Closed-Loop Output Error Identification of Nonlinear Plants using Kernel representations

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Abstract

In this paper, we extend the family of algorithms presented in [3, 4] for the identification of continuous time nonlinear plants operating in closed-loop. The main novelty is that the identification of unstable plants is covered in its generality.

Keywords: closed-loop identification, nonlinear systems, adaptive systems, output-error, kernel representations

1 Introduction

One of the successful ways to develop algorithms for identification in closed-loop is to consider "closed-loop output error" schemes. A closed-loop output-error-type predictor parameterized in terms of the existing controller and the estimated plant is used. The algorithm minimizes a quadratic criterion in terms of the closed-loop output error or drives the closed-loop output error to zero; see [5] for results in a linear framework.

In [3, 4], the authors derive several algorithms for the recursive identification of *nonlinear* plants operating in closed-loop with a *nonlinear* controller using a closed-loop output error identification scheme. The reference [4] also presents a variant of the algorithm for the identification of nonlinear unstable plants that have a left coprime fractional description. As not every nonlinear plant has a left coprime fractional description, the theory developed in [4] is not fully general. In this paper, we consider the most general case by using the notion of a kernel representation of a nonlinear system. As noted in [7], a stable kernel representation can be seen as a generalization of a left coprime factorization. Also, we have used the notion of a stable image representation to show the resemblance of our algorithm with the one derived in [5] for the linear case.

The paper is organized as follows. Background material on stable kernel representations and stable image representations is presented in Section 2 and the problem setting is given in Section 3. In Section 4, the derivation of the algorithms is done in continuous time and in a deterministic noise free environment assuming that the plant is in the model set for a particular value of the unknown parameter vector and that one can neglect the terms of power higher than one in certain Taylor expansions. Section 5 connects the more general results derived in this paper with the ones derived in [3, 4]. Section 6 illustrates the theory with a simulation example. We conclude in Section 7.

2 Background material

A nonlinear generalization of the results in [4] hinges on a concept that generalizes the notion of a stable left coprime factorisation of a nonlinear system. This generalization is provided by the notion of a stable kernel representation of a nonlinear system. We also use the notion of a stable image representation to show the similarity of our algorithm with the one derived in [5]. Kernel and image representations of a nonlinear system are described in [7].

For simplicity, let us consider affine systems of the type

$$P := \begin{cases} \dot{x} &= f(x) + g(x)\,u, \quad x \in \mathcal{X}, u \in \mathbb{R}^m \\ y &= h(x), \quad y \in \mathbb{R}^p \end{cases} \tag{2.1}$$

Suppose that P can be stabilized by an output-to-state feedback law $k(x)$. Then the system

$$z_l = G(u, y) \tag{2.2}$$

$$G := \begin{cases} \dot{x} = [f(x) - k(x)h(x)] + g(x)\,u + k(x)\,y \\ z_l = y - h(x) \end{cases} \tag{2.3}$$

is a stable kernel representation of P. Here $z_l = 0$ means that u and y are trajectories of P. Of course when P is stable, one can choose $k(.) = 0$. The construction above specialized to the linear case gives rise to a left coprime factorization; a nonlinear system kernel representation however does not normally yield a left coprime realization.

Also the system

$$\begin{bmatrix} y \\ u \end{bmatrix} = G\,z_r \tag{2.4}$$

$$G = \begin{cases} \dot{x} = [f(x) - g(x)k(x)] + g(x)\,z_r \\ y = h(x) \\ u = z_r - k(x) \end{cases} \tag{2.5}$$

is a stable image representation of P with $k(x)$ a stabilizing feedback law for P. The construction above specialized to the linear case gives rise to a right coprime factorization.

3 The Basic Equations and Problem Setting

The objective is to estimate the parameters of a single input single output (SISO) nonlinear time invariant system described by

$$S: \quad y = P_0(u, v) \tag{3.1}$$

where P_0 is an unknown causal nonlinear operator, u is the control input signal, y is the achieved output signal and v is the disturbance signal allowed to enter the system nonlinearly. It is not assumed that the output y can be expressed linearly in terms of some parameter vector θ_0. For ease of notation the time argument will be omitted when there are no ambiguities.

