Design of Variable Structure Controller
– *From Sliding Mode to Sliding Sector* –

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Abstract

In general, a variable structure (VS) control system is designed with a sliding mode. But recently a sliding sector designed using algebraic Ricatti equation has been proposed to replace the sliding mode for chattering-free VS controllers and for discrete-time VS controllers. In this paper we design a new sliding sector based on the sliding mode and propose a VS controller with the sliding sector. The proposed VS control system is quadratically stable even if there exists parameter uncertainties and external disturbances. The experimental results on an inverted pendulum show the effectiveness of the proposed VS control algorithm with the sliding sector.

1 Introduction

The Variable Structure Control (VSC) system has been mainly considered for continuous-time systems in the form of sliding mode (Utkin, 1992). The VS control law, where a sign function is often used with sliding mode

$$s(x) = Sx \tag{1}$$

is designed such that the state moves to the sliding mode in a finite time and stays there since then, and such that the reduced order system in the sliding mode is stable.

When the VSC with sliding mode is implemented in practical systems or realized by digital controllers, not only the chattering around the sliding mode may be generated because of the finite switching frequency, but also the stable sliding mode designed for continuous-time systems may become unstable after discretizing (Furuta and Pan, 1994). To resolve this problem some VS controllers with boundary or sliding sector have been proposed in literature (Drakunov and Utkin, 1989), (Furuta, 1990), (Yu and Potts, 1991) and(Bartolini *et al.*, 1992).

In the VS controller with boundary, a saturation function is introduced to replace the sign function and inside the boundary

$$|s(x)| \leq \Phi, \tag{2}$$

a continuous feedback rule is used, where $s(x)$ is the sliding mode defined in (1) and Φ is a positive constant. With this kind of VS control law, the chattering is removed but it is difficult to show the stability inside the boundary.

As an alternate design algorithm of the Variable Structure Control (VSC), a sliding sector has been proposed to replace the sliding mode in the design of VSC for a chattering free controller and for the implementation in discrete-time control systems (Furuta, 1990)(Furuta and Pan, 2000). The PR-sliding sector in (Furuta and Pan, 2000) is a subset on the state space, inside which some norm of state decreases with zero control. The corresponding VS controller was designed such that the state moves from the outside to the inside of the sector with some VS control law, and the norm of the state keeps decreasing in the state space with specified negativity of its derivative. The sliding sector is designed with using Riccati equation as

$$\mathcal{S} = \{x|\ |s(x)| \leq \delta(x), x \in R^n\ \} \tag{3}$$

where $s(x)$ is a linear function and $\delta(x)$ the square root of a quadratic function. The linear function $s(x)$ in the sliding sector (3) is similar with the sliding mode (1). Therefore it is considerable to design a sliding sector based on the sliding mode.

In this paper, we will design a generalized sliding sector as

$$\tilde{\mathcal{S}} = \{x|\ |s(x)| \leq \delta(x,t), x \in R^n\ \} \tag{4}$$

where $s(x)$ is the sliding mode (1) which is designed such that the reduced order system is stable, $\delta(x,t)$

is a known positive function on x and t. With the generalized sliding sector (4), a VS control law will be designed such that the state moves into the generalized sliding sector in a finite time without chattering and some Lyapunov function keeps decreasing in the state space. Therefore the proposed VS control system is quadratically stable. A rotational inverted pendulum apparatus will be used to evaluate the proposed VS control algorithm.

The organization of the paper is as follows. Section 2 defines the generalized sliding sector and then designs it based on the sliding mode; Section 3 proposes a VS controller with the generalized sliding sector; Section 4 gives the experimental results on the rotational inverted pendulum apparatus with the proposed VS control and LQR control algorithms.

