

Identification and fault diagnosis of nonlinear dynamic processes using hybrid models

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Abstract

This work addresses a novel approach for fault diagnosis of industrial processes using hybrid models. A nonlinear dynamic process can, in fact, be described as a composition of different affine submodels selected according to the process operating conditions. This paper concerns the identification of hybrid model parameters through input-output data affected by additive noise. The fault detection scheme adopted to generate residuals uses the estimated hybrid model. In order to show the effectiveness of the developed technique, the results obtained in the fault diagnosis of a real industrial plant are reported.

Keywords: multiple models, hybrid systems, nonlinear identification, fault diagnosis, noise rejection.

1 Introduction

There is an increasing interest in the development of model-based fault detection and fault diagnosis methods [1, 2]. The main disadvantage of this class of methods is that, being based on the mathematical model, it can be very sensitive to modelling errors, parameter variations, noise and disturbances, etc. The majority of real industrial processes are nonlinear [3] and cannot be modelled by using a single model for all operating conditions. Instead of exploiting complicated nonlinear models obtained by modelling techniques, it is also possible to approximate the plant by a collection of local affine models [4] obtained by identification procedures.

Residual are signals representing inconsistencies between the model and the actual system being monitored. Any inconsistency will indicate a fault in the system. However, the deviation between the model and the plant is influenced not only by the presence of the fault but also the modelling error.

Several techniques had been proposed for FDI in dy-

namic systems [2]. In particular, in this work, hybrid model [5] identification is combined with the model-based method to formulate a diagnosis technique using the estimated model itself for residual generation. Under such an identification and diagnosis scheme, a number of local affine models are designed and the estimate of outputs is given by a composition of local outputs. The diagnostic signal (residual) is the difference between the estimated and actual system output.

In this paper, the different operating points can be selected by means of clustering method [6]. On the basis of knowledge of the operating point regions, the identification of the structure and the parameters of each local model composing the hybrid system can be performed [7, 8].

2 Model description

The main idea underlying the mathematical description of non-linear dynamic systems is based on the interpretation of single input-single output, non-linear, time-invariant regression models [9, 10] such as:

$$\begin{aligned} y(t+n) &= F(y(t+n-1), \dots, y(t), \\ &u(t+n-1), \dots, u(t)), \quad t = 0, 1, \dots \end{aligned} \quad (1)$$

where $u(\cdot)$ and $y(\cdot)$ belong respectively to the bounded input \mathcal{U} and output \mathcal{Y} sets, n is the finite system memory (i.e. the model order) and $F(\cdot)$ is a continuous non-linear function defining a hypersurface from a \mathcal{A}_n to \mathcal{Y} , being \mathcal{A}_n the Cartesian product $\mathcal{U}^n \times \mathcal{Y}^n$.

The identification of the non-linear system can be translated to the approximation of its mathematical model (1) using a parametric structure, such as an hybrid model, which exhibits arbitrary accuracy interpolation properties.

The hybrid model is formed by a collection of paramet-

ric submodels of the type

$$y(t+n) = \sum_{j=0}^{n-1} \alpha_j^{(i)} y(t+j) + \sum_{j=0}^{n-1} \beta_j^{(i)} u(t+j) + b^{(i)}, \quad t = 0, 1, \dots \quad (2)$$

in which the system operating point is described by the input and output samples $y(t+n-1), \dots, y(t)$ and $u(t+n-1), \dots, u(t)$, that can be collected with a vector $\mathbf{x}_n(t) = [y(t), \dots, y(t+n-1), u(t), \dots, u(t+n-1)]^T$. The *switching* function $\chi_i(\mathbf{x}_n(t))$, $i = 1, \dots, M$ is

$$\chi_i(\mathbf{x}_n(t)) = \begin{cases} \chi_i(\mathbf{x}_n(t)) = 1 & \text{if } \mathbf{x}_n(t) \in \mathcal{A}_n^{(i)} \\ \chi_i(\mathbf{x}_n(t)) = 0 & \text{otherwise} \end{cases} \quad (3)$$

where $\{\mathcal{A}_n^{(1)}, \dots, \mathcal{A}_n^{(M)}\}$ is a partition of \mathcal{A}_n , whose structure will be characterised in the following.