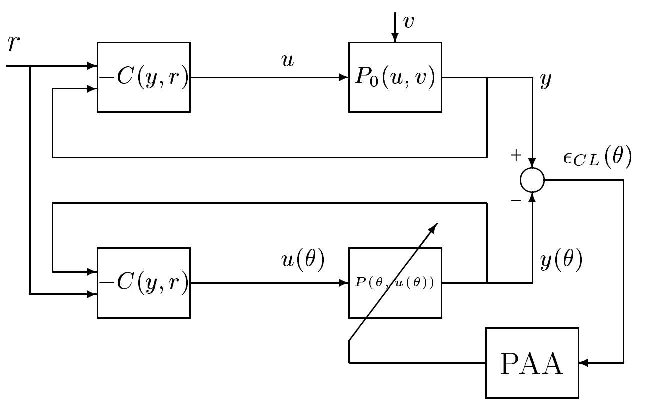

Figure 3.1: Closed-loop output error identification scheme

The plant is operated in closed-loop with a known nonlinear controller, i.e.

$$C: \quad u = -C(y, r) \tag{3.2}$$

where r is an external reference which is assumed to be quasi-stationary and uncorrelated with v. The controller C is a causal nonlinear operator acting on r and y.

The closed-loop operator from the measured reference signal r to the measured output signal y, as defined in Figure 3.1, is denoted by

$$y = T_0(r, v). \tag{3.3}$$

It is required that the closed-loop system is Bounded Input Bounded Output (BIBO) stable. In the sequel we often make use of linearizations of some nonlinear operators around their operating trajectories. We therefore require that the plant, the model (to be defined subsequently), the controller and all closed-loop operators are smooth functions of the reference signal, the input signal, the output signal and the disturbance signal. This means that if the closed-loop operator is linearized around any (stable) trajectory, the resulting linear (time-varying) system is BIBO stable. See [1] for more details.

We consider the following adjustable predictor for the closed-loop system defined by (3.1) and (3.2) (See also Figure 3.1)

$$y(\theta) = P(\theta, u(\theta)) \tag{3.4}$$

$$u(\theta) = -C(y(\theta), r) \tag{3.5}$$

where $P(\theta, u)$ defines the adjustable plant model, $y(\theta)$ is the output of the closed-loop predictor and $u(\theta)$ is the plant model input. It is also assumed that one can construct a stable kernel representation of

$$y_u(\theta) = P(\theta, u) \tag{3.6}$$

(see Section 2) as

$$z_l(\theta) = G(\theta, u, y_u(\theta)). \tag{3.7}$$

Further, it is assumed that one can construct a stable image representation of

$$u = -C(y, r) \tag{3.8}$$

(see Section 2) as

$$\begin{bmatrix} u \\ \begin{bmatrix} y \\ r \end{bmatrix} \end{bmatrix} = \begin{bmatrix} N \\ M \end{bmatrix} z_r. \tag{3.9}$$

Note that if the controller is stable a trivial image representation is $N = -C(.)$, $M = I$ (I is identity) and $z_r = [y^T \;\; r^T]^T$.

The closed-loop prediction error is defined as

$$\varepsilon_{CL} = y - y(\theta). \tag{3.10}$$

The following assumptions will be made in the sequel:

(i) $\exists\,\theta_0$ such that $P(\theta_0, u) = P_0(u, 0)$ for all $u \in \mathcal{L}_{2e}$.

(ii) **Notation:**

The operators $\partial G_u(\theta, u, y)$ and $\partial G_y(\theta, u, y)$ are the linearization of $G(\theta, u, y)$, respectively, in response to a perturbation in u and y along the trajectories produced by u and y.