2 Generalized Sliding Sector

In the paper, a linear time-invariant continuous-time single input system with parameter uncertainties and external disturbances described by the state equation is taken into consideration.

$$\dot{x}(t) = Ax(t) + Bu(t) + Bd(x,t) \tag{5}$$

where $x(t) \in R^n$ and $u(t) \in R^1$ are state and input vectors, respectively, A and B are constant matrices of appropriate dimensions, the pair (A, B) is controllable, and $d(x,t)$ represents parameter uncertainties and external disturbances. It is assumed that the absolute value of $d(x,t)$ is bounded by a known positive function $f(x,t)$, i.e.

$$|d(x,t)| \le f(x,t), \forall x \in R^n, \forall t \in [0, +\infty) \tag{6}$$

and the derivative function of $f(x,t)$ satisfies the following inequalities:

$$-x^T P_1 x \le \frac{d}{dt} f(x,t) \le x^T P_2 x, \forall x \in R^n, \forall t \in [0, +\infty). \tag{7}$$

$P_1 \in R^{n\times n}$ and $P_1 \in R^{n\times n}$ are positive definite symmetric matrices.

Similar with the definition (Furuta and Pan, 2000) of the PR-sliding sector, a generalized PR-sliding sector for the plant (5) with parameter uncertainties and external disturbances is defined at first.

Definition 1 *The* Generalized PR-Sliding Sector *for the plant (5) is defined on the state space R^n as*

$$\mathcal{S} = \{x |\ |s(x)| \le \delta(x,t), x \in R^n \ \} \tag{8}$$

inside which the P-norm decreases with some control law and the derivative of a Lyapunov function candidate $L(t)$ satisfies that

$$\dot{L}(t) = \frac{d}{dt}(x^T(t)Px(t)) \le -x^T(t)Rx(t), \quad \forall x(t) \in \mathcal{S}$$

where $P \in R^{n\times n}$ and $R \in R^{n\times n}$ are positive definite symmetric matrices, $s(x)$ is a linear function on x, $\delta(x,t)$ is a positive nonlinear function on x and t satisfying:

$$\begin{aligned} s(x) &= Sx, \quad S \in R^{1\times n} & (9)\\ \delta(x,t) &= \eta f(x,t). & (10)\end{aligned}$$

$\eta > 0$ is a positive constant, $f(x,t)$ is the boundary function of the parameter uncertainties and external disturbances, and the P-norm and the Lyapunov function $L(t)$ are respectively defined as

$$\begin{aligned} ||x||_P &= \sqrt{x^T(t)Px(t)} & (11)\\ L(t) &= ||x||_P^2 = x^T(t)Px(t), & (12)\end{aligned}$$

In (Furuta and Pan, 2000), the PR-sliding sector has been designed by using the algebraic Riccati equation. The generalized sector defined above can also be designed in the same way. But in this paper, we will give an alternate design algorithm for the generalized PR-sliding sector where a sliding mode is first designed.

2.1 Sliding Mode

A sliding mode defined as

$$s(x) = Sx(t), \quad S \in R^{1\times n} \tag{13}$$

should be designed such that the reduced order system in the sliding mode

$$\begin{cases} \dot{x}(t) = Ax(t) + B(u(t) + d(x,t)) \\ s(x) = Sx(t) = 0 \end{cases} \tag{14}$$

is stable.

As it is assumed that the pair (A, B) is controllable, there exists a nonsingular matrix $T_1 \in R^{n\times n}$ that converts the plant (5) into the following controllable canonical form:

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}(u(t) + d(x,t)) \tag{15}$$

which can be rewritten as the following block form:

$$\frac{d}{dt}\left[\begin{array}{c}\bar{x}_1(t)\\ \hline \bar{x}_2(t)\end{array}\right] = \left[\begin{array}{c|c}\bar{A}_{11} & \bar{A}_{12}\\ \hline \bar{A}_{21} & \bar{A}_{22}\end{array}\right]\left[\begin{array}{c}\bar{x}_1(t)\\ \hline \bar{x}_2(t)\end{array}\right] + \left[\begin{array}{c}\bar{B}_1\\ \hline \bar{B}_2\end{array}\right][u(t)+d(x,t)], \tag{16}$$

where $\bar{x}_1(t) \in R^{n-1}$, $\bar{x}_2(t) \in R^1$, and

$$\begin{aligned} \bar{A} &= T_1^{-1}AT_1 \\ &= \left[\begin{array}{cccc|c} 0 & 1 & & & 0\\ \vdots & & \ddots & & \vdots \\ 0 & & & 1 & 0\\ 0 & 0 & \cdots & 0 & 1\\ \hline -a_0 & -a_1 & \cdots & -a_{n-2} & -a_{n-1}\end{array}\right], \\ \bar{B} &= T_1^{-1}B = \left[\begin{array}{cccc|c} 0 & \cdots & 0 & 0 & 1\end{array}\right]^T. \end{aligned}$$