Thus output $y(t+n)$ of the non-linear dynamic system (1) can be approximated by the hybrid piecewise linear model $f(\cdot)$ in the form

$$y(t+n) = f(\mathbf{x}_n(t)) = \sum_{i=1}^M \chi_i(\mathbf{x}_n(t)) [\mathbf{x}_n(t), 1]^T \mathbf{a}_n^{(i)} \quad (4)$$

where the model parameters are collected in the vector $\mathbf{a}_n^{(i)} = [\alpha_0^{(i)}, \dots, \alpha_{n-1}^{(i)}, \beta_0^{(i)}, \dots, \beta_{n-1}^{(i)}, b^{(i)}]^T$. It is worthwhile noting that the model is affine in each $\mathcal{A}_n^{(i)}$, $\mathbf{a}_n^{(i)}$ being the affine submodel parameters.

Since the model (1) is supposed continuous, $f(\cdot)$ is forced to be continuous over the whole \mathcal{A}_n . In such a case the parameter vectors are constrained to satisfy the following relation

$$\bar{\mathbf{x}}_n(t)^T \mathbf{a}_n^{(i')} = \bar{\mathbf{x}}_n(t)^T \mathbf{a}_n^{(i'')}. \quad (5)$$

$\bar{\mathbf{x}}_n$ being an accumulation point for both $\mathcal{A}_n^{(i')}$ and $\mathcal{A}_n^{(i'')}$.

The straightforward application of Equation (5) to all the accumulation points common to neighbouring regions leads to an infinite number of constraints. Yet, in [11, 12] it is shown that the adoption of regions with straight borders guarantees that only a finite number of them is linearly independent. Under these assumptions, the continuity constraints (one for each simplex vertex) can be collected in a finite matrix C_n such that:

$$C_n \mathbf{A}_n = \mathbf{0}. \quad (6)$$

being $\mathbf{A}_n^T = \begin{bmatrix} \mathbf{a}_n^{(1)} & \dots & \mathbf{a}_n^{(M)} \end{bmatrix}$.

3 Dynamic System Identification

Let us assume that the input-output data $u(t)$ and $y(t)$, ($t = 0, 1, \dots, L_i$) generated by a system of the type (2) are available.

Restricting our investigation to find order n and parameters $\mathbf{a}_n^{(i)}$ for local model (2) in region $\mathcal{A}_n^{(i)}$, the following matrix should be defined:

$$X_k^{(i)} = \begin{bmatrix} y(k) & \mathbf{x}_k^T(0) & 1 \\ y(k+1) & \mathbf{x}_k^T(1) & 1 \\ \vdots & \vdots & \vdots \\ y(k+N_i-1) & \mathbf{x}_k^T(N_i-1) & 1 \end{bmatrix} \quad (7)$$

$$\Sigma_k^{(i)} = \left(X_k^{(i)} \right)^T X_k^{(i)}$$

with $k + N_i - 1 \leq L_i$ and N_i large enough.

To determine the model order n in region $\mathcal{A}_n^{(i)}$, it is possible to consider the sequence of increasing-dimension positive definite or positive semidefinite $((2k+2) \times (2k+2))$ matrices $\Sigma_k^{(i)}$ testing their singularity as k increases. As soon as a semidefinite positive matrix $\Sigma_k^{(i)}$ is found then $n = k$, and the parameters $\mathbf{a}_n^{(i)}$ describe the dependence relationship of the first vector of $\Sigma_n^{(i)}$ on the remaining ones as

$$\Sigma_n^{(i)} \mathbf{a}_n^{(i)} = 0 \quad (8)$$

In order to solve the problem in a mathematical framework, it is necessary to characterise the noise affecting the input-output data. Following common assumptions [7, 13, 8], the noises $\tilde{u}(t)$ and $\tilde{y}(t)$ are assumed additive on input-output data $u^*(t)$ and $y^*(t)$ and region independent, so that

$$\begin{cases} u(t) &= u^*(t) + \tilde{u}(t) \\ y(t) &= y^*(t) + \tilde{y}(t). \end{cases} \quad (9)$$

Obviously, only $u(t)$ and $y(t)$ are available for the identification procedure, and moreover every noise term $\tilde{u}(t)$ and $\tilde{y}(t)$ is modelled with a zero-mean white process and is supposed to be independent of every other term. These structures are also commonly known as Error-In-Variables (EIV) models.

