

Modeling and Control of Military Operations Against Adversarial Control

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Abstract

In this paper we present a nonlinear state space mathematical model for a class of dynamical systems that can serve as the basis for a simulation test bed for the investigation of enterprise control. Dynamic complex enterprises generally include multiple control agents of a decision team. In addition, the enterprise is generally imbedded in a larger environment that has competing and even hostile decision teams that affect the enterprise. In such situations it is appropriate to model an *extended enterprise* that includes the competing decision teams. For example an enterprise might be a military command and control hierarchy with several levels of command. If a command and control enterprise is deployed in a military operation, the enterprise states may be affected by non-friendly commands. In order to develop acceptable and even optimal control strategies, it is important to consider the effect of the adversarial controls even at the control design stage. Before these control strategies can be designed or investigated, a model for the extended enterprise "plant" is needed. This extended plant should have inputs from the competing decision team, in addition to the decision team inputs to the enterprise. In our model the command hierarchy will be designated as the Blue Forces. The enterprise is imbedded in a larger system that includes a hostile command designated as the Red Forces. This extended enterprise will be designated as "Military Operations". In this paper, a discrete-time nonlinear state space model of a Military Operation is formulated and an example illustrating the implementation of the Nash strategies from non-zero sum game theory is presented.

1. Introduction

Attrition models for modern warfare have received considerable attention in recent years [1-4]. In this paper, we present a dynamic state-space attrition-type model of a complex military operation that involves two opposing forces. We will label the attacking forces a *Blue* and the defending forces as *Red*. The blue forces consist of Blue Weasels (BWs) and Blue Bombers (BBs). The weasels are

essentially SEAD¹ units whose purpose is to attack and suppress the red air defenses, and the purpose of the bombers is to attack the red units. The red forces consist of Red Troops (RTs) such as tanks and mobile vehicles and Red Defense units (RDs) such as SAM's². In addition, we will assume that there are Fixed Targets (FTs) such as bridges, refineries, air bases, etc. that the blue forces would attack and the red forces would defend.

Let N^{BW} , N^{BB} , N^{RT} , N^{RD} , and N^{FT} denote the number of units of each type involved in the operation. Although the model can be derived in the continuous time-space domain, we will initially assume that time is sampled into stages $k = 0, 1, 2, \dots, K$ and that the scenario is taking place on a two-dimensional terrain sampled in the x-y directions into square blocks. Continuous time and three-dimensional continuous space will be considered as an extension of this work at a later time.

2. The Unit's State Vector

Consider the i^{th} unit of type X where

$$X = \{BW, BB, RT, RD\}. \quad \text{Let } \xi_i^X(k) = \begin{bmatrix} x_i^X(k) \\ y_i^X(k) \end{bmatrix}$$

denote its location vector at time k , where x is the horizontal coordinate and y is the vertical coordinate. Let $p_i^X(k)$ denote the number of platforms and let $w_i^X(k)$ denote the average number of weapons per platform at time k in that unit. Thus, for each moving unit in the theatre of operations, we will define a 4-dim state variable

$$z_i^X(k) = \begin{bmatrix} \xi_i^X(k) \\ p_i^X(k) \\ w_i^X(k) \end{bmatrix}, \quad X = \{BW, BB, RT, RD\},$$

¹ SEAD = Suppressing Enemy Air Defenses

² SAMs = Surface to Air Missiles

$$i = 1, 2, \dots, N^X, \quad k = 0, 1, 2, 3, \dots, K$$

Combining all the state variables for each type of forces into one vector, we can write:

$$z^X(k) = \begin{bmatrix} z_1^X(k) \\ \cdot \\ \cdot \\ z_{N^X}^X(k) \end{bmatrix}.$$

The overall state vectors corresponding to the Blue and Red forces are defined as:

$$z^B(k) = \begin{bmatrix} z^{BW}(k) \\ z^{BB}(k) \end{bmatrix}, \text{ and } z^R(k) = \begin{bmatrix} z^{RT}(k) \\ z^{RD}(k) \end{bmatrix}$$

Now, for the fixed targets, we will assume that their fixed

positions are determined by the vectors $\xi_i^{FT} = \begin{bmatrix} x_i^{FT} \\ y_i^{FT} \end{bmatrix}$,

