Game Theoretic Campaign Modeling and Analysis[1]

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Abstract

In this paper we address the modeling and analysis issues associated with a generic theater level campaign where two adversaries pit their military resources against each other over a long series of engagements. Specifically, we use the scenario of an air raid campaign using SEAD aircraft and bombers against enemy troops and air defense units. The problem is decomposed into a temporal and a spatial resource allocation problem. The temporal resource allocation problem is formulated as a multiple resource interaction problem in a game-theoretical framework and solved for linear attrition functions. The spatial resource allocation problem is posed as a risk minimization problem in which the two adversaries decide on the corridor of ingress and movement of the ground troops and air defense units. These two solutions are integrated using an aggregation/deaggregation approach to evaluate resource strengths and distribute losses.

1 Introduction

Campaign modeling and analysis in the context of military command and control poses a variety of challenges. Several important issues – such as modeling tactical, operational, and strategic levels of conflict, integration of lower level conflict models to create higher level mission models, interactions between force allocations, strategies, and system capabilities, etc., need to be addressed. The overall military campaign problem is too complex to be tackled by a single analytical or computational model, approach, or tool. The real challenge lies in integrating these disparate entities into an integrated whole. The specific lines of investigation pursued in this paper include issues related to the integration of optimal campaign level temporal and spatial resource allocations in an air campaign.

In large scale military operations, air campaigns are frequently used as the preferred mode of invasion of the enemy territory [1]. They are also effective measures of defense against enemy invasion into friendly territory. One of the earliest papers that addresses the air campaign problem is the tactical air war game formulated by Berkovitz and Dresher [2], where the two adversaries are evenly matched in terms of their resources, imposing a certain symmetry in their resource types, capabilities, and decision variables. Solution is sought in terms of an optimal partitioning of a single resource on each side among several tasks. Specifically, both players have several aircraft in their arsenal, that have to be assigned different roles of counter air, air defense, and ground support. It is shown that when the players resources are distributed between two tasks, the game has a pure strategy solution, but for more than two tasks the optimal solution is obtained in terms of mixed strategies. The spatial dimension of the problem is completely suppressed in this model.

Another classical paper that addressed a similar problem is by Blackwell [3] on multi-component attrition games. The game is defined in normal form where each player selects an action from a finite set of actions. Each action pair leads to an attrition to the resources of the players. The game is played several times till at least one of the resources is reduced to zero. Thus, the game is essentially defined through the attrition matrices associated with each resource. The strategy of each player is defined in terms of a probability distribution on the action set (mixed strategy). This paper focuses only on the attrition suffered by each player and does not address the decision-making problem in the spatial dimension. Moreover, the solution obtained does not prescribe strategies for the players, but obtains certain asymptotic results on the probability of winning by a player given an initial relative strength of resources. While mathematically elegant, these results are difficult to interpret or use in a practical air campaign.

Our model differs from both these papers in that we consider different types of resources as having distinct roles. We model the air campaign planning problem as an asymmetric game where one of the adversaries carries out an air campaign against the other adversary who mounts a ground invasion. Further, we retain

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the game-theoretical premise underlying an air campaign while incorporating both its temporal and spatial dimensions. Our objective is to develop a realistic but simple model that addresses important issues arising in an air campaign. We do this by decoupling the spatial dimension from the temporal dimension of the problem. The problems addressed in the spatial dimension include route planning and movement of resources, while in the temporal dimension we decide how much of the available resource strengths each adversary should use during a particular mission. Finally, we put these together to obtain an integrated decision making capability for the two adversaries.