The operators $\partial N(z_r)$ and $\partial M(z_r)$ are, respectively, the linearization of $N(z_r)$ and $M(z_r)$ in response to a perturbation in $z_r = M^{-1}y$ along the trajectories produced by z_r.

It is assumed that the operators $\partial G_u(\theta, u, y)$,

$\partial G_y(\theta,u,y)$, $\partial N(z_r)$ and $\partial M(z_r)$ exist for all allowable u, y and r. They are linear time-varying operators along the trajectories of the closed-loop system.

Note that G, N and M are stable operators. Using the smoothness assumptions made earlier, we can conclude that their linearization around any stable trajectory are BIBO stable systems.

(iii) **Notation:**

The partial derivative of $G(\theta,u,y)$ with respect to θ_j is denoted by $G'_{\theta_j}(\theta,u,y)$ for $j=1,\cdots,d$ where d is the dimension of the parameter vector θ.

The operator $G'_{\theta_j}(\theta,u,y)$ and its time derivatives exist and are norm-bounded $\forall j$ along the trajectories of the closed-loop predictor which requires $\dot{r}$ to be bounded. This assumption is not particularly restrictive as $P(\theta,.)$ is assumed to be a smooth operator for all θ.

(iv) Let us define

$$P_{CL}(\theta)=[\partial G_y(\theta,u(\theta),y(\theta))\partial M(z_r(\theta)) + \partial G_u(\theta,u(\theta),y(\theta))\partial N(z_r(\theta))]. \quad (3.11)$$

It is assumed that $P_{CL}=P_{CL}(\theta_0)$ and its inverse P_{CL}^{-1} exist along every trajectory of the closed-loop system encountered during the identification process. Both operators are BIBO linear time-varying operators. Note that P_{CL}^{-1} is BIBO stable precisely because of the stability of the closed-loop system and the smoothness assumption on the closed-loop operator.

(v) The reference r and the stochastic disturbance v are independent.

Assumption (i) means that at least for $\theta=\theta_0$, the plant is in the model set.

The generic parameter adaptation algorithm (PAA) which will be used throughout the paper is the continuous time version of the general PAA used in [6]:

$$\dot{\theta}(t)=F(t)\phi(t)\varepsilon_{CL}(t) \quad (3.12)$$

$$\dot{F}^{-1}(t)=-\left[1-\lambda_1(t)\right]F^{-1}(t)+\lambda_2(t)\phi(t)\phi^T(t) \quad (3.13)$$

$$0<\lambda_1(t)\le 1,\ 0\le\lambda_2(t)<2,\ F(0)>0,$$
$$F^{-1}(t)>\alpha F^{-1}(0),\ 0<\alpha<\infty$$

where $\theta(t)$ is the estimated parameter vector, $\varepsilon_{CL}(t)$ is the closed-loop output error, $\phi(t)$ is the observation vector, $F(t)$ is the adaptation gain matrix, $\lambda_1(t)$ is a time-varying forgetting factor and $\lambda_2(t)$ allows one to weight the rate of decrease of the adaptation gain. The two functions $\lambda_1(t)$ and $\lambda_2(t)$ allow one to have different laws of evolution of the adaptation gain.

We will consider subsequently that the assumptions (i) through (v) are valid and furthermore, for some analysis, that $v\equiv 0$. This will allow us to implement the appropriate parameter estimation algorithm, i.e. it allows us to derive the observation vector $\phi(t)$.

4 Nonlinear Closed-loop Output Error identification using stable image and kernel representations

In this section, we present the main theorem of this paper. We will only provide the derivations of the algorithm. A stability analysis in a deterministic environment (assuming that the system can be modeled exactly and that one can neglect terms of power higher than one in certain Taylor series expansions) can easily be obtained by mimicking the proofs in [4].