Then the sliding mode in (13) can be rewritten as

$$s(x) = Sx(t) = \bar{S}\bar{x}(t) = \bar{S}_1\bar{x}_1(t) + \bar{x}_2(t) \tag{17}$$

where $\bar{S} = ST_1 = \begin{bmatrix} \bar{S}_1 & 1 \end{bmatrix}$ and $\bar{S}_1 \in R^{n-1}$.

The equivalent control guaranteeing $\dot{s}(x) = 0$ is given by

$$u_{eq}(t) = -(SB)^{-1}SAx(t) = -SAx(t), (SB = 1) \tag{18}$$

Take a nonsingular transformation as

$$z(t) = T_2\bar{x}(t) = \begin{bmatrix} \bar{x}_1(t) \\ s(x) \end{bmatrix} \tag{19}$$

where

$$T_2 = \begin{bmatrix} I_{n-1} & 0 \\ \bar{S}_1 & 1 \end{bmatrix}.$$

Then the system (16) with the equivalent control $u_{eq}(t)$ (18) becomes

$$\dot{z}(t) = \left[\begin{array}{c|c} \tilde{A}_{11} & \bar{A}_{12} \\ \hline 0 & 0 \end{array}\right] z(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [v(t) + d(x,t)], \tag{20}$$

where $\tilde{A} = \bar{A}_{11} - \bar{A}_{12}\bar{S}_1$, and $v(t)$ is an alternate control input satisfying

$$u(t) = u_{eq}(t) + v(t)$$

In the sliding mode $s(x) = 0$, the reduced order system (14) becomes

$$\dot{\bar{x}}_1(t) = (\bar{A}_{11} - \bar{A}_{12}\bar{S}_1)\bar{x}_1(t). \tag{21}$$

It is obvious that the pair $(\bar{A}_{11},\ \bar{A}_{12})$ is controllable. Therefore it is easy to design a feedback gain $\bar{S}_1$ such that the above reduced order system is stable, for example, using pole assignment algorithm, LQR control algorithm, and so on.

2.2 Design of Generalized Sliding Sector

With the sliding mode designed in the last subsection, the generalized PR-sliding sector (8) can be easily designed with choosing the sliding mode $s(x)$ in (13) as the linear function $s(x)$ in (8). In this case, the problem is how to determine the matrices P and R and also the control law.

As the sliding mode is designed so that the reduced order system (21) is stable, there exists a positive definite symmetric matrix $\tilde{P}$ such that the following Lyapunov equation holds for some positive definite symmetric matrix $\tilde{Q}$:

$$\tilde{Q} = \tilde{A}_{11}^T\tilde{P} + \tilde{P}\tilde{A}_{11}. \tag{22}$$

Choose positive definite symmetric matrices $\bar{P}$ and $\bar{R}$ as

$$\bar{P} = \begin{bmatrix} \tilde{P} & 0 \\ 0 & \frac{h}{2} \end{bmatrix} \tag{23}$$

$$\bar{R} = \begin{bmatrix} \tilde{Q} & -\tilde{P}\bar{A}_{12} \\ -\bar{A}_{12}^T\tilde{P} & h \end{bmatrix} \tag{24}$$

where h is a large enough positive constant such that the matrix R is positive definite. Then for the Lyapunov function

$$L(t) = z^T(t)\bar{P}z(t),$$

the following holds:

$$\dot{L}(t) = -z^T(t)\bar{R}z(t) + hs(x)(s(x) + v(t) + d(x,t))$$

Therefore inside the generalized PR-sliding sector, i.e. $s^2(x) \le \delta^2(x,t)$, if the control law is given by