2 The Air Campaign Problem

The problem addressed in this paper is that of an air campaign in which the friendly (BLUE) forces attempt to thwart the invasion of its territory by the enemy (RED) forces. BLUE has two types of resources at its disposal – SEAD (suppression of enemy air defense) aircraft and bombers (BMB). RED has air defense (AD) units and ground troops (GT). The SEAD aircraft detect and destroy enemy air defenses and create a safe corridor for the bombers to penetrate into territory defended by the RED air defense units and destroy the invading ground troops. The campaign is carried out in the form of several SEAD/BMB missions, each lasting for perhaps half a day or even a complete day, while the RED forces plan the movement of their AD units in order to afford maximum protection to their invading ground troops. This scenario has all the elements of a two player game in which players take decisions regarding the utilization of available resources in each mission, the route to be selected for attack, the movement of various resources, and on other relevant aspects. Some of the complications that arise in this problem are due to several apparently unrelated objectives that each adversary has at different stages of the game. For instance, the BLUE forces have to decide the optimum route of ingression into the enemy territory and the number of SEAD aircraft and bombers that should be used in each mission. The RED forces, on the other hand, have to decide on how to move its AD units and GT units, and also how many of the AD units to be used for defense purposes. This imposes an asymmetry among the adversary's decision variables.

The objectives of the various resources are as follows: (i) GT attempts to advance through the gameboard till it reaches a well-defined BLUE border with a certain minimum strength. (ii) ADs attempt to destroy SEADs and BMBs to protect themselves and the GT. (iii) SEADs precede BMBs to sanitize a corridor of AD threats. (iv) BMBs fly through the sanitized corridor to target GTs.

The air campaign is modeled as a multi-stage game where each stage, comprising of a mission, is of about half a day duration. A single mission in the air campaign by the BLUE forces consists of a single ingression by SEADs followed by the BMBs. ADs have the choice of remaining hidden (passive) or they may have their radars in the track mode (active). When the ADs are active they may be assumed to remain stationary. On the other hand, when they are passive they are undetected and can be moved to adjacent sectors. At the beginning of each stage both players need to take the following decisions:

BLUE decisions: (i) Determination of the ingress corridor. (ii) Selection of the number of BMBs and SEADs to be used in a given mission.

RED decisions: (i) The number of active AD units; (ii) Identification of active and passive ADs; (iii) Movement of AD units; (iv) Movement of GT units.

The actual locations of the GTs and ADs determine the losses that the SEAD missions and BMB missions suffer and hence determine their effectiveness. The spatial dimension of this problem concerns the movement and location of RED GTs and ADs and the selection of the string of sectors that defines an ingress corridor for the BLUE SEADs, and gives rise to the *spatial resource allocation problem.* The temporal dimension of the game is concerned with the decision of how many of the SEAD and BMB of the BLUE forces and how many of the ADs of the RED forces should participate in a mission. This gives rise to the *temporal resource allocation problem.*

3 Spatial Resource Allocation

The spatial resource allocation problem basically addresses the problem of creating an optimal corridor of ingress for the BLUE SEADs and BMBs. It also addresses the problem of the movement of the GTs and the ADs of the RED forces. The air campaign is assumed to take place on a battlefield modeled as a gameboard consisting of several sectors assumed to be hexagonal in shape, each having a maximum of six neighboring sectors.

3.1 Notations

We will use the following notations: Let the gameboard be denoted by $\mathcal{G} = \{g_1, \ldots, g_m\}$ where each g_i identifies a sector in the gameboard. Let $N(g_i)$ denote the set of neighboring sectors of (or sectors adjacent to) a sector g_i. Let a corridor in $\mathcal{G}$ at the k-th stage be denoted by $u_k^c = \{c_1, \ldots, c_r\}$ where each $c_i \in \mathcal{G}$ and $c_{i+1} \in N(c_i)$. Also, sector c_r contains a GT unit and sector c_1 contains the SEAD and BMB bases. In its most general form r could either be a fixed integer (which would imply that each SEAD/BMB sortie must be of a certain fixed length in terms of number of sectors) or it could be bounded above by $r \leq r_{\max}$ (which would imply a constraint on the SEAD/BMB capability in terms of fuel or endurance) or it could be an arbitrarily large integer (implying no such constraint on the SEAD/BMB capability). Note that u_k^c is the control of the BLUE forces in the k-th stage of the game.