$i = 1, 2, \dots, FT$. Let $p_i^{FT}(k)$ denote the number of platforms in the i^{th} fixed target at time k . These platforms carry no weapons and are subject to attack by the Blue forces. We can define a state vector for the fixed targets as:

$$z^{FT}(k) = \begin{bmatrix} p_1^{FT}(k) \\ \cdot \\ \cdot \\ p_{N^{FT}}^{FT}(k) \end{bmatrix}, \quad k = 0, 1, 2, 3, \dots, K$$

Combining the state vectors for the Blue and Red forces as well as the state vector for the fixed targets, we can define a state vector for the entire operation as:

$$z(k) = \begin{bmatrix} z^B(k) \\ z^R(k) \\ z^{FT}(k) \end{bmatrix}$$

This will be a $4 \times (N^{BW} + N^{BB} + N^{RT} + N^{RD}) + N^{FT}$ dimensional vector.

3. The Command Variables

We will assume that each moving unit has the following command (or control) variables at each time k :

i) Relocate: A unit can decide to relocate (move) to another adjacent point on the grid. The corresponding control command is:

$$r_i^X(k) = \begin{bmatrix} a_i^X(k) \\ b_i^X(k) \end{bmatrix},$$

where $a_i^X(k) \in \{-1, 0, +1\}$ and $b_i^X(k) \in \{-1, 0, +1\}$, and where a corresponds to the move in the x-direction and b corresponds to the move in the y-direction. There are eight neighboring locations that each unit can relocate to.

The $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ option corresponds to the unit deciding to remain in its current location.

ii) Fire Control: Each unit has an option to fire or not to fire. When a unit decides to fire, it must decide on the salvo size. There is a finite set of options for salvo size at each time k . Thus the corresponding control is

$$c_i^X(k) \in \{0, 1, 2, 3, \dots, C_i^X(k)\}$$

where $C_i^X(k)$ is the largest salvo size that can be fired at time k . Note that if a unit decides not to fire, then $c_i^X(k) = 0$

iii) Choice of Target: Each unit can fire only at one target of the opposing forces. If $d_i^X(k)$ denotes the choice of target for unit i at time k , then

$$d_i^{BB}(k) = \{RT_j, RD_j, \text{ or } FT_j \text{ for some } j\}$$

$$d_i^{BW}(k) = \{RT_j, RD_j, \text{ or } FT_j \text{ for some } j\}$$

$$d_i^{RT}(k) = \{BW_j, \text{ or } BB_j \text{ for some } j\}$$

$$d_i^{RD}(k) = \{BW_j, \text{ or } BB_j \text{ for some } j\}$$

Combining all the command variables into one 4-dimensional control vector, we have the following control vector for each unit

$$u_i^X(k) = \begin{bmatrix} a_i^X(k) \\ b_i^X(k) \\ c_i^X(k) \\ d_i^X(k) \end{bmatrix}.$$

We will now define a composite control vector for each type of forces:

$$u^{BW}(k) = \begin{bmatrix} u_1^{BW}(k) \\ u_2^{BW}(k) \\ \vdots \\ u_{N^{BW}}^{BW}(k) \end{bmatrix}, \text{ and } u^{BB}(k) = \begin{bmatrix} u_1^{BB}(k) \\ u_2^{BB}(k) \\ \vdots \\ u_{N^{BB}}^{BB}(k) \end{bmatrix}$$

for the Blue units and

$$u^{RT}(k) = \begin{bmatrix} u_1^{RT}(k) \\ u_2^{RT}(k) \\ \vdots \\ u_{N^{RT}}^{RT}(k) \end{bmatrix}, \text{ and } u^{RD}(k) = \begin{bmatrix} u_1^{RD}(k) \\ u_2^{RD}(k) \\ \vdots \\ u_{N^{RD}}^{RD}(k) \end{bmatrix}$$

for the Red units. The overall control vectors for the Blue and Red forces can be represented as:

$$u^B(k) = \begin{bmatrix} u^{BW}(k) \\ u^{BB}(k) \end{bmatrix}, \text{ and } u^R(k) = \begin{bmatrix} u^{RT}(k) \\ u^{RD}(k) \end{bmatrix}.$$

The dimensionality of these vectors will be $4 \times (N^{BW} + N^{BB})$ and $4 \times (N^{RT} + N^{RD})$ respectively.