Under these assumptions, and $\bar{\sigma}_u$ and $\bar{\sigma}_y$ being the input and output noise variances respectively, the generic positive definite matrix $\Sigma_k^{(i)}$ associated with the input-output noise-corrupted sequences can always be expressed as the sum of two terms $\Sigma_k^{(i)} = \Sigma_k^{*(i)} + \bar{\Sigma}_k$ where

$$\bar{\Sigma}_k = \text{diag}[\bar{\sigma}_y I_{k+1}, \bar{\sigma}_u I_k, 0] \geq 0. \quad (10)$$

Thus, it is again possible to determine the order and parameters of the model in region $\mathcal{A}_n^{(i)}$ from the analysis of the sequence of increasing-dimension $((2k+2) \times (2k+2))$ symmetric positive definite matrices $\Sigma_k^{(i)}$. The solution of the above identification problem requires the computation of the unknown noise covariances $\bar{\sigma}_u$ and $\bar{\sigma}_y$, that can be achieved solving the following relation:

$$\Sigma_k^{*(i)} = \Sigma_k^{(i)} - \tilde{\Sigma}_k \geq 0 \quad (11)$$

in the variables $\bar{\sigma}_u, \bar{\sigma}_y$, where $\tilde{\Sigma}_k = \text{diag}[\bar{\sigma}_y I_{k+1}, \bar{\sigma}_u I_k, 0]$.

Unfortunately the relation (11) admits for any k an infinite solution set describing a curve $\Gamma_k^{(i)}(\bar{\sigma}_y, \bar{\sigma}_u) = 0$ in the first orthant of the noise plane whose concavity faces the origin.

Since determination of the system order requires the increasing values of k to be tested, it is relevant to analyse the behaviour of the associated curves when k varies. As proven in [8], the solution sets of condition (11) for different values of k are non-crossing curves in the noise plane.

It is also important to observe that, since we assume that a system of type (2) has generated the noiseless data, for $k \geq n$ all the curves of type (11) have necessarily at least one common point, i.e. point $(\bar{\sigma}_u, \bar{\sigma}_y)$ corresponding to the true variances of the noise affecting the input and the output data. The search for a solution for the identification problem can be achieved on the basis of the $\Sigma_k^{*(i)}$ matrix properties. With reference to the diagonal non-negative definite matrices $\tilde{\Sigma}_k$, if $k < n$ the matrices $\Sigma_k^{*(i)}$ are positive definite. If $k > n$ the dimension of the null space of $\Sigma_k^{*(i)}$ and consequently, the number of eigenvalues equal to zero is $(k - n + 1)$. For $k = n$, matrix $\Sigma_k^{*(i)}$ is characterised by a linear dependence relation among its $2k + 2$ vectors, and the coefficients which link the first vector of $\Sigma_k^{*(i)}$ to the remaining ones are the parameters $\mathbf{a}_n^{(i)}$, of the system (2) which has generated the noiseless sequences. For $k \geq (n + 1)$, all the $k - n + 1$ linear dependence relations among the vectors of the matrix $\Sigma_k^{*(i)}$ are characterised by the same $2n + 2$ coefficients $\mathbf{a}_n^{(i)}$.

If the noise characteristics are common to all the regions $\mathcal{A}_n^{(i)}$, since the physical nature of the process generating the noise is independent of the model structure and of the partition of \mathcal{A}_n , and all assumptions regarding the Frisch scheme are fulfilled, a common point $(\bar{\sigma}_y, \bar{\sigma}_u)$ in the noise plane exists for the singularity curves.

When the order n has been determined, the parameters $\mathbf{a}_n^{(i)}, i = 1, \dots, M$ can be identified solving the following

equation

$$(\Sigma_n^{(i)} - \tilde{\Sigma}_n) \mathbf{a}_n^{(i)} = \mathbf{0} \quad \text{for } i = 1, \dots, M. \quad (12)$$

The previous result can be fully applied when the assumptions behind the Frisch scheme are satisfied (independence between input-output sequences, additive noise, noise whiteness).

In real applications, we are forced to relax these assumptions, thus no common point can be determined among curves $\Gamma_n^{(i)} = 0$ in the noise plane. A criterion to select a different noisy point for each region as best approximation of the ideal case has to be introduced.

