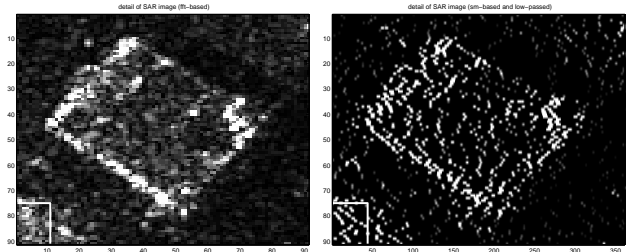


Figure 2: SAR image using (a) fft techniques, (b) canonical spectral analysis in [6]

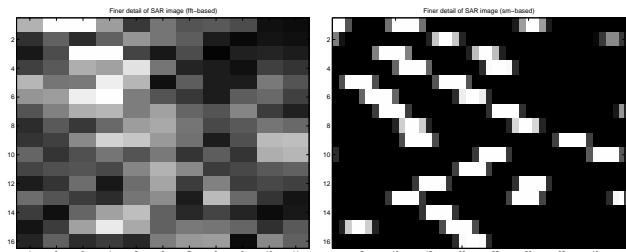


Figure 3: Detail of the SAR image using (a) fft techniques, (b) canonical spectral analysis in [6]

### 3 SAR Imaging and Image Processing

We have been working on a knowledge based segmentation method for SAR imagery. This is an application of a new approach combining anisotropic diffusion and the Bayesian paradigm for the segmentation of SAR (synthetic aperture radar) images. See [7] and the references therein.

One of the key ideas is the introduction of *a priori* knowledge about the number of objects present in the image, e.g., *target*, *shadow*, and *background terrain* via Bayes’ rule. Posterior probabilities obtained in this way are then anisotropically smoothed, and the image segmentation is obtained via MAP classifications of the smoothed data.

The model we employ begins with the assumption that the image is composed of  $n$  classes of objects. The goal of our segmentation is to determine to which class each pixel in the image belongs. We assume that the value of each pixel in a given class can be thought of as a random variable with a known normal distribution, and that these variables are independent across pixels. Thus, the likelihood of a particular pixel  $i$  having a certain value  $v$  given that it is in a given class  $c$  is:

$$\Pr(V_i = v | C_i = c) = \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left(-\frac{1}{2} \frac{(v - \mu_c)^2}{\sigma_c^2}\right) \quad (3)$$

where  $i$  is an index ranging over all pixels in the image,  $V_i$  is the value of the pixel, and  $C_i$  is its class.  $\mu_c$  and

$\sigma_c$  denote the mean and standard deviation of class  $c$ ; these are assumed known. In practice, these parameters are estimated from a set of sample images.

Given a set of intensity distributions  $\Pr(V_i = v|C_i = c)$  and priors  $\Pr(C_i = c)$ , we can apply Bayes' Rule from elementary probability theory to calculate the posterior probability that a given pixel belongs to a particular class, given its intensity:

$$\Pr(C_i = c|V_i = v) = \frac{\Pr(V_i = v|C_i = c) \Pr(C_i = c)}{\sum_{\gamma} \Pr(V_i = v|C_i = \gamma) \Pr(C_i = \gamma)} \quad (4)$$

(The denominator is just regarded as a normalization constant and can be ignored.) For simplicity, assume a homogeneous prior.

We can then calculate the posteriors  $P_i^c := \Pr(C_i = c|V_i = v)$  using (3) and (4) above, and then to apply **anisotropic smoothing** to each  $P^c$ . Specifically, we have chosen to smooth by evolving  $P^c$  according to a discretized version of the partial differential equation

$$\frac{\partial P^c}{\partial t} = ((P_y^c)^2 P_{xx}^c - 2P_x^c P_y^c P_{xy}^c + (P_x^c)^2 P_{yy}^c)^{1/3} \quad (5)$$

This equation defines the affine geometric heat flow, under which the level sets of  $P^c$  undergo affine curve shortening [9]. This particular diffusion equation was chosen because of its affine invariance, because it preserves edges well, and because of its numerical stability. The final segmentation is obtained using the maximum *a posteriori* probability estimate after anisotropic smoothing.