$$v(t) = -k\delta(x,t)\mathrm{sgn}(s(x)), \tag{25}$$

where k is a large enough positive constant satisfying

$$k > 1 + \frac{1}{\eta}$$

then we have

$$\dot{L}(t) \le -z^T(t)\bar{R}z(t) \tag{26}$$

The following theorem concludes the above discussion,

Theorem 1 *The generalized PR-sliding sector defined in (8) can be designed as*

$$\mathcal{S} = \{x|\ |s(x)| \le \delta(x,t), x \in R^n\ \} \tag{27}$$

where the linear function $s(x)$ is the sliding mode in (13) and the positive definite symmetric matrices P and R are respectively determined by

$$P = T_1^{-T}T_2^T\bar{P}T_2T_1^{-1} \tag{28}$$

$$R = T_1^{-T}T_2^T\bar{R}T_2T_1^{-1} \tag{29}$$

$\bar{P}$ are $\bar{R}$ are defined in (23) and (24), respectively, and T_1 and T_2 are transformation matrices defined in the last subsection. Then inside the generalized PR-sliding sector $\mathcal{S}$ with the control law (25), the P-norm decreases and

$$\frac{d}{dt}L(t) = \frac{d}{dt}(x^T(t)Px(t)) \le -x^T(t)Rx(t)$$

3 VS Controller with Generalized Sliding sector

With the generalized PR-sliding sector $\mathcal{S}$ (8) a VS control law should be designed such that the state moves into the sector in a finite time. Together with the control law used inside the sector, the VS control law with the generalized PR-sliding sector is given as follows.

$$u(t) = \begin{cases} -SAx - ks(x) & x(t) \in \mathcal{S} \\ -SAx - k\delta(x,t)\mathrm{sgn}(s(x)) & x(t) \bar{\in} \mathcal{S} \end{cases} \tag{30}$$

Theorem 2 *The VS control law (30) with the generalized PR-sliding sector $\mathcal{S}$ (8) ensures that the state moves into the sector in a finite time and the resulted VS control system is quadratically stable.*

Proof: It is assumed that the initial state is outside the generalized PR-sliding sector, i.e., $|s(x)| > \delta(x,t)$. In this case the VS control law is determined by

$$u(t) = -(SAx(t) + ks(x)). \tag{31}$$

which will be active until the system state is moved into the inner sector. With the above control, it can be shown by some calculating that

$$\begin{aligned} \frac{d}{dt}s^2(x) &= 2s(x)(d(x,t) - ks(x)) < -2s^2(x) \\ \frac{d}{dt}L(t) &= -x^T(t)Rx(t) + hs(x)(s(x) - ks(x) + d(x,t)) \\ &< -x^T(t)Rx(t) \end{aligned}$$

i.e., $s^2(x)$ and the Lyapunov function $L(t)$ in (12) decrease with the VS control law (31). The decreasing of $s^2(x)$ means that the state moves toward the inside of the generalized PR-sliding sector. And also it can be shown that the state will move into the sector with the decreasing of $s^2(x)$ if the positive constant k is chosen large enough although it will cost infinite time for the state to converge to the sliding mode $s(x) = 0$.

After being moved into the sector and until moving out of it, the control input is determined by

$$u(t) = -SAx(t) - k\delta(x,t)\mathrm{sgn}(s(x)) \tag{32}$$

with which it has been shown in the last section that the Lyapunov function decreases as

$$\dot{L}(t) \le -x^T(t)Rx(t).$$

As long as the state with the control law (32) moves out of the generalized PR-sliding sector, the control input (31) will let it back to the inside of the sector in a finite time again while the Lyapunov function $L(t)$ keeps decreasing.