Let $\mathcal{A} = \{a_1, \ldots, a_s\}$ denote the collection of AD units. At a given stage k let the placement of AD units on the gameboard be given by $P_k = \{d_{1k}, \ldots, d_{sk}\}$ with $d_{jk} \in \mathcal{G}$ implying that the AD unit a_j is placed in the sector $d_{jk} \in \mathcal{G}$. Let $g_k^t \in \mathcal{G}$ denote the location of the GT units on the gameboard.

3.2 Creation of ingress corridor

A location of ADs imposes a risk profile on the gameboard for BLUE aircraft. It quantifies the risk or danger to a BLUE aircraft in passing through a sector. This risk is a function of the locations and capabilities of ADs used by RED forces. It also depends on the kill probability of the AD unit, the terrain, and also the type of aircraft. An optimal corridor would be one with the minimum risk associated with it. In the following discussion, we omit the stage index 'k'.

Let the risk on a sector g_i on the gameboard due to the location of the AD unit a_j in the sector d_j (given by placement P) be denoted by $r_{ij}|_P$. Then the risk profile on the gameboard due to a_j in d_j is given by,

$$r_j|_P = \{r_{1j}, \ldots, r_{mj}\}|_P \tag{1}$$

Then the risk profile on the gameboard due to the placement P of all the AD units is given by,

$$\mathcal{R}(P) = \{\mathcal{R}(P, g_1), \ldots, \mathcal{R}(P, g_m)\} \tag{2}$$

where, $\mathcal{R}(P, g_i)$ is the risk at sector g_i, due to the AD unit placement P, and is defined as,

$$\mathcal{R}(P, g_i) = \sum_{a_j \in \mathcal{A}} r_{ij}|_P \tag{3}$$

The risk on a corridor u^c due to an AD unit placement P is denoted by $\rho(P, u^c)$ and is defined as,

$$\rho(P, u^c) = \sum_{c_i \in u^c} \mathcal{R}(P, c_i) \tag{4}$$

For a given placement P of AD units, the corridor creation problem may be formulated as,

$$\min_{u^c} \rho(P, u^c) \tag{5}$$

Dijkstra's algorithm [4] is used to compute the minimum risk corridor.

3.3 Movement of GTs

The GT movement by the RED forces have the objective of getting to the BLUE border with minimum losses, which means that the strength of BMB units undergoes the maximum possible attrition as it attacks each sector through which the GT units pass. Based on this requirement the problem is defined as follows: (i) For a given AD distribution pattern determine the risk profile on the gameboard. (ii) For each sector in the gameboard find the minimum cumulative risk that a BMB undergoes as it flies to this sector. (Use the corridor creation algorithm to obtain these values) (iii) Suppose the BMB strength at the current stage is B_i. Then find the BMB strength that remains after it flies through the minimum risk corridor to this sector and is confronted with the corresponding cumulative risk associated with this corridor. (iv) Assign this remaining strength to the sector. Repeat for all sectors on the game board. (v) Find the minimum "remaining strength of BMBs" path from the current location of the GT to the BLUE border. Dijkstra's algorithm is used for this task. (vi) The first step in this path is the GT movement for the RED forces.

3.4 Movement of AD units

The decision about how an AD unit should move during a stage in the game is solved under the following assumptions: (i) The ingress corridors of the BLUE aircraft are known. (ii) The temporal resource allocation problem is solved for the current stage. (iii) The GT movement for the current stage is known.

The objective of the AD unit movement is to maximize the risk to the BLUE aircraft flying through the corridor. The AD movement can be determined through the following procedure: (i) Obtain the new corridor for the new GT position with all ADs assumed to be active. Note that we have to take into account the damage suffered by the ADs at the current stage. (ii) Now compute the risk imposed on this corridor (that is, the air defense strength) by the present AD locations when all of them are assumed to be active. Each movement of an AD to an adjacent sector changes this strength. Select the set of movements that collectively maximizes this strength. Hence, the objective is to move ADs in a way that maximizes the available AD strength on a corridor.