An example of the method from [7] is shown in Figures 4, 5.

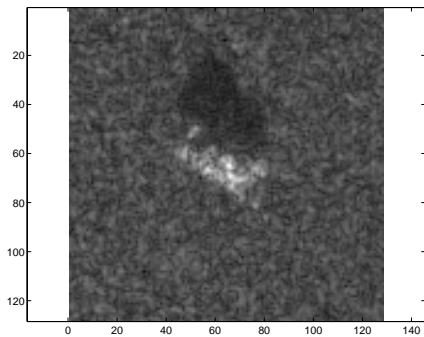


Figure 4: SAR Image - A Tank

#### 4 Concluding remarks

Spectral analysis and image processing tools such as the ones we discuss above may have a wide range of applications. However, the nature of SAR imaging render the synergy of these techniques especially promising.

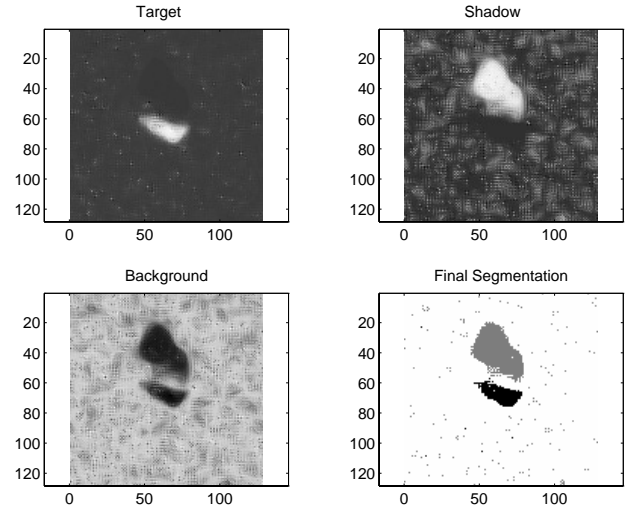


Figure 5: Smoothed Posteriors and Segmentation of Figure 3.

#### References

- [1] Alaska SAR facility <http://www.asf.alaska.edu>
- [2] C. I. Byrnes, T.T. Georgiou, and A. Lindquist, "A generalized entropy criterion for Nevanlinna-Pick interpolation," *IEEE Trans. on Aut. Contr.*, to appear.
- [3] C. I. Byrnes, T.T. Georgiou, and A. Lindquist, "A new approach to spectral estimation: A tunable high-resolution spectral estimator," *IEEE Trans. on Signal Proc.*, to appear.
- [4] J.C. Curlander and R. N. McDonough, **Synthetic Aperture Radar**, Wiley, 1991.
- [5] T.T. Georgiou, "Signal estimation via selective harmonic amplification: MUSIC, Redux," *IEEE Trans. on Signal Processing*, **48(3)**: 780-790, March 2000.
- [6] T.T. Georgiou, "Spectral estimation via selective harmonic amplification," *IEEE Trans. on Aut. Contr.*, to appear, February 2001.
- [7] S. Haker, G. Sapiro, and A. Tannenbaum, "Knowledge-based segmentation of SAR data with learned priors," *IEEE Trans. Image Processing* **9** (2000), pp. 298-302.
- [8] D.H. Johnson and D.E. Dudgeon, *Array Signal Processing*, Prentice-Hall, 1993.
- [9] G. Sapiro and A. Tannenbaum, "Affine invariant scale-space," *International Journal of Computer Vision* **11** (1993), 25-44.
- [10] M. Soumekh, "Reconnaissance with ultra wide-band UHF synthetic aperture radar," *IEEE Signal Processing Magazine*, **12(4)**: 21-40, 1995.