With reference to the identification of the system order n in the i -th region $\mathcal{A}_n^{(i)}$, it must be noted that the $\Gamma_{n+1}^{(i)} = 0$ curve has a single point in common with the $\Gamma_n^{(i)} = 0$ curve in ideal conditions, which corresponds to a double singularity of the matrix $\Sigma_{n+1}^{*(i)}$.

In real cases, the order n can be computed finding the point $(\bar{\sigma}_u, \bar{\sigma}_y) \in \Gamma_{n+1}^{(i)} = 0$ that makes $\Sigma_{n+1}^{*(i)}$ closer to the double singular condition (i.e. minimal eigenvalue equal to zero and the second minimum eigenvalue near to zero). As n is unknown, increasing system orders k must be tested, and the value of k associated to the minimum of the second eigenvalue of the matrix $\Sigma_{k+1}^{*(i)}$ corresponds to the order n .

Once the model order n is selected, the parameters $\mathbf{a}_n^{(i)}, i = 1, \dots, M$ cannot be computed from (12), because the curves $\Gamma_n^{(i)} = 0$ do not share the common point $(\bar{\sigma}_u, \bar{\sigma}_y)$. In this case, for each region a different noise $(\bar{\sigma}_u^{(i)}, \bar{\sigma}_y^{(i)})$ must be considered and relation (11) should be rewritten as

$$\Sigma_n^{*(i)} = \Sigma_n^{(i)} - \tilde{\Sigma}_n^{(i)} \geq 0 \quad (13)$$

where $\tilde{\Sigma}_n^{(i)} = \text{diag}[\bar{\sigma}_u^{(i)} I_{n+1}, \bar{\sigma}_y^{(i)} I_n, 0]$. The values $(\bar{\sigma}_u^{(i)}, \bar{\sigma}_y^{(i)})$ can be computed by solving an optimisation problem which minimises both the distances between $(\bar{\sigma}_u^{(i)}, \bar{\sigma}_y^{(i)})$ and $(\bar{\sigma}_u^{(j)}, \bar{\sigma}_y^{(j)})$ with $i \neq j$ and the continuity constraints proposed in (6)

$$\begin{aligned} J((\bar{\sigma}_u^{(1)}, \bar{\sigma}_y^{(1)}), \dots, (\bar{\sigma}_u^{(M)}, \bar{\sigma}_y^{(M)})) = \\ d \left((\bar{\sigma}_u^{(1)}, \bar{\sigma}_y^{(1)}), \dots, (\bar{\sigma}_u^{(M)}, \bar{\sigma}_y^{(M)}) \right) + \\ + (C_n A_n)^T H C_n A_n \end{aligned} \quad (14)$$

H being a definite positive weighting matrix and d a

distance defined as

$$d\left((\bar{\sigma}_u^{(1)}, \bar{\sigma}_y^{(1)}), \dots, (\bar{\sigma}_u^{(M)}, \bar{\sigma}_y^{(M)})\right) = \sum_{i=1}^M \sum_{j=i+1}^M (\bar{\sigma}_u^{(i)} - \bar{\sigma}_u^{(j)})^2 + (\bar{\sigma}_y^{(i)} - \bar{\sigma}_y^{(j)})^2. \quad (15)$$

It's worthwhile observing that the matrix A_n collects the parameters $\mathbf{a}_n^{(i)}, i = 1, \dots, M$ which depend on $(\bar{\sigma}_u^{(i)}, \bar{\sigma}_y^{(i)})$. Minimisation of cost function (14) can be computationally difficult, as it depends on $2M$ independent variables. Therefore, in order to decrease the complexity of the problem, a common parametrisation can be defined for all the curves. In such a way, the cost function has the form

$$J(q) = d\left((\bar{\sigma}_u^{(1)}(q), \bar{\sigma}_y^{(1)}(q)), \dots, (\bar{\sigma}_u^{(M)}(q), \bar{\sigma}_y^{(M)}(q))\right) + (C_n A_n)^T H C_n A_n. \quad (16)$$

The parametrisation chosen to simplify the minimisation problem leads to consistent results. In fact, when the data are generated by a continuous piecewise-affine dynamic system, all assumptions regarding the Frisch scheme being fulfilled and noise being region-independent, the curves $\Gamma_n^{(i)} = 0$ share a common point in the noise plane. In these conditions, cost function $J(q) = 0$ and the variances $(\bar{\sigma}_u, \bar{\sigma}_y)$ are identified exactly.