Thus the system state will be moved from the outside into the inside of the generalized PR-sliding sector in a finite time and the Lyapunov function (12) keeps decreasing in the state space with the VS control law (30), i.e.

$$\dot{L}(t) \le -x^T(t)Rx(t), \quad \forall x \in R^n$$

which means that the VS control law (30) quadratically stabilize the plant (5). ∎

4 Experimental Results

4.1 Apparatus system

4.1.1 Derivation of dynamic equation: Our experimental system is shown in Fig.1. It is a rotational type pendulum, which is known as so-called Furuta-pendulum. The link of a pendulum part is equipped on a DD-motor's output axis, and the link can rotate around a pivot freely. The angle of a pendulum-link is detected by an encoder mounted on a rotated base. The DD-motor generates torque according to commands from a computer, and its rotational angle is measured by an encoder inside the motor. The range of torque of the motor is $\pm$ 9.8$[N \cdot m]$, and resolution of each sensors are 409600 and 144000 $[pulse/rev]$, respectively.

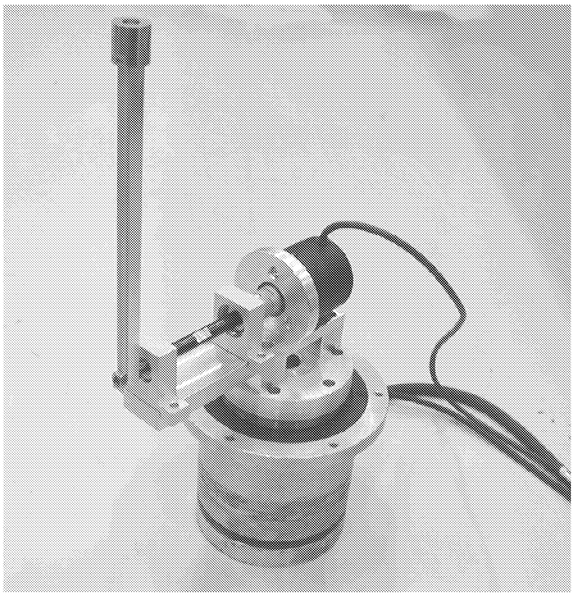

Figure 1: furuta pendulum

To derive geometric relation for modeling, a coordinate system is attached by DH-convention as shown in Fig. 2. The DH-parameters of each link are described in Tab.1. Mechanical parameters and their meanings are listed in Tab.2

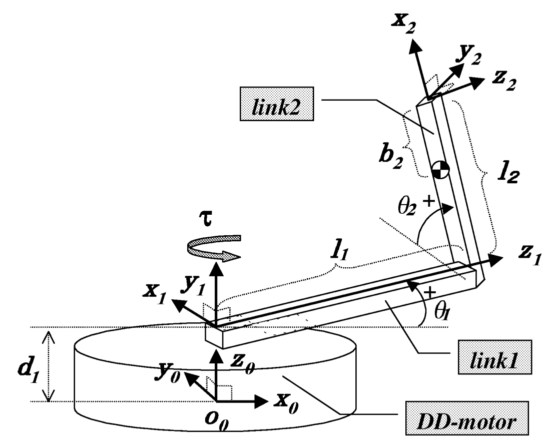

Figure 2: Modeling and its parameters

Table 1: DH parameters

link	a_i	d_i	α_i	θ_i
1	0	d_1	$+\frac{\pi}{2}$	θ_1
2	l_2	l_1	0	θ_2

Table 2: Mechanical parameters of a pendulum

	value		meanings
l_1	0.245	m	length of a link-1
J_1	1.69×10^{-2}	$kg{\cdot}m^2$	inertia of link-1 around its pivot
c_1	3.60×10^{-2}	Nms/rad	viscous coefficient of link-1
l_2	0.30	m	length of a link-2
b_2	0.183	m	distance from tip to center of gravity of link-2
m_2	0.084	kg	mass of a link-2
J_2	7.16×10^{-3}	$kg{\cdot}m^2$	inertia of a link-2 around its center of gravity
c_2	8.29×10^{-4}	Nms/rad	viscous coefficient of link-2

Transformation matrices ${}^iA_j$ from j-coordinate system to i-coordinate one are derived as follows.