4. The Command Constraints

There are numerous constraints that the above command variables must satisfy:

i) Relocate-Fire constraint. For simplicity, we will assume that a unit cannot relocate and fire at the same time.

ii) Fire-Target constraint: We will assume that that no two units of the same force can fire at the same target of the opposing force.

iii) Salvo size constraint: We will assume that ammunitions are not being replenished during the course of the operation.

5. The Highest Level Commands

We will assume that the blue and red forces each have a highest level of commands. Its purpose is to define:

i) The initial states: The initial positions $\xi_i^X(0)$, numbers of platforms $p_i^X(0)$, and weapons $w_i^X(0)$ for each moving unit.

ii) The corridor: Any constraints on the paths of each unit.

The highest level commands may also be able to provide an incentive for the lower level commands to cooperate as a team [5].

6. The State Equations

The state vector for each moving unit is a 4-dimensional vector consisting of the position subvector ξ_i^X , the number of platforms p_i^X , and the number of weapons per platforms w_i^X in that unit. The state vector for each fixed target is p_i^{FT} . We will now derive equations that relate the state variables at time $k+1$ to the state and control variables at time k .

i) The position subvectors for all moving units in $X = \{BB, BW, RT, RD\}$ change according to the equation of motions:

$$\xi_i^X(k+1) = \xi_i^X(k) + r_i^X(k)$$

ii) The number of platforms for the moving units changes according to a nonlinear attrition model. For example, for BW,

$$p_i^{BW}(k+1) = p_i^{BW}(k) \left[1 - \sum_{j=1}^{N^{RT}} Q_{ij}^{BWRT}(k) P_{ij}^{BWRT}(k) \delta(\xi_i^{BW}(k), \xi_j^{RT}(k)) \delta(BW_i, d_j^{RT}(k)) - \sum_{j=1}^{N^{RD}} Q_{ij}^{BWRD}(k) P_{ij}^{BWRD}(k) \delta(\xi_i^{BW}(k), \xi_j^{RD}(k)) \delta(BW_i, d_j^{RD}(k)) \right]$$

For other units the equations are similar. In the above expression the terms $Q_{ij}^{XY}(k)$ and $P_{ij}^{XY}(k)$ represents the engagement factor and attrition factor between the attacking unit (j^{th} unit of Y) and the unit being attacked (i^{th} unit of X). These factors are computed from the expressions:

$$Q_{ij}^{XY}(k) = \beta_{p_{ij}}^{XY} \left(1 - e^{-\frac{p_j^Y(k)}{p_i^X(k)}} \right)$$

$$P_{ij}^{XY}(k) = \left[1 - (1 - \beta_w PK_{ij}^{XY})^{c_j^Y(k)} \right]$$

where $p_i^X(k)$ and $p_j^Y(k)$ are the number of platforms in the i^{th} unit of X and j^{th} unit of Y respectively,

$\beta_{p_{ij}}^{XY}$ represents the probability that platform p_j of Y acquires platform p_i of X as a target, β_w represents the weapon probability to acquire the target, PK_{ij}^{XY} represents the probability of kill for a single weapon (i.e., a salvo size of 1) for the type of weapon used by unit j against the type of platform in unit i , and $c_j^Y(k)$ is the salvo size of the weapons fired by the j^{th} unit of Y at time k . The Kronecker delta, which appears in the above expressions, is defined as

$$\delta(V, W) = \begin{cases} 0 & \text{if } V \neq W \\ 1 & \text{if } V = W \end{cases}$$

In concise form, the four equations for the number of platforms can be written as:

$$p_i^X(k+1) = f_i^X(z(k), u^B(k), u^R(k), k)$$

iii) The number of weapons per platform for each moving unit changes according to the following expression:

$$w_i^{BW}(k+1) = w_i^{BW}(k) - c_i^{BW}(k) \times \left[\sum_{j=1}^{N^{RT}} Q_{ij}^{BWR T}(k) \delta(d_i^{BW}(k), RT_j) + \sum_{j=1}^{N^{RD}} Q_{ij}^{BWR D}(k) \delta(d_i^{BW}(k), RD_j) + \sum_{j=1}^{N^{FT}} Q_{ij}^{BWR FT}(k) \delta(d_i^{BW}(k), FT_j) \right]$$