Finally, one should note how once the parameter q minimising the cost function (16) is computed, the matrices $\tilde{\Sigma}_n^{(i)}$ can be built and the model parameter $\mathbf{a}_n^{(i)}, i = 1, \dots, M$ determined by means of relation

$$(\Sigma_n^{(i)} - \tilde{\Sigma}_n^{(i)})\mathbf{a}_n^{(i)} = \mathbf{0} \quad \text{for } i = 1, \dots, M. \quad (17)$$

This completes the multiple model identification procedure.

4 Fault Diagnosis of Dynamic Systems

The problem treated in this section regards the detection and isolation of the output sensor faults on the basis of the knowledge of the measured noisy sequences $u(t)$ and $y(t)$.

In the following it is assumed that the monitored system can be described by a model of the type (4). $y(t) \in \mathbb{R}^m$ is the system output vector and $u(t) \in \mathbb{R}^r$ the input vector.

In real applications variables $u^*(t)$ and $y^*(t)$ are measured by means of sensors whose outputs are affected by noise (see relations (9)) and faults.

In the first stage of this work, neglecting sensor dynamics, faults on the measured output signals $y(t)$ are considered and they can be modelled as

$$y(t) = y^*(t) + \hat{y}(t) + f_y(t) \quad (18)$$

in which, the vector $f_y(t) = [f_{y_1}(t) \dots f_{y_m}(t)]^T$ is composed of additive signals assuming values different from zero only in the presence of faults. Usually these signals are described by step and ramp functions representing, respectively, abrupt and incipient faults (bias or drift).

As depicted in Figure (1), residuals can be generated by the comparison of measured $y(t)$ and estimated $\hat{y}(t)$ outputs, $r(t) = \hat{y}(t) - y(t)$, where $\hat{y}(t)$ is the estimate generated by the identified model (4) of the process under investigation.

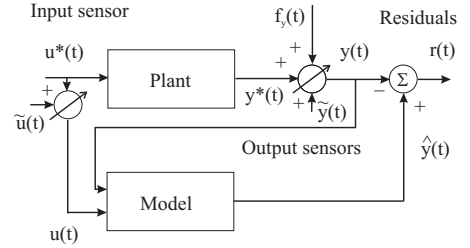


Figure 1: Residual generation.

Therefore, faults can be detected by using a simple thresholding logic.

5 Identification and Fault Diagnosis of the Plant

The proposed methodology was applied to the identification and fault diagnosis of an industrial fermenter which contains 25l of water.

The structure of the plant is shown in Figure (2). At the bottom of the fermenter, air is fed into the water at a specified flow rate which is kept at a desired value by a local mass-flow controller. The air pressure in the head space can be controlled by the position of an outlet valve at the top of the fermenter.

This process has two inputs: the position of the outlet valve, denoted by $u_1(t)$ and the inlet air flow rate, denoted by $u_2(t)$, and one output: the pressure in the head space, denoted by $y(t)$. The inlet flow rate can be kept constant, in which case the process is a single-input, single-output (SISO) system. Because the underlying physical mechanism and because of the nonlinear characteristics of the outlet valve, the process has a strongly nonlinear behaviour, both static and dynamic.

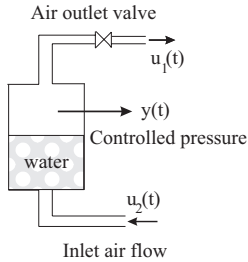


Figure 2: Fermentation tank.

By keeping the air flow rate constant ($u_2(t) = 22.5$ l/s), the available data from the input $u_1(t)$ ($r = 1$) were 482 samples. The data from the output sensor ($m = 1$) were the corresponding values of pressure $y(t)$. The sampling time was of $T = 5$ s.

In order to determine process working points, a clustering algorithm was exploited with $M = 5$ regions (operating conditions) and $n = 1$ the number of shifts of input and output. After clustering, the system parameters $\mathbf{a}_n^{(i)}$, with $i = 1, \dots, M$ for each output, were estimated using the scheme identification method proposed in Section (3).

In fault-free conditions, the mean square value (MSE) of the output estimation error $r(t)$ given by using the prediction $\hat{y}(t)$ of a single model for all operating conditions [14, 15] is $\text{MSE} = 37$. This value is large and it cannot be used to detect faults reliably.