One has the following result (the NL-CLOE algorithm):

Theorem 4.1 *Under the assumptions (i) through (iv), assuming that $v(t)\equiv 0$ and neglecting the higher terms in certain Taylor expansions around the trajectories of the system one has for*

$$\begin{aligned}\phi(t) &= [-G'(\theta,u(\theta),y(\theta))]^T\\ &= -[G'_{\theta_1}(\theta,u(\theta),y(\theta)) \quad \cdots \quad G'_{\theta_d}(\theta,u(\theta),y(\theta))]^T\end{aligned}$$

that

$$\lim_{t\to\infty}\varepsilon_{CL}(t)=0 \quad (4.1)$$

if the linear time-varying operator

$$H=\partial M(z_r)\,P_{CL}^{-1}-\frac{\lambda(t)}{2}I;\quad \lambda(t)>\lambda_2(t),\quad \forall t \quad (4.2)$$

is strongly strictly passive.

If furthermore the linear time-varying operator P_{CL}^{-1} has a state-space representation (see Remark 1) one has also

$$\lim_{t\to\infty}\phi^T(t)\,(\theta(t)-\theta_0)=0. \quad (4.3)$$

Remark 1:

1. It is assumed that P_{CL}^{-1} has a state-space representation $\{A(t),B(t),C(t),D(t)\}$ with $A(t)$, $B(t)$, $C(t)$ and $D(t)$ continuous in t.
2. As the theorem proof will make clear, the Taylor series referred to in the theorem statement involve expansions in powers of $(u-u(\theta))$, $(y-y(\theta))$ and $(\theta_0-\theta)$.
3. The condition (4.2) assures that the prediction error goes asymptotically to zero, and that the estimated parameter vector θ, converges to a set defined as

$$\mathcal{D}_c=\left\{\theta:\phi^T(t)\,(\theta-\theta_0)=0\right\}\cdot \quad (4.4)$$

If

$$\phi^T(t)\,(\theta-\theta_0)=0 \quad (4.5)$$

has a unique solution $\theta=\theta_0$, the parameter vector will converge toward this value. In fact this condition is a "persistence of excitation" condition for the nonlinear case.

Proof of Theorem 4.1: In this paper, we provide Step 1 of the proof; Step 2 can easily be obtained by mimicking the proof in [4].

Step I: Establishing the expression $\varepsilon_{CL} = \mathbf{f}(\theta_0 - \theta(t))$
One has the following lemma:

Lemma 4.1 *Neglecting the higher order terms in the Taylor expansion around the trajectories of the closed-loop system and higher order terms in the Taylor expansion in powers of $(\theta - \theta_0)$, the closed-loop output error is given by*

$$\varepsilon_{CL} = \partial M(z_r)\, P_{CL}^{-1}\, [-G'(\theta, u(\theta), y(\theta))]\; [\theta_0 - \theta(t)]\,. \tag{4.6}$$

Proof: Let u and y denote trajectories from the closed-loop system (3.1)-(3.2) with $v \equiv 0$; one gets $y = P(\theta_0, u)$. From (3.7), $z_l = G(\theta_0, u, y)$ is a stable kernel representation of $P(\theta_0, u)$. Note that $z_l = 0$ as u and y are trajectories from $y = P(\theta_0, u)$. It follows that

$$\begin{aligned} z_l &= 0 = G(\theta_0, u, y) \\ &= G(\theta_0, u(\theta), y(\theta)) + [G(\theta_0, u, y) - G(\theta_0, u(\theta), y(\theta))] \end{aligned} \tag{4.7}$$

and, using series expansions around u and y while neglecting higher order terms in $(u - u(\theta))$ and $(y - y(\theta))$, one gets

$$\begin{aligned} &G(\theta_0, u, y) - G(\theta_0, u(\theta), y(\theta)) \\ &\quad = G(\theta_0, u, y) - [G(\theta_0, u, y) + \partial G_u(\theta_0, u, y)(u(\theta) - u) \\ &\qquad + \partial G_y(\theta_0, u, y)(y(\theta) - y)] \\ &\quad = \partial G_u(\theta_0, u, y)(u - u(\theta)) + \partial G_y(\theta_0, u, y)\, \varepsilon_{CL} \end{aligned}$$