$$
\begin{aligned}
{}^0A_1 &= \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^1A_2 &= \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & l_2\sin\theta_2 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Now it is assumed that p_2 is center of gravity of link-2 on frame-2, say $p_2 = [-b_2, 0, 0, 0]^T$, the another expression on global coordinate system is

$$ {}^0p_2 = {}^0A_1 \cdot {}^1A_2 \cdot {}^2p_2. $$

Therefore the kinematics' energy $\mathcal{K}$ is calculated as

$$
\begin{aligned}
\mathcal{K} &= \frac{J_1\dot{\theta}_1^2}{2} + \frac{J_2\dot{\theta}_2^2}{2} + \frac{m_2}{2}\left\{ (\frac{d}{dt}\,{}^0p_{2x})^2 \right. \\
&\left. +(\frac{d}{dt}\,{}^0p_{2y})^2 + (\frac{d}{dt}\,{}^0p_{2z})^2 \right\}, \qquad (33)
\end{aligned}
$$

here ${}^0p_{2x}$ means the x-component of vector p_2, and the others are in the same manners. The potential energy $\mathcal{U}$ is described as

$$ \mathcal{U} = m_2 g\,{}^0p_{2z}. \qquad (34) $$

Considering viscous of the joints, the disappeared term is

$$ \mathcal{R} = \frac{1}{2}c_1\dot{\theta}_1^2 + \frac{1}{2}c_2\dot{\theta}_2^2. \qquad (35) $$

Substituting Eq.(33), Eq.(34) and Eq.(35) into the Lagrange equation:

$$ \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\theta}_i}\right) - \frac{\partial\mathcal{L}}{\partial\theta_i} + \frac{\partial\mathcal{R}}{\partial\dot{\theta}_i} = Q, \qquad (36) $$

then the dynamic equations are obtained. Here $\mathcal{L}$ is the Lagrangian: $\mathcal{L} = \mathcal{K} - \mathcal{U}$, and Q is the generalized force term. For convenience, changing θ_2 as $\theta_2 \to \theta_2 + \frac{\pi}{2}$,and setting $l_b := l_2 - b_2$, then the result equations are

$$
\begin{aligned}
p_{11}\ddot{\theta}_1 + p_{12}\ddot{\theta}_2 + q_1 &= \tau \\
p_{12}\ddot{\theta}_1 + p_{22}\ddot{\theta}_2 + q_2 &= 0,
\end{aligned} \qquad (37)
$$

$$
\begin{aligned}
p_{11} &= J_1 + m_2 l^2 + \frac{1}{2}m_2 l_b^2(1-\cos 2\theta_2) \\
p_{12} &= -m_2 l_1 l_b \cos\theta_2 \\
p_{22} &= J_2 + m_2 l_b^2 \\
q_1 &= c_1\dot{\theta}_1 + m_2 l_b(l_b \sin 2\theta_2\dot{\theta}_1\dot{\theta}_2 + l_1\sin\theta_2\dot{\theta}_2^2) \\
q_2 &= c_2\dot{\theta}_2 - m_2 l_b\left(\frac{1}{2}l_b\sin 2\theta_2\dot{\theta}_1^2 + g\sin\theta_2\right).
\end{aligned}
$$

To design an observer and a controller, Eq.(37) is linearized around unstable equilibrium $\theta_i \simeq 0, \dot{\theta}_i \simeq 0$ $(i = 1, 2)$, then dynamic matrix equations is obtained as follows.

$$ E\dot{x} = Fx + Gu \qquad (38) $$

$$
\begin{aligned}
x &= \begin{bmatrix} \theta_1 & \theta_2 & \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}^T \\
E &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & J_1 + m_2 l_1^2 & -m_2 l_1 l_b \\ 0 & 0 & -m_2 l_1 l_b & J_2 + m_2 l_b^2 \end{bmatrix} \\
F &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -c_1 & 0 \\ 0 & m_2 l_b g & 0 & -c_2 \end{bmatrix} \\
G &= \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T
\end{aligned}
$$

As E is always full rank, the state space model is gained after substituting each numerical parameters in Tab.2 to Eq.(38) and inverse operation.