The residual $r(t)$ generated a single dynamic model ($M = 1$) is depicted in Figure (3(a)). One can note the peaks due to the changes in the working conditions of the process. A single model cannot describe the behaviour of the plant.

A meaningful improvement has been obtained by using the identification technique presented in this paper where the process is described as a hybrid system collecting affine submodels obtained using the Frisch scheme method. The output $y(t)$ of the plant can be estimated by means of a multiple model (4) with $r = 1$, $m = 1$ and $n = 1$. The corresponding mean square value of the output estimation error $r(t)$, under no-fault conditions, is $\text{MSE} = 0.017$. The residual is shown in Figure (3(b)).

One can note how the hybrid model approximates the real process very accurately.

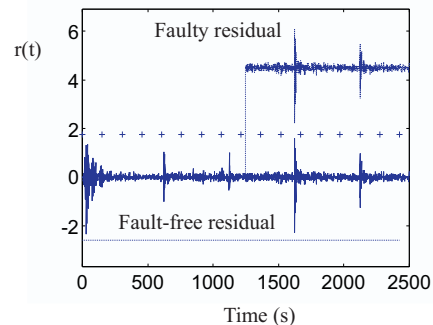
According to relation (18), faults in the output sensor were simulated by adding variations (step functions of different amplitudes) in the output measurement [16, 17, 15].

The fault occurring on the single output sensor causes

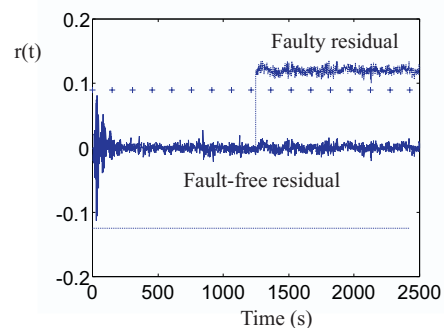
alteration of the sensor signal $y(t)$ and of the residual $r(t)$ given by the predictive model (4) using $u(t)$ and $y(t)$ as inputs. Residual indicates fault occurrence whether its value is lower or higher than the thresholds fixed in fault-free conditions.

In order to determine the thresholds above which the faults are detectable, the simulation of different amplitude faults in the sensor signal was performed. The threshold value depends on the residual error amount due to the model approximation. These thresholds were settled on the basis of fault-free residuals. A margin of 10% between the thresholds and the residual values was imposed.

To summarise the performance of the FDI technique, residuals generated using a single auto-regressive model and the identified hybrid system are reported in Figures (3(a)) and (3(b)), respectively. Fault-free thresholds were marked by using '-' and '+'.



(a) Single model faulty residual



(b) Hybrid model faulty residual

Figure 3: (a) single model and (b) multiple model faulty residuals $r(t)$

Using a single model, (Figure (3(a))) the minimal detectable faults on the sensor is 30%, expressed as per cent of the mean values of the relative signal. This value is relative to the case in which the fault must be

detected as soon as it occurs. The results were obtained by using a single model for all operating conditions.

An improvement of the FDI performance has been obtained by using the hybrid multiple system (Figure 3(b)). Model parameters were identified under the assumptions of the Frisch Scheme. The minimal detectable fault is 1.5%. The residual obtained by using multiple model approach is more sensitive to a fault occurring on the output sensor, since the corresponding output estimation error is smaller. Noise rejection is, in fact, achieved by means of the dynamic Frisch scheme identification method. Moreover, smaller thresholds can be placed on the residual signals to declare the occurrence of faults.

6 Conclusions

In this paper an off-line procedure was proposed for the identification and fault diagnosis of a dynamic system using a hybrid model identified from noisy input-output measurements. An hybrid model consists of several local affine models each for different operating point of the process. The identification algorithm requires the determination of the regions in which measured data can be approximated by affine dynamic submodels. Parameters and structure of submodels were estimated using a technique based on the rules of the Frisch scheme, traditionally exploited to analyse economic systems. This identification approach gives a reliable model of the plant under investigation which can be exploited to generate redundant residuals for fault diagnosis. The diagnosis system presented uses geometrical analysis of residuals for fault detection and isolation. The effectiveness of these procedures were tested on real data acquired from a dynamic nonlinear process.

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