where the definition of ε_{CL} given in (3.10) has been used. Using (3.9), it follows that

$$\begin{aligned} u - u(\theta) &= N(z_r) - N(z_r(\theta)) = \partial N(z_r)(z_r - z_r(\theta)), \\ y - y(\theta) &= M(z_r) - M(z_r(\theta)) = \partial M(z_r)(z_r - z_r(\theta)) \end{aligned}$$

which yields

$$u - u(\theta) = \partial N(z_r)\, \partial M(z_r)^{-1}\, (y - y(\theta)). \tag{4.8}$$

It is now straightforward to see that

$$\begin{aligned} G(\theta_0, u, y) - G(\theta_0, u(\theta), y(\theta)) &= [\partial G_y(\theta_0, u, y) \\ &+ \partial G_u(\theta_0, u, y) \partial N(z_r) \partial M(z_r)^{-1}]\, \varepsilon_{CL}. \end{aligned}$$

It now follows that

$$\begin{aligned} G(\theta_0, u, y) &= G(\theta_0, u(\theta), y(\theta)) + [\partial G_y(\theta_0, u, y) \partial M(z_r) \\ &+ \partial G_u(\theta_0, u, y) \partial N(z_r)] \partial M(z_r)^{-1}\, \varepsilon_{CL}. \end{aligned}$$

Let $y(\theta)$ and $u(\theta)$ denote trajectories from the closed-loop system (3.4)-(3.5). From (3.7), it follows that $z_l(\theta) = G(\theta, u(\theta), y(\theta))$ is a stable kernel representation of $P(\theta, u(\theta))$. Here $z_l(\theta) = 0$ as $u(\theta)$ and $y(\theta)$ are trajectories from $y(\theta) = P(\theta, u(\theta))$. Let us subtract $G(\theta, u(\theta), y(\theta))$ from $G(\theta_0, u, y)$. One obtains

$$\begin{aligned} 0 &= G(\theta_0, u, y) - G(\theta, u(\theta), y(\theta)) \\ &= G(\theta_0, u(\theta), y(\theta)) - G(\theta, u(\theta), y(\theta)) \\ &\quad + P_{CL}\; \partial M(z_r)^{-1}\, \varepsilon_{CL}, \\ &= G'(\theta, u(\theta), y(\theta))\, (\theta_0 - \theta) + P_{CL}\; \partial M(z_r)^{-1}\, \varepsilon_{CL}. \end{aligned}$$

from which one obtains

$$\varepsilon_{CL} = \partial M(z_r)\, [P_{CL}]^{-1}\, [-G'(\theta, u(\theta), y(\theta))]\, (\theta_0 - \theta).$$

Here $G'(\theta, u(\theta), y(\theta))$ has to be read as $G'(\theta, u, y)|_{u = u(\theta),\; y = y(\theta)}$. ■

Remarks:

1. It can easily be seen that the results derived here simplify to the ones derived in [5] for the linear case.

2. The passivity condition derived in this paper is less restrictive than the one derived in [3, 4] as there is an additional degree of freedom when constructing the kernel representation. This is illustrated in Section 6 by using the example of [4].

3. Note that the passivity condition implies the stability of the controller; a similar observation had been made for the linear case in [5].

4. One can also derive robustness results (that take into consideration the influence of noise and second order terms in the Taylor expansions) by mimicking the results obtained in [3, 4].

Variant on the algorithm: Neglecting the swapping correction terms which anyway become negligible when one uses decreasing adaptation gains, (4.6) can be also written as

$$\begin{aligned} \varepsilon_{CL} &= \partial M(z_r)\, P_{CL}^{-1}\, P_{CL}(\theta)\; \partial M(z_r(\theta))^{-1} \times \\ &\quad [-\partial M(z_r(\theta))\, P_{CL}(\theta)^{-1}\, G'(\theta, u(\theta)]\, [\theta_0 - \theta(t)]). \end{aligned}$$

In this case, one has to choose

$$\phi(t) = -\partial M(z_r(\theta)) P_{CL}(\theta)^{-1}\, G'(\theta, u(\theta). \tag{4.9}$$

In this case one filters $G'(\theta, u(\theta))$ through a linear time-varying closed-loop system which depends upon the current estimate.