There are similar equations for the other units. In concise form, these expressions can be written as:

$$w_i^X(k+1) = f_i^X(z(k), u^B(k), u^R(k), k)$$

Combining the state equations for all forces, we get the final expression for the state equation

$$z(k+1) = f(z(k), u^B(k), u^R(k), k)$$

where z is a $4 \times (N^{BS} + N^{BB} + N^{RT} + N^{RD}) + N^{FT}$ dimensional state vector, u^B is an $4 \times (N^{BS} + N^{BB})$ dimensional control vector of the Blue forces and u^R is an $4 \times (N^{RT} + N^{RD})$ dimensional control vector of the Red forces. The function f is a

$4 \times (N^{BS} + N^{BB} + N^{RT} + N^{RD}) + N^{FT}$ vector of functions.

7. Illustrative Example

We consider a scenario where the mission of the Blue forces is to attack and inflict at least 40% damage to the air base that is being defended by Red forces. We consider an attacking group formed of one unit of BW followed by two units of BB . The air base is defended by two RD batteries. In addition, two RT units are located between the base and the Blue attacking group. Let the grid size over which the attack is taking place consist of 10×10 square units, each with 40×40 nautical miles in size. The controls and the states are being updated every 5 minutes and we consider a run of 24 updates corresponding to mission duration of 2 hours. The description of forces is as follows:

Fixed Target (FT): An air base with a total of 10 platforms (command center, runways, hangars, etc..)

- Initial location: $x_i^{FT} = 2, y_i^{FT} = 2$;
- Initial value of the platform state variable: $p_1^{FT}(0) = 10$.

Defending Forces (RED):

Red Defense (RD): Two Fixed SAM batteries consisting of 6 launchers with 3 fixed SAMs each (SAM-F) and one radar.

- Initial Location: $x_1^{RD}(0) = x_2^{RD}(0) = 2, y_1^{RD}(0) = y_2^{RD}(0) = 2$;
- Initial values of platform state variables (launchers + radar): $p_1^{RD}(0) = p_2^{RD}(0) = 7$;
- Initial values of weapon state variables (Average # of SAMs per platform): $w_1^{RD}(0) = w_2^{RD}(0) = 2.57$.

Red Troops (RT): Two mechanized regiments consisting of 50 armored vehicles and equipped with 3 shoulder launched SAMs (SAM-H) per armored vehicle.

- Initial Locations: $x_1^{RT}(0) = 5, y_1^{RT}(0) = 5, x_2^{RT}(0) = 5, y_2^{RT}(0) = 4$
- Initial values of platform state variables (armored vehicles): $p_1^{RT}(0) = p_2^{RT}(0) = 50$.
- Initial values of weapon state variables (Average # of SAMs per platform): $w_1^{RT}(0) = w_2^{RT}(0) = 3$.

Attacking Forces (Blue):

Blue Weasels (BW): Two F2-E fighter planes each equipped with 4 AGM2 (air to ground) missiles.

- Initial Location: $x_1^{BW}(0) = 8, y_1^{BW}(0) = 6$;
- Initial value of platform state variable (F2-E fighters):

$$p_1^{BW}(0) = 2;$$

- Initial value of weapon state variable (Ave. # of missiles) per platform: $w_1^{BW}(0) = 4$.

Blue Bombers (BB): Two units consisting of ten F4 bomber planes each equipped with 4 MK2s (guided bombs).

- Initial Location:
 - $x_1^{BB}(0) = x_2^{BB}(0) = 8$, $y_1^{BB}(0) = y_2^{BB}(0) = 7$;
- Initial values of platform state variables (F4 bombers):
 - $p_1^{BB}(0) = p_2^{BB}(0) = 10$;
- Initial values of weapon state variables (Ave. # of bombs per platform): $w_1^{BB}(0) = w_2^{BB}(0) = 4$.

The initial locations and number of platforms of all units are shown on the grid and bar chart in Figure 1.