4 Temporal Resource Allocation

A stage in a game is defined as a single mission in which SEADs and BMBs participate. At any given stage k of the game the BLUE forces have an available SEAD strength of S_k^s and a bomber strength of S_k^b. Similarly, the RED forces have an available air defense strength of S_k^a and GT strength of S_k^g. The quantities S_k^s, S_k^b, S_k^a, and S_k^g are known to the players at the beginning of a stage. These strengths may not be measured in terms of numbers (of SEAD, BMB, or AD), but rather they are derived from an aggregation process that models strength as capabilities that each resource group has in terms of its mission objectives. This aspect is closely related to the spatial dimensions of the problem which determines the corridor of operation and which, in turn, defines the effectiveness of specific resources against adversary's resources through loss functions.

The objective of the BLUE forces is to minimize the effect of the surviving target strength over a specified number of stages while the objective of the RED forces is to maximize this effect.

4.1 Problem Formulation

At any given stage k of the game, the BLUE forces partition S_k^s and S_k^b as,

$$S_k^s = u_k^s + r_k^s, \quad S_k^b = u_k^b + r_k^b \tag{6}$$

where, $u_k^s \in [0, S_k^s]$ and $u_k^b \in [0, S_k^b]$ are used by the BLUE forces in the campaign at the k-th stage and $r_k^s = S_k^s - u_k^s$ and $r_k^b = S_k^b - u_k^b$ are kept in reserve or "rest" for later use. Thus, the decision that the BLUE forces need to take at the beginning of each stage is how much of the SEAD and BMB force strengths should be used for the campaign at that stage and how much of these strengths are to be kept in reserve. Similarly, at each stage, the RED forces have the option of keeping some of its air defenses "hidden" (or passive) while the rest can be switched on (or made active) to track and engage SEADs and BMBs. Thus, the RED forces partition its air defense strength as,

$$S_k^a = v_k^a + r_k^a \tag{7}$$

where, $v_k^a \in [0, S_k^a]$ is the AD strength that is used to engage SEADs and BMBs and $r_k^a = S_k^a - v_k^a$ is the AD strength that is kept in reserve for later use. Thus, the decision variables of BLUE forces at the beginning of stage k in the temporal resource allocation game is (u_k^s, u_k^b) and for the RED forces it is v_k^a.

First, the SEADs fly along a designated corridor and engage ADs located on it. The ADs and the SEADs inflict damage on each other.

$$s_k^1 = \text{Surviving SEADs} = \max\{0, u_k^s - L_a^s(v_k^a, u_k^s)\} \tag{8}$$

$$a_k^1 = \text{Surviving ADs} = \max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\} \tag{9}$$

where, $L_a^s(.,.)$ defines the damage that the SEAD strength suffers when it is confronted with an AD force strength, and $L_s^a(.,.)$ defines the damage that the AD force strength suffers in its interaction with a SEAD force strength.

Next, the BMBs fly through the corridor and are engaged by ADs defending the corridor.

$$\begin{aligned} b_k^2 &= \text{Surviving BMBs} = \max\{0, u_k^b - L_a^b(a_k^1, u_k^b)\} \\ &= \max\{0, u_k^b - L_a^b(\max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\}, u_k^b)\} \end{aligned} \tag{10}$$

$$\begin{aligned} a_k^2 &= \text{Surviving ADs} = \max\{0, a_k^1 - L_b^a(a_k^1, u_k^b)\} \\ &= \max\{0, \max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\} \\ &\quad - L_b^a(\max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\}, u_k^b)\} \end{aligned} \tag{11}$$

where, $L_a^b(.,.)$ defines the damage that the ADs inflict on the BMBs and $L_b^a(.,.)$ defines the damage that the BMBs inflict on the ADs.

Finally, BMBs engage GTs at the end of the corridor.

$$\begin{aligned} g_k^3 &= \text{Surviving GTs} = \max\{0, S_k^g - L_b^g(b_k^2, S_k^g)\} \\ &= \max\{0, S_k^g - L_b^g(\max\{0, u_k^b - L_a^b(\max\{0, v_k^a \\ &\quad -L_s^a(v_k^a, u_k^s)\}, u_k^b)\}, S_k^g)\} \end{aligned} \tag{12}$$

where, $L_b^g(.,.)$ defines damage that BMBs inflict on GTs.

At the next stage $k+1$ the two players have the following force strengths available:

$$\begin{aligned} S_{k+1}^s &= r_k^s + s_k^1, \quad S_{k+1}^b = r_k^b + b_k^2 \\ S_{k+1}^a &= r_k^a + a_k^2, \quad S_{k+1}^g = g_k^3 \end{aligned} \tag{13}$$

The state equations are as follows,

$$S_{k+1}^s = \max\{0, u_k^s - L_a^s(v_k^a, u_k^s)\} + (S_k^s - u_k^s) \tag{14}$$

$$\begin{aligned} S_{k+1}^b &= \max\{0, u_k^b - L_a^b(\max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\}, u_k^b)\} \\ &\quad + (S_k^b - u_k^b) \end{aligned} \tag{15}$$

$$\begin{aligned} S_{k+1}^a &= \max\{0, \max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\} - L_b^a(\max\{0, v_k^a \\ &\quad - L_s^a(v_k^a, u_k^s)\}, u_k^b)\} + (S_k^a - v_k^a) \end{aligned} \tag{16}$$

$$\begin{aligned} S_{k+1}^g &= \max\{0, S_k^g - L_b^g(\max\{0, u_k^b \\ &\quad - L_a^b(\max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\}, u_k^b)\}, S_k^g)\} \end{aligned} \tag{17}$$

with the controls of the two players as $u_k^s \in [0, S_k^s]$, $u_k^b \in [0, S_k^b]$, $v_k^a \in [0, S_k^a]$.

We define the payoff as the sum of the surviving GT strengths at each stage, maximized by RED and minimized by BLUE,

$$J = \sum_{k=1}^{n} S_{k+1}^g \tag{18}$$

A fundamental minimax theorem by Fan [5] can be used to show that in the single stage game each player has optimal pure strategies if the loss functions satisfy certain monotonicity properties [6]. Conditions under which the multi-stage game also has pure strategy solutions have been obtained in [6].

4.2 Linear Loss Functions

In this section we consider linear loss functions that are 'linear' in the sense that the loss to a player's given resource is proportional to the adversary's resource strength with which this resource interacts, but within the bounds of resource availability. Let,

$$\begin{aligned} l_a^s(v_k^a) &= \alpha v_k^a, \quad l_s^a(u_k^s) = \beta u_k^s \\ l_a^b(a_k^1) &= \gamma a_k^1, \quad l_b^a(u_k^b) = \eta u_k^b, \quad l_b^g(b_k^2) = \theta b_k^2 \end{aligned} \tag{19}$$

where, α, β, γ, η, and θ are non-negative scalars. The first equation means that α SEAD strength is destroyed by one unit of AD strength. The other loss parameters have a similar interpretation. For this linear model the optimal strategies for the players at each stage can be obtained as [6]:

If $(S_k^a) \in \mathcal{M}$ then $v_k^{a*} = S_k^a$; else (if $(S_k^a) \notin \mathcal{M}$) then $v_k^{a*} \in \{[0, S_k^a] \backslash \mathcal{M}\}$.

If $(S_k^s, S_k^b) \in \mathcal{N}$ then $u_k^{b*} = S_k^b$. Further, if $S_k^a/\beta > S_k^s$ then $u_k^{s*} = S_k^s$ else $u_k^{s*} \in [S_k^a/\beta, S_k^s]$.

If $(S_k^s, S_k^b) \notin \mathcal{N}$ then $(u_j^{s*}, u_k^{b*}) \in \{\{[0, S_k^s] \times [0, S_k^b]\} \backslash \mathcal{N}\}$.

The sets $\mathcal{N}$ and $\mathcal{M}$ are defined as,

$$\mathcal{N}_1 = \{(x^{us}, x^{ub}) : x^{us} \geq S_j^a/\beta;\ x^{ub} < S_k^g/\theta\} \quad (20)$$

$$\mathcal{N}_2 = \{(x^{us}, x^{ub}) : x^{us} < S_k^a/\beta; \quad x^{ub} < \gamma (S_k^a - \beta x^{us}) + S_j^g/\theta\} \quad (21)$$

$$\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2 \quad (22)$$

$$\mathcal{M} = \{y^{va} : y^{va} < S_k^b/\gamma + \beta S_j^s\} \quad (23)$$

where, x^{us}, x^{ub}, and y^{va} are variables that correspond to the SEAD, BMB, and AD resource strengths, respectively.