The corresponding strongly strictly passive condition will become

$$\begin{aligned} H &= \partial M(z_r)\; P_{CL}^{-1}\, P_{CL}(\theta)\, \partial M(z_r(\theta))^{-1} - \frac{\lambda(t)}{2} I; \\ &\quad \lambda(t) > \lambda_2(t),\; \forall\, t > t_0 \end{aligned}$$

should be strongly strictly passive for every θ encountered during the identification procedure.

Clearly in the vicinity of θ_0, this condition is much more likely to be satisfied than condition (4.2) for NL-CLOE. The preceding algorithm is the generalization of the AFNL-CLOE algorithm presented in [3, 4].

5 Connections with previous results

In this section, we consider three special cases:

1. The controller is stable,
2. the plant has a left coprime factorization and the controller is stable,
3. both the plant and the controller are stable.

Suppose the controller is stable. Then one can choose (without loss of generality) $N = -C(.)$ and $M = I$. This yields

$$\varepsilon_{CL} = P_{CL}^{-1}\left[-G'(\theta, u(\theta), y(\theta))\right]\left[\theta_0 - \theta(t)\right]$$

with

$$P_{CL} = [\partial G_y(\theta, u(\theta), y(\theta)) - \partial G_u(\theta, u(\theta), y(\theta))\, \partial C_y(r, y)].$$

Suppose that, in addition to C being stable, one can also construct a stable left coprime description of

$$y_u(\theta) = P(\theta, u) \tag{5.10}$$

(see [2] for further details) as

$$D_l(\theta, y_u(\theta)) = N_l(\theta, u).$$

Thus $P = D_l^{-1} N_l$. Then it follows that

$$z_l(\theta) = G(u, y_u(\theta)) = D_l(\theta, y_u(\theta)) - N_l(\theta, u)$$

is a stable kernel representation of (5.10). It now follows that

$$\begin{aligned}
\varepsilon_{CL} &= P_{CL}^{-1}\, \phi(t)\, [\theta_0 - \theta(t)] \\
\phi(t) &= [-G'(\theta, u(\theta), y(\theta))]^T \\
&= [N_l'(\theta, u(\theta)) - D_l'(\theta, y(\theta))]^T \\
P_{CL} &= [\partial G_y(\theta_0, u, y) - \partial G_u(\theta_0, u, y)\ \partial C_y(r, y)] \\
&= [\partial D_{l_y}(\theta_0, y) + \partial N_{l_u}(\theta_0, u)\, \partial C_y(r, y)]
\end{aligned}$$

as originally derived in [4].

Suppose that, in addition to C being stable, (5.10) is a stable plant. Then it follows that

$$z_l(\theta) = G(u, y_u(\theta)) = y_u - P(\theta, u)$$

is a stable kernel representation of (5.10). It now follows that

$$\begin{aligned}
\varepsilon_{CL} &= P_{CL}^{-1}\, \phi(t)\, [\theta_0 - \theta(t)] \\
\phi(t) &= [-G'(\theta, u(\theta), y(\theta))]^T = [P'(\theta, u(\theta)]^T \\
P_{CL} &= [\partial G_y(\theta_0, u, y) - \partial G_u(\theta_0, u, y)\ \partial C_y(r, y)] \\
&= [I + \partial P_u(\theta_0, u)\, \partial C_y(r, y)]
\end{aligned}$$

as originally derived in [4].

6 An Example

In this section, we consider the same example as the one used in [4]. However, in this paper, we will use it to illustrate the recursive identification of an unstable nonlinear plant. Consider a plant described by

$$\dot{x} = u + \theta_0\, x^2 \tag{6.1}$$
$$y = x + v \tag{6.2}$$

with x, u, y in $\mathbf{R}^1$. Note that, contrary to what was done in [4], we do not impose any restrictions on θ_0.