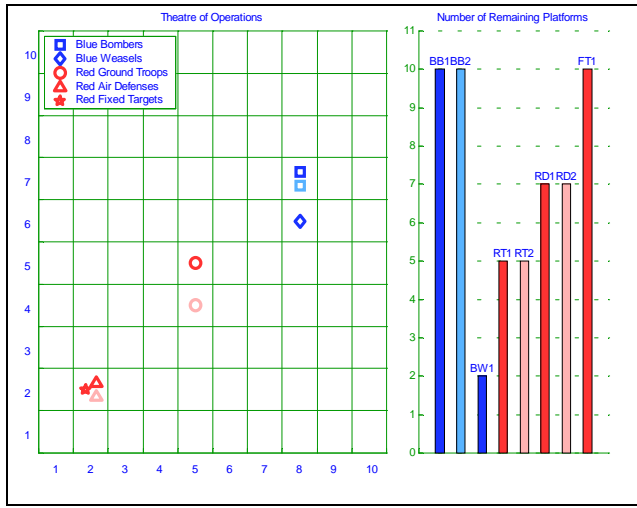


Figure 1: Initial states

We will assume that the probabilities of kill are as follows:

$$PK^{RTBB} = 0.6; PK^{RDBB} = 0.7; PK^{FTBB} = 0.3; PK^{RDBW} = 0.7; PK^{BBRT} = 0.2; PK^{BWRT} = 0.2; PK^{BBRD} = 0.5; PK^{BWRD} = 0.2.$$

For simplicity, we will assume that each force has only one objective function that is common to all units in that force. At time k , the objective function that the Blue Forces are trying to maximize is of the form:

$$J^B(k) = 0.4 \sum_{i=1}^2 \hat{p}_i^{BB}(k) + 0.2 \hat{p}_1^{BW}(k) - 0.1 \sum_{i=1}^2 \hat{p}_i^{RT}(k) - 0.2 \sum_{i=1}^2 \hat{p}_i^{RD}(k) - \hat{p}_1^{FT}(k)$$

Similarly, the objective function for the Red Forces is given as:

$$J^R(k) = -0.7 \sum_{i=1}^2 \hat{p}_i^{BB}(k) - 0.4 \hat{p}_1^{BW}(k) + 0.1 \sum_{i=1}^2 \hat{p}_i^{RT}(k) + 0.5 \sum_{i=1}^2 \hat{p}_i^{RD}(k) + 0.5 \hat{p}_1^{FT}(k)$$

where $\hat{p}_i^{XY}(k) = p_i^{XY}(k) / p_i^{XY}(0)$.

The objective functions over the entire time horizon (K steps) are summations of their corresponding single-step functions:

$$J^B = \sum_{k=1}^K J^B(k); \quad J^R = \sum_{k=1}^K J^R(k).$$

In this extended enterprise, we are dealing with a system with two-controllers (or players) having different (possibly conflicting) objective functions. Hence, we will investigate control strategies adopted from game theory. More specifically, we will compute the Nash equilibrium strategies.

The Nash strategy in a non-zero sum game is an equilibrium solution in the sense that neither player can improve his performance by changing his strategy (i.e. cheating) if the other player sticks to the Nash strategy. Mathematically, it is defined by the two inequalities:

$$J^B(u_N^B, u_N^R) \geq J^B(u^B, u_N^R) \quad \forall u^B \in U^B$$

$$J^R(u_N^B, u_N^R) \geq J^R(u_N^B, u^R) \quad \forall u^R \in U^R$$

where u_N^B and u_N^R are the Nash strategies for Blue and Red respectively, U^B is set of admissible strategies for Blue and U^R is set of admissible strategies for Red.

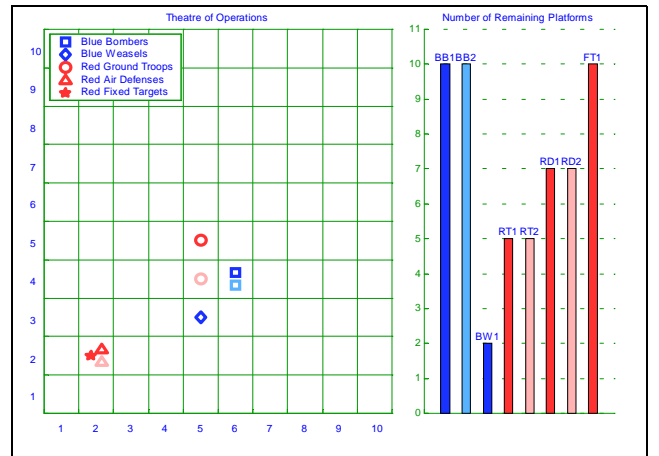


Figure 2: Avoiding RT

For the sake of computational simplicity, instead of calculating the Nash strategies over the entire time horizon we consider a one-step moving horizon optimization. At every time k we will compute a pair of strategies