$$ \dot{x} = Ax + Bu \qquad (39) $$

where matrices A and B are determined by

$$
\begin{aligned}
A &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 7.65\times10^{-3} & -2.21\times10^{-1} & -9.13\times10^{-5} \\ 0 & 6.03 & -3.33\times10^{-2} & -7.20\times10^{-2} \end{bmatrix} \\
B &= \begin{bmatrix} 0 & 0 & 2.86 & 0.43 \end{bmatrix}^T
\end{aligned}
$$

The eigenvalues of the open loop system are

$$ \{-2.49, 2.42, -0.22, 0\}. $$

4.1.2 Experiment with LQR Control Algorithm: We choose the parameters of the LQR controller as

$$Q = diag\{5000, 400, 20, 10\} \quad (40)$$
$$R = [6.0] \quad (41)$$

to minimize the following quadratic performance index:

$$J = \int_0^\infty (x^T(t)Qx(t) + Ru^2(t))dt$$

The eigenvalues of the resulted closed-loop system are

$$\{-2.78497508, -3.21387208, -15.4962061, -353.984216\} \quad (42)$$

4.1.3 Experiment with Proposed VS Control Algorithm: Choose the first three eigenvalues in (42) as the ones of the reduced order system (21), other parameters of the VS controller are given as follows.

$$S = [-0.081550, 1.627677, -0.063939, 0.505242]$$
$$\delta(x,t) = \sqrt{x^T(t)diag\{0.1, 0.1, 0.01, 0.01\}x(t)}$$
$$k = 30.0, \quad \eta = 1.0$$

The experiment results with the LQR and the proposed VS control algorithms are shown in Fig. 4. and Fig. 3, respectively.

With LQR control method, chattering happens because an observer is used and the friction of the pendulum apparatus is rather large. With the proposed VS control method, the control performance is satisfactory and no chattering happens because the VS control law proposed in the paper is smooth and its gain becomes smaller when the state is inside the generalized *PR*-sliding sector.

5 conclusion

In this paper, we proposed a new VS control algorithm with a generalized *PR*-sliding sector. The sector is designed based on a sliding mode. The resulted VS control system is quadratically stable even if there exist parameter uncertainties and external disturbances. We took a rotational inverted pendulum apparatus to evaluate the proposed VS control algorithm. The experiment results show that the VS controller with a generalized *PR*-sliding sector is chattering-free and is with good control performance.

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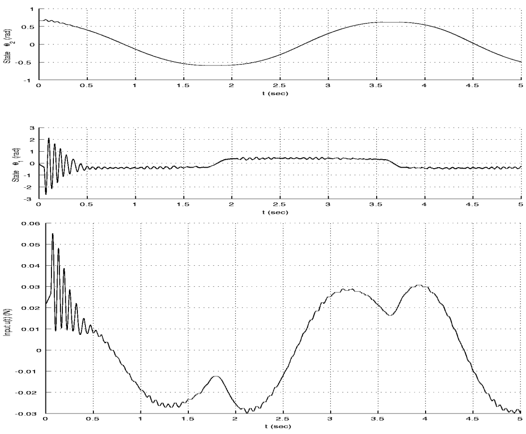

Figure 3: Experiment Result with Proposed VS Control (without Chattering)

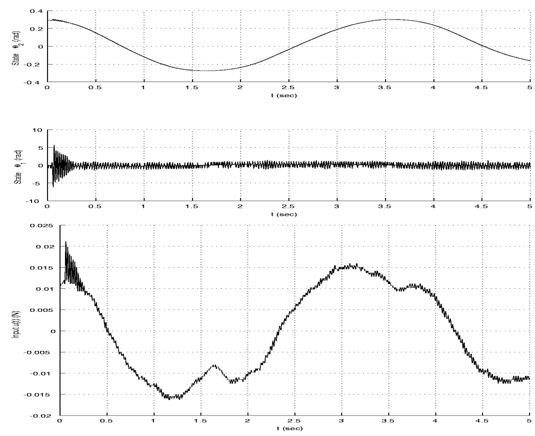

Figure 4: Experiment Result with LQR Control (with Chattering)