Consider the BIBO stable controller

$$u = -\,(y^3 + by^2) + r = -C(y) + r. \tag{6.3}$$

Note that the controller is stable as it is memoryless. It is assumed that θ_0 is unknown but b is known. Observe that b might well be chosen as an estimate of θ_0. For if $b = \theta_0$ and $v \equiv 0$, the closed-loop system equation becomes

$$\dot{x} = -x^3 + r$$

and the closed-loop system is asymptotically stable.

The equations of the estimated closed-loop system are as follows: the estimated plant model will be described by

$$\dot{x}(\theta) = u(\theta) + \theta\, x(\theta)^2 \tag{6.4}$$
$$y(\theta) = x(\theta) \tag{6.5}$$

and the estimated control will be given by

$$u(\theta) = -[y(\theta)^3 + by(\theta)^2] + r = -C(y(\theta)) + r. \tag{6.6}$$

A kernel representation of

$$\dot{x}(\theta) = u + \theta\, x(\theta)^2 \tag{6.7}$$
$$y_u(\theta) = x(\theta) \tag{6.8}$$

is

$$G(\theta, u, y_u(\theta)) : \begin{cases} \dot{x}(\theta) = [\theta\, x(\theta)^2 - k(x(\theta))\, x(\theta)] + k(x(\theta))\, y_u(\theta) + u \\ z_l(\theta) = y_u(\theta) - x(\theta) \end{cases}$$

where $k(.)$ is chosen such that

$$\dot{x}(\theta) = [\theta\, x(\theta)^2 - k^2(x(\theta))\, x(\theta)] + q$$

is BIBO stable, with q as input.

One gets (with $p = \frac{d}{dt}$)

$$\begin{aligned}
G'(\theta, u(\theta), y(\theta)) &= [p - 2\theta\, y(\theta) + k(y(\theta))]^{-1}\, y(\theta)^2, \\
\partial G_u(\theta, u(\theta), y(\theta)) &= -[p - 2\theta\, y(\theta) + k(y(\theta))]^{-1}, \\
\partial G_u(\theta_0, u, y) &= \partial G_u(\theta_0, u(\theta_0), y(\theta_0)) \\
&= -[p - 2\theta_0\, y + k(y)]^{-1}, \\
\partial G_y(\theta, u(\theta), y(\theta)) &= \frac{p - 2\theta\, y(\theta) + 2\, k(y(\theta))}{p - 2\theta\, y(\theta) + k(y(\theta))}, \\
\partial G_y(\theta_0, u, y) &= \frac{p - 2\theta_0\, y + 2\, k(y)}{p - 2\theta_0\, y + k(y)}, \\
\partial C_y(r, y(\theta)) &= 3\, y^2(\theta) + 2b\, y(\theta), \\
\partial C_y(r, y) &= \partial C_y(r, y(\theta_0)) = 3\, y^2 + 2b\, y.
\end{aligned}$$

Here we have used the formulae of Section 5 as the controller is BIBO stable. One can express now $P_{CL}(\theta)$ and P_{CL}:

$$P_{CL}(\theta) = \frac{p + [3\,y^2(\theta) - 2(\theta - b)\,y(\theta) + 2\,k(y(\theta))]}{p - 2\theta\,y(\theta) + k(y(\theta))},$$
$$P_{CL} = \frac{p + [3\,y^2 - 2(\theta_0 - b)\,y + 2\,k(y)]}{p - 2\theta_0\,y + k(y)}.$$

The convergence condition requires that $P_{CL}^{-1}(\theta_0) - \frac{\lambda}{2}$ be strongly strictly passive where $P_{CL}(\theta)$ is given above. In this example one should make the assumption that $-2\theta_0\,y + k(y) > 0\ \forall t$, as well as the assumption that $3y^2 - 2(\theta_0 - b)\,y + 2\,k(y) > 0\ \forall t \geq t_0$. This can always be achieved using an appropriate choice of $k(.)$ in the kernel representation G defined above. Note that the results simplify to the ones obtained in [4] when $k(.) = 0$, but their validity presupposes that the plant (6.1)-(6.2) is stable.