$(u_N^B(k), u_N^R(k))$ that will satisfy the above two inequalities with $J^B(k+1)$ and $J^R(k+1)$ as performance indices. This is, in effect, a repetitive static two-person non-zero sum game Nash solution.

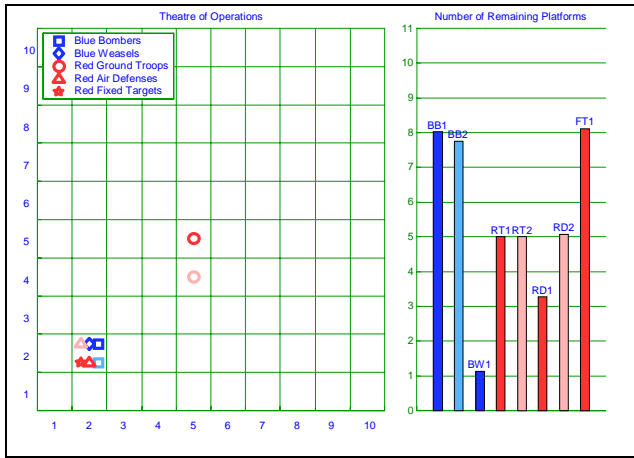


Figure 3: Attrition in the full engagement

In the simulation, the movement of the Blue attacking forces is programmed to be straight towards the air base, avoiding the RT units along the way, as they are not being a part of their mission to damage the air base by at least 40%. A snapshot of the location of all units before engagement is shown in Figure 2. When the Blue Forces approach the air base, the BW unit engages the RD units while attacking the air base. Both sides will incur damage to their platforms. In the next step, all Blue units engage the enemy. They follow the Nash strategy to decide whether to attack or not, and if so, which unit to attack and with what strength. With the set of parameters described earlier, the Nash strategy suggests that both RD units attack the BB units instead of the BW . This is so because a heavier weight is assigned to destroying the BB rather than the BW platforms in J^R . For the same reason the BB units decide to attack the air base instead of the RD units. This is illustrated in the bar chart shown in Figure 3.

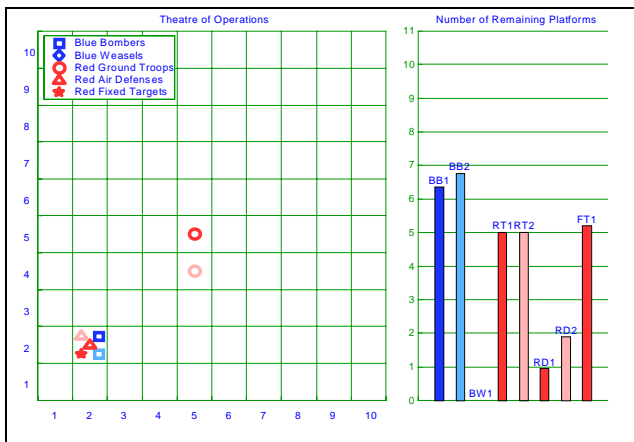


Figure 4: Blue Forces' mission accomplished

After two more steps the air base is destroyed by more than 40% and the Blue Forces decide to return to their base with their mission accomplished. Notice that the RD units sustained considerable damage as illustrated in Figure 4. This is highly dependent on the weights assigned in the objective functions. It may be better for the Blue Forces to assign higher weights to their primary mission target in order to accomplish it faster and with smaller losses.

8. Concluding Remarks

A nonlinear dynamic model for military operations as a basis for a simulation test bed has been developed. The model is an example of an extended enterprise used to investigate the application of Nash strategies. Simulation tests were performed and the results proved that the model is sound and that it can be readily used to investigate the effectiveness of various game theoretic control strategies applied to a complex system in an intelligent hostile environment.

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