Although, depending on the available resource levels, the game admits multiple saddle points in pure strategies, it is logical for the players to avoid using excessive resources. This implies that the RED forces will use,

$$v_k^{a*} = \min\{S_k^b/\gamma + \beta S_k^s, S_k^a\} \quad (24)$$

and the BLUE forces will select a Pareto minimum point from its solution set. A clear interpretation of these solutions are given in [6]. We omit details.

5 Integrated Spatio-Temporal Solution

The solutions to the spatial and temporal resource allocation problems are integrated through a aggregation/de-aggregation process.

5.1 Aggregation and de-aggregation

In the following we suppress 'k', which denotes the k-th stage, from the variables for simplicity. Define a vector E that stores the effectiveness of an AD unit against an aircraft, that is, an AD unit inflicts E_0 units of damage on an aircraft located in the same sector as the AD unit, E_1 units of damage on an aircraft located one sector away from the AD location, and so on. For computational simplicity we assume that only finite number of E_j's are non-zero. Define a vector H that stores the vulnerability of an AD to a SEAD, that is, an AD sustains H_j units of damage from a SEAD located j sectors away from the AD location. Define parameters $\{r_j\}_{j=1}^s$, where $r_j \in (0, 1)$ denotes the strength reduction of j-th AD unit due to the previous interactions.

With this information, one can calculate the variables required for making the temporal resource allocation decisions. Calculate the vulnerability of j-th AD unit to the damage inflicted by the aircraft passing along the corridor as $h_j = \sum_i H_i n^i(d_j)$. Here, the integer $n^i(d_j)$ is the number of sectors that belong to the corridor and are exactly i steps away from sector d_j). Next, we obtain the total available AD strength by assuming that the risk inflicted by ADs on sectors nearby can be added linearly. So,

$$a_j = r_j \sum_i E_i n^i(d_j), \quad (25)$$

is the strength of the j-th AD unit. Then the total available AD strength can be calculated by summing all the individual AD strengths as,

$$A_{u^c}(P) = S^a = \sum_{j=1}^{N} a_j. \quad (26)$$

Note that, in contrast with the BLUE resource strengths, where the allocations are continuous, the increments in the AD strength achieved by activating additional AD units are discrete. We pick a subset of $\mathcal{A}$ such that the total strength of the ADs in this subset is close to the required v^a. This subset will constitute the *active* ADs.

The total damage to ADs inflicted by the SEADs during the interaction is $\Delta S^a = \min(v^a, L_s^a(v^a, u^b))$. The attrition ΔS^a is then distributed among the active AD units in proportion to their vulnerability as follows: The damage sustained by the j-th AD is,

$$\Delta a_j = \frac{h_j}{\sum_j h_j}(\Delta S^a) \quad (27)$$

5.2 The algorithm

The complete algorithm is as follows:

(i) Assume a distribution of ADs on the gameboard. (ii) Assume all the ADs are active and create a risk profile as a function of the kill probabilities, terrain, and other relevant factors. (iii) Obtain SEAD/BMB corridors from the aircraft base to the known GT unit location. (iv) Compute the AD strength using the aggregation process given above. (v) Solve the temporal resource allocation problem. The solution to the temporal resource allocation problem is used to obtain the number of SEADs and BMBs to be employed for the current stage of the campaign. It is also used to specify the number and location of ADs to be made active. (vi) Distribute damages (or losses) to the ADs using the deaggregation process given above. (vii) Solve the GT and AD movement problem and repeat the process.

6 Simulation Study

Suppose that the air campaign takes place on a 7×7 hexagonal gameboard, and the Blue territory is the

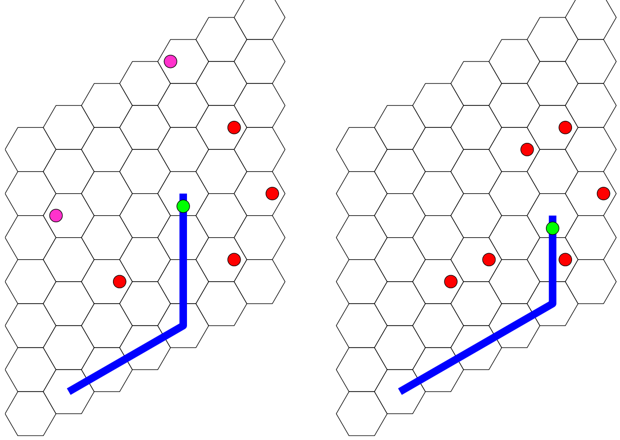

Figure 1: *Stage 1:* $S^g = 20$, $S^a = 33$, $S^s = 20$, $S^b = 10$. *Stage 2:* $S^g = 12.7$, $S^a = 38.3$, $S^s = 13.4$, $S^b = 7.3$. Even though some AD units have reduced in strength, their total strength in Stage 2 is greater than in Stage 1. This is due to the fact that, in the spatial setting, the AD strength strongly depends on the location of the AD unit.

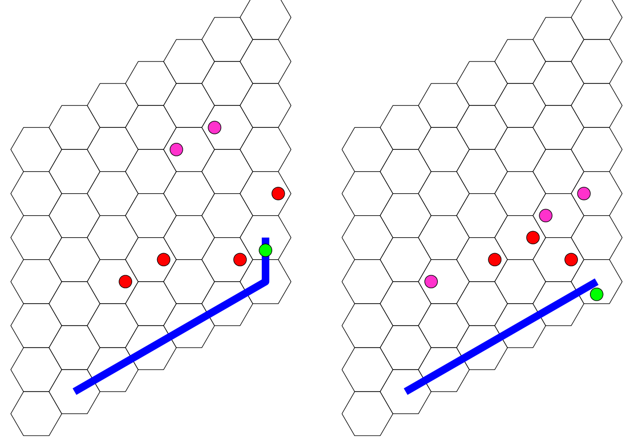

Figure 2: *Stage 3:* $S^g = 8.8$, $S^a = 26$, $S^s = 5.7$, $S^b = 3.8$. Since all the ADs were active during the previous stage, none of them moved. *Stage 4:* $S^g = 7.4$, $S^a = 30$, $S^s = 0.5$, $S^b = 1.5$. Since $v^a = 14.6 < S^a$, some of the ADs, imposing risk on the corridor, are switched off.

lowest row of the game board. We assume that only the passive ADs are allowed to move $\lambda = 2$ steps per stage whereas the active ADs do not move. At the beginning of the game the players have the following resources: (i) BLUE has 20 SEADs and 10 bombers. (ii) RED has 20 GT units and 6 AD units.

The effectiveness of ADs against blue aircraft is defined as $E_0 = 0.3$, $E_1 = 0.8$, $E_2 = 0.5$ (note that an AD is not very effective in the sector where it is located, because the missiles are difficult to control during the initial boost stage). The vulnerability of the ADs to the SEAD threat is defined as $H_0 = 0.7$, $H_1 = 0.5$, $H_2 = 0.3$, $H_3 = 0.1$. The coefficients used for the game-theoretical temporal resource allocation are $\alpha = 0.2$, $\beta = 0.3$, $\gamma = 0.1$, $\eta = 0$, $\theta = 1$.

Figures 1 and 2 show the location of the resources at the beginning of each stage of the game. Active ADs are darker in shade than the passive ADs. The thick line shows the corridor of ingress. The position of the SEAD/BMB base is at the starting sector of the corridor and the GT location at each stage is at the end of the corridor.

7 Concluding Remarks

In this paper we presented a game theoretical formulation of an air campaign incorporating both the spatial and temporal dimensions in the model. The solutions to these two resource allocation problems are obtained separately and integrated through an aggregation/deaggregation technique. The air campaign problem addressed has certain generic elements that are applicable to other theater-level campaign scenarios too.

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