Note that the old theory ($k = 0$) would have implied that, for $\theta_0 > 0$, $y(t) < 0$ for all t. We show below that the new theory can handle this situation with an parropriate choise of k.

We apply the NLCLOE algorithm using the previous example with $b = 0.4$, $r = 2 + 0.5\sin(0.1\,t)$ and v zero mean white Gaussian noise with variance σ^2. Note that this choice of reference signal yields a positive $y(t)$. The parameter which is to be identified recursively is given by

$$\theta_0(t) = \begin{cases} 0.5 \quad \text{for } t \leq 105 \\ 0.5 + 0.25\sin(0.03\,t) \quad \text{for } t > 105, \end{cases} \tag{6.9}$$

i.e. the parameter θ_0 is first held constant and then allowed to vary sinusoidally. We adopt a least squares strategy with forgetting factor ($\lambda_1 = 0.5$, $\lambda_2 = 1$) and the algorithm is initialized with $\theta(0) = -0.5$. Note that the open-loop system is unstable at all t whereas the initial $\theta(0)$ corresponds to an open loop stable model for $x > 0$. We have used a stable kernel representation G (as computed above) with $k(.) = 3\,[(.)^2 + 1]$. This choice of $k(.)$ ensures that the convergence condition is satisfied.

The top picture of Figure 6.1 shows the identification results in a noiseless situation. The NLCLOE algorithm allows a consistent identification of θ_0 with very good tracking results. The bottom picture of Figure 6.1 shows the appearance of a systematic small bias on the estimate in a noisy situation with the NLCLOE algorithm. The tracking results remain very good. Note that the sensitivity to noise can be be improved (at the expense of the tracking performance) by increasing the value of λ_1.

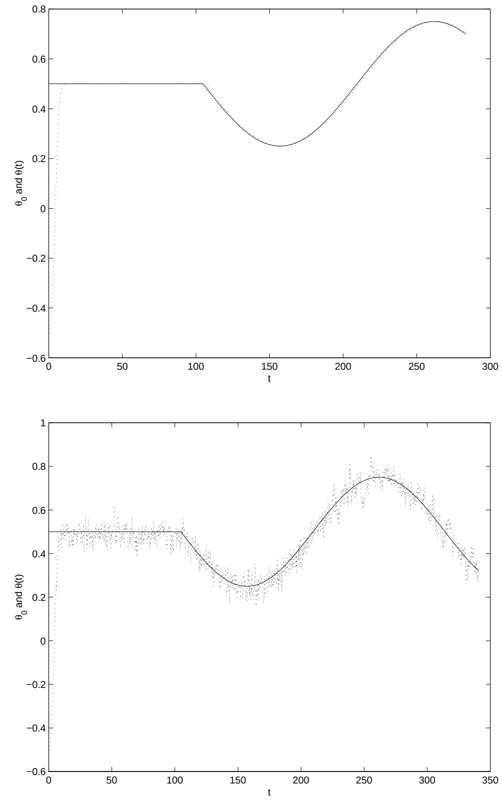

Figure 6.1: Identification of $\theta_0(t)$ (—) in a noiseless case ($\sigma^2 = 0$, top picture) and in a noisy situation ($\sigma^2 = 0.01$, $\lambda_1 = 0.5$ and $\lambda_2 = 1$, bottom picture) using the NLCLOE ($\cdots$) algorithm.

7 Conclusion

The main contribution of this paper is to show that the applicability of a number of closed-loop output-error identification algorithms can be pushed from stable nonlinear systems (or unstable nonlinear systems with a left coprime description) to nonlinear systems